nary spontaneous decay, with all five types of the coupling in each case.

For very slow mesons w' vanishes in the case of charge retention with scalar or pseudovector coupling in the approximation $m=0$. The optimum numerical value of R', for slow mesons, is provided by the theory of charge retention with pseudoscalar coupling and turns out to be (with $m=0$):

$$R'=192\pi^2(\hbar/\mu c)^3 N.$$

Here N is the number of electrons per cm^3 of the medium. For $N\sim 10^{24}/\text{cm}^3$ the ratio R' is only of the order of magnitude 10^{-11}. Neither can the preparatory formation of the mesotronium yield significantly larger values for R'. The absence of the positron track at the end of some μ^+-meson tracks must therefore be accounted for in another way.[5]

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[2] J. Tiomno and J. A. Wheeler, Revs. Modern Phys. **21**, 144 (1949).
[3] This second event might also be observable (in spite of having presumably smaller probability than the first one) because it may be technically easier to detect in the photographic emulsion a splitting of the μ-meson track into two electron tracks than to find an unusually fast decay electron.
[4] The value w_0 given by Ogawa and Kamefuchi, reference 1, is too large by a factor of 2.
[5] C. Franzinetti, Phil. Mag. **41**, 86 (1950); see especially p. 101. Further, C. F. Powell, *Cosmic Radiation* (Colston Papers, 1949), Vol. I, p. 85.

A Note on Time Reversal and the Dirac Equation*

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(Received December 18, 1950)

IT has been known for some time that in the customary formulation of the one-particle Dirac theory the bilinear form $\psi^\dagger\psi(\psi^\dagger\equiv i\gamma^4\psi^*)$ is invariant under continuous Lorentz transformations and space inversion but not under time reflection.[1] For a properly quantized field theory, however, $\psi^\dagger\psi$ can be made invariant to time reflection.[2] This behavior causes no difficulty in the one electron theory [it is necessary, in fact, in order that the normalization integral $\int(\psi^\dagger\gamma^4\psi)dx^1dx^2dx^3$ be a scalar under the extended group]. It might be of consequence, however, for interaction with other spin $\frac{1}{2}$ particles (e.g., β-decay).

If the invariance of $\psi^\dagger\psi$ to the extended group is postulated in addition to the usual postulates of the Dirac theory, some generalization must be made. A possible generalization is to consider equations with all real elements in which ψ is an 8-rowed column vector, and the matrices 8×8. Motivation for this is the observation that the nonlinear operation[3] of complex conjugation, K_0, is adjointed to the Dirac matrix operations and plays an intimate role in the usual theory. By adjoining to the 4×4 Dirac matrices (multiplied by i if imaginary) the four matrices

$$I=\begin{pmatrix}1&0\\0&1\end{pmatrix},\quad K_0=\begin{pmatrix}1&0\\0&-1\end{pmatrix},\quad K_1=\begin{pmatrix}0&-1\\1&0\end{pmatrix},\quad K_2=\begin{pmatrix}0&1\\1&0\end{pmatrix},$$

we linearize all matrix operations. The algebra generated by this process has 64 elements, is irreducible, and is quite similar to the usual Dirac algebra. It is generated by the commutation relation:

$$\Gamma^\mu\Gamma^\nu+\Gamma^\nu\Gamma^\mu=2g^{\mu\nu}(\mu,\,\nu=0\cdots 5;\quad g^{\mu\nu}=\pm\delta^{\mu\nu},\\ -\text{ for }\mu=0,4,5;\quad +\text{ for }\mu=1,2,3).$$

It is found possible to maintain all the postulates leading to the Dirac theory plus the additional requirement of the invariance of $\psi^\dagger\psi$. The wave equation is

$$(\Gamma^\lambda\partial_\lambda+\kappa)\psi=0(\lambda=0\cdots 3;\quad \partial_\lambda=\partial/\partial x^\lambda;\quad x^0=t;\quad \hbar=c=1);$$

the adjoint equation is $\psi^\dagger(\Gamma^\lambda\partial_\lambda-\kappa)=0$ with the adjoint uniquely determined to be $\psi^\dagger=\text{transpose }(\Gamma^0\Gamma^4\Gamma^5\psi)$. To every wave function ψ there correspond three "conjugate" functions generated by the operators $\Gamma^6\Gamma^4$, $\Gamma^6\Gamma^5$, and $\Gamma^4\Gamma^5(\Gamma^6\equiv\Gamma^0\Gamma^1\Gamma^2\Gamma^3\Gamma^4\Gamma^5)$. These conjugates are analogous to the usual charge conjugate solution; interactions with external fields distinguish among them. Three anti-commuting operators are available for interactions $\Gamma^4\Gamma^5$, Γ^4, Γ^5. The proper operator for vector interactions is $\Gamma^4\Gamma^5$, and with this we get gauge invariance and a continuity equation. Gauge invariance (of the first kind) is lost, however, if the operators Γ^4 and/or Γ^5 appear in the wave equation with nonzero mass. The spin of the particles is readily shown to be $\frac{1}{2}$.

The time reversal operator in this formulation is $\Gamma^0\Gamma^4$ (or $\Gamma^0\Gamma^5$ equivalently). In the representation where $\Gamma^4\Gamma^5$ corresponds to i (the usual Dirac formalism), the time reversal operator becomes $i\sigma_yK_0$.[3-5] The reversal in sign of $\psi^\dagger\psi$ under time reflection in the Dirac theory is therefore due to the incorrect identification of $\gamma^1\gamma^2\gamma^3$ as the time reversal operator.

Because transpose $(\psi^\dagger O\psi)=\psi^\dagger O\psi$, the expectation value of many operators is identically zero. In particular, we may have only the S, A, P covariants[5] with I (the identity); V, T with $\Gamma^4\Gamma^5$; and T, A with Γ^4 and Γ^5. As a result, the only nonvanishing vector is $j^\mu=\psi^\dagger\Gamma^4\Gamma^5\Gamma^\mu\psi(\mu=0\cdots 3)$. Since $\Gamma^4\Gamma^5$ is a scalar to all Lorentz transformations except time reversal, where it changes sign, the proper behavior for j^0 is obtained. Considering $\Gamma^4\Gamma^5$ as the charge operator, one is led to the conclusion that changing sign under time reversal is a basic property of charge.

Besides the Dirac equation the proposed generalization includes other equations describing spin $\frac{1}{2}$ particles, the significance of which is yet to be explored.

The writer wishes to thank Dr. M. E. Rose and Professor E. P. Wigner for helpful discussions.

* This document is based on work performed for the Atomic Energy Project at Oak Ridge.
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Neutron Yield for the Li+T Reactions*

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(Received January 17, 1951)

A PRELIMINARY investigation of neutron production from bombardment of natural metallic lithium by tritons with energies between 0.25 and 2.10 Mev has been made using the Los Alamos 2.5-Mev electrostatic accelerator.

Targets were made by evaporating lithium on thin aluminum or tungsten disks which could be attached to the target end of the accelerating tube. Thicknesses of the lithium films were obtained by measuring the neutron yields from $Li^7(p,n)Be^7$ for 2-Mev protons and using a value[1] of 0.025 barn for the differential cross section at 0°. Targets were about $\sim$50 kev thick for 2-Mev protons.

Neutron yields were measured with a BF_3 "long counter"[2] placed about one meter from the target and at angles of 0° to 135°

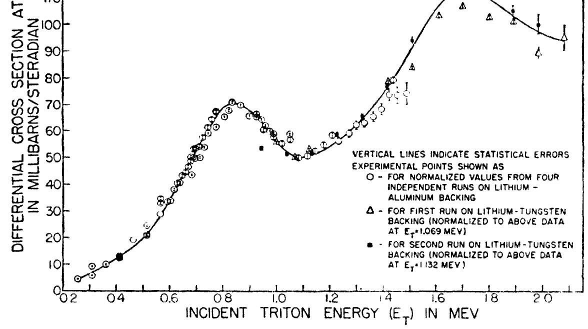

FIG. 1. Neutron yield at 0° in the laboratory system, expressed as a cross section, for the Li+T reactions.

with respect to the triton beam. The efficiency of the counter was obtained by measuring the yield from a standard RaBe source placed at the target. Errors up to 20 percent in total yield measurements may arise from a difference in sensitivity of the counter to neutrons from Li+T and from the RaBe source. The neutron energy-spectrum from Li+T is not known.

Data were obtained in a series of eight runs using beams of mass 6 (TT^+), mass 4 (HT^+), and mass 3 (principally T^+), which covered a triton energy range from 0.258 to 2.080 Mev. One of each of the following targets was used: lithium on aluminum, lithium on tungsten, aluminum blank, tungsten blank.

Background measurements were obtained from runs on the metal blanks. For triton energies below 1.0 Mev, backgrounds were negligible. Above 1.5 Mev, tritons on aluminum produced backgrounds which were twice as large as those for tritons on tungsten. At angles greater than 90°, backgrounds were at least twice as large as those at 0°.

For the machine energies available, probable neutron-producing reactions are:

$$Li^6+T\rightarrow 2He^4+n+16.02 \text{ Mev}, \tag{1}$$
$$Li^6+T\rightarrow Be^8+n+15.97 \text{ Mev}, \tag{2}$$
$$Li^7+T\rightarrow Be^9+n+10.52 \text{ Mev}, \tag{3}$$
$$Li^7+T\rightarrow 2He^4+2n+8.88 \text{ Mev}, \tag{4}$$
$$Li^7+T\rightarrow Be^8+2n+8.83 \text{ Mev, and} \tag{5}$$
$$Li^7+T\rightarrow He^5+He^4+n+8.08 \text{ Mev}. \tag{6}$$

The compound nucleus from Li^7+T is Be^{10} with an excitation energy of 17.324 Mev. Contributions from the 7.4 percent Li^6 present in the targets are assumed to be small.

The neutron yield at 0°, expressed as a cross section, is shown in Fig. 1. Apparent resonances occur at triton energies of 0.84 and 1.70 Mev. One may assume these peaks to indicate virtual states in Be^{10} of 17.91 Mev and 18.51 Mev.

Figure 2 shows two angular distributions of neutron yields, expressed in terms of differential cross sections. Numerical integration of the data at 0.728 Mev gives a total cross section of

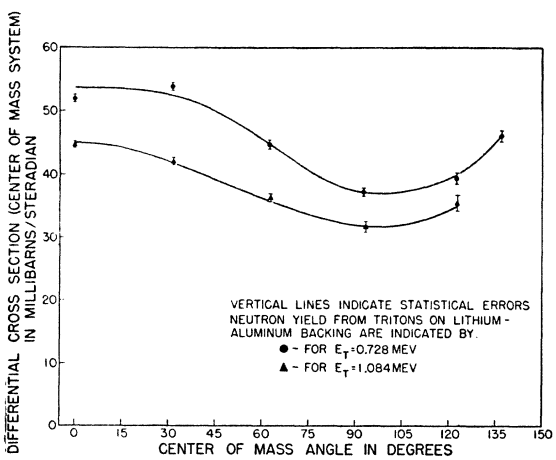

FIG. 2. Angular distribution of neutron yields in the center-of-mass system, expressed as cross section values, for the Li+T reactions.

approximately 0.5 barn. An average total cross section ~0.7 barn is obtained for a triton energy range of 0.25 to 2.0 Mev. Almqvist[3] reports an average total cross section of 1.5 barns for a thick target and a maximum triton energy of 2.6 Mev.

The author is indebted to the other members of the Los Alamos electrostatic accelerator group for their assistance during the experiment.

* Work done under the auspices of the AEC.

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On the Spin of the Nuclear Ground State in the *jj*-Coupling Scheme

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RECENTLY, some interest has arisen in the problem concerning the spin of the ground state of the j^n configuration predicted for Majorana forces in the *jj*-coupling scheme.

Racah[1] showed, on the basis of the "long range" approximation, in which one assumes that the potential between a pair of nucleons can be approximated by a square well in the region where the amplitudes of the wave functions are important, that the ground state has the minimum value of spin J allowed by the Pauli principle. On the other hand, results of Mayer[2] show that in the case of δ-type interaction between the nucleons, the spin of the ground state is j if n is odd and 0 if n is even. This case should be referred to as the "short range" approximation, because here one considers an interaction potential which differs from zero only in a region so small that the change of the wave function in it is small compared to the wave function itself.

In order to see how these results depend on the approximations used, it is useful to develop the interaction potential in a series of Legendre polynomials.[3]

$$V(|r_1-r_2|)=\sum_{k=0}^{\infty} f_k(r_1, r_2)P_k(\cos\omega).$$

The energy levels are then given as finite sums of the radial integrals

$$F^k=\int_0^\infty\int_0^\infty R_l^2(r_1)R_l^2(r_2)f_k(r_1, r_2)dr_1dr_2,$$

the coefficients of which are integrals of products of spherical harmonics. The term values of the Majorana interaction for the levels in the *jj*-coupling are easily found by the method of sums in a way similar to that used for ordinary forces,[4] and the relative order can be determined as a function of the parameters F^k.

It is easy to see that in the "long range" limit

$$f_k(r_1, r_2)=\frac{2k+1}{2}\int_{-1}^{+1} V(|r_1-r_2|)P_k(\cos\omega)d\cos\omega$$
$$=\frac{2k+1}{2}V\int_{-1}^{+1} P_k(\cos\omega)d\cos\omega,$$

where V is the (fixed) depth of the potential well; therefore, only F^0 does not vanish. This approximation is accomplished by setting

$$F^k \ll F^0 \quad k\geqslant 1.$$

In the approximation of Mayer we simply have the expansion of the δ-function in a series of orthogonal functions

$$\delta(\cos\omega-1)=\sum_{k=0}^{\infty}\Theta_{k0}(\cos\omega)\Theta_{k0}(1)=\tfrac{1}{2}\sum_{k=0}^{\infty}(2k+1)P_k(\cos\omega),$$

and the limit is reached by putting

$$F^k=(2k+1)F^0.$$

The simplest configuration in which the two approximations give different order of levels is $(d_{5/2})^3$, where we obtain

$$E_{5/2}=4/5F^0-(56/245)F^2-(217/735)F^4,$$
$$E_{3/2}=3/5F^0+(12/245)F^2-(209/735)F^4.$$

Assuming a special form of potential and of wave functions, one is able to calculate the F^k. Kurath[5] took a gaussian potential and harmonic oscillator wave functions without nodes and obtained the result that a "cross-over" of these levels occurred at a value of r_l, which lies in a region of physical interest. The gaussian potential yields the two extreme limits: if r_0 tends to infinity, the potential approximates a square well; and if r_0 tends to zero and V is appropriately increased (for the well to have a fixed volume), the δ-potential is obtained.