Determination of the Number of Neutrons Associated with the Stoppage of Negative μ-Mesons in Lead*

GERHART GROETZINGER,† MARTIN J. BERGER, AND GORDON W. MCCLURE‡
Institute for Nuclear Studies, University of Chicago, Chicago, Illinois

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Negative mesons (at sea level and therefore assumed to be μ-mesons) were focused onto a lead absorber by means of a magnetic cosmic-ray spectrometer. The number of μ-mesons which stopped in the absorber was determined with an anticoincidence arrangement. Neutrons issuing from the lead absorber were registered during intervals of from 5 to 135 μsec after each such stoppage by a counting system consisting of enriched-BF_3 counters surrounded by 8 cm of paraffin, whose efficiency was measured with the help of a calibrated Na-Be source of $\approx$1-Mev photo-neutrons.

For 5500 stopped μ-mesons, the average number of neutrons emitted per μ-meson stoppage was found to be 1.96 ± 0.72. This result is consistent with the assumption that a neutrino (or light neutral meson) is emitted in the interaction between a μ-meson and a nucleon.

I. INTRODUCTION

IN a previous paper[1] we described an experiment, carried out at sea level, in which the emission of neutrons following the capture of negative μ-mesons in lead was found. The capture of mesons as the initiating event for neutron emission was established through the use of a G-M counter spectrometer capable of discriminating against all particles except negative mesons. The number of neutrons emitted per captured meson was given as roughly two. Shortly before the publication of this paper Sard *et al.*[2] had reported the emission of neutrons following the stoppage of charged penetrating radiation for which the negative meson component was held to be responsible. In a later paper[3] the number of neutrons per captured μ-meson was stated to be most likely two to three, and certainly smaller than five.

The purpose of the present experiment was to determine the neutron multiplicity with greater accuracy than in our preliminary experiment, using the same cosmic-ray spectrometer, but an improved arrangement for counting neutrons. The hard component of the cosmic radiation at sea level contains a considerable number of slow protons;[4,5] furthermore, locally produced penetrating showers are accompanied by many neutrons.[6] Even after the phenomenon of neutron emission subsequent to μ-meson capture had been established, it was therefore essential to continue the use of the spectrometer in order to obtain the correct number of stopped mesons and the neutrons associated with them.

The number of neutrons emitted per μ-meson capture provides significant information concerning the interaction between μ-mesons and nucleons. Marshak and Bethe[7] proposed the interaction

$$(\mu^{-}\text{meson})+(\text{proton})=(\text{neutron}),$$

while Sakata[8] proposed the interaction

$$(\mu^{-}\text{meson})+(\text{proton})=(\text{neutron})+(\text{neutrino}).$$

It follows from the conservation of energy and momentum that if a neutrino is emitted, it will carry away most of the energy made available by the disappearance of the rest mass of the μ-meson (108 Mev). Tiomno and Wheeler,[9] and Rosenbluth[10] showed, on the basis of the free particle model of the nucleus, that the most probable excitation energy given to the nucleus in this case will be $\sim$15 Mev, and the maximum energy $\sim$22 Mev. It was pointed out further[9] that if a light neutral meson were emitted instead of a neutrino, or if the binding of the nucleons were taken into account, the energy given to the nucleus would be somewhat, but not much, smaller. Under the most favorable assumptions the emission of an average number of between one and two neutrons could be expected. Taketani and Sasaki,[11] on the other hand, come to the conclusion that the nuclear excitation energy would be 6 Mev for large Z and approximately 14 Mev for small Z.

If no neutrino is emitted, enough energy is available for the emission of a larger number of neutrons. A small number of fast nucleons will be emitted within a very short time ($\approx10^{-21}$ sec) after the absorption of the μ-meson, taking with them a considerable fraction of the energy. The residual energy is distributed among the remaining nucleons, some of which will eventually evaporate. Table I gives the approximate numbers of neutrons expected to evaporate from a lead nucleus

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† Now at the Lewis Laboratory, NACA, Cleveland, Ohio.
‡ Now at the Bartol Research Foundation, Swarthmore, Pennsylvania.
[1] G. Groetzinger and G. W. McClure, Phys. Rev. **74**, 341 (1948). G. W. McClure and G. Groetzinger, Phys. Rev. **75**, 340 (1949).
[2] Sard, Ittner, Conforto, and Crouch, Phys. Rev. **74**, 97 (1948).
[3] Sard, Conforto, and Crouch, Phys. Rev. **76**, 1134 (1949).
[4] Merkle, Goldwasser, and Brode, Phys. Rev. **78**, 92 (1950).
[5] G. W. McClure, dissertation (University of Chicago), unpublished.
[6] Cocconi, Cocconi-Tongiorgi, and Greisen, Phys. Rev. **74**, 1867 (1948).
[7] R. E. Marshak and H. A. Bethe, Phys. Rev. **72**, 508 (1947).
[8] S. Sakata and T. Inoue, Prog. Theor. Phys. **1**, 143 (1946). Taketani, Nakamura, Ono, and Sasaki, Phys. Rev. **76**, 60 (1949).
[9] J. Tiomno and J. A. Wheeler, Revs. Modern Phys. **21**, 153 (1949).
[10] M. N. Rosenbluth, Phys. Rev. **75**, 532 (1949).
[11] M. Taketani and M. Sasaki, Phys. Rev. **76**, 852 (1949).

TABLE I. Number and energy of evaporated neutrons as a function of the excitation energy.

Residual excitation (Mev)	% of the meson mass	Number of neutrons	Neutron energies (Mev)
32.4	30	3	2.7–1.6
27	25	2	2.5–1.6
21.6	20	2	2.2–1.6
16.2	15	1	1.9
13.5	12.5	1	1.8

for several moderate excitation energies. In making this estimate, it was assumed that because of the large coulomb barrier no charged particles are emitted. The relation between excitation energy U and temperature T was taken to be

$$U = T^2 A/12,$$

where A is the atomic number of the nucleus. The numerical factor 12 was found by Le Couteur[12] to be consistent with the characteristics of evaporation stars (silver and bromine) as observed in photographic emulsions.

On the basis of the Marshak-Bethe type of interaction one would expect charged particles to be evaporated at least from light and medium nuclei after the capture of a μ-meson. But according to many authors[13] μ-mesons do not produce stars in photographic emulsion, or if so, only in a small fraction of the cases [(10±10) percent].[14] Experiments with a cloud chamber,[15] a crystal counter,[16] and an ionization chamber[5] have shown that the emission of charged particles subsequent to the capture of μ-mesons in lead, silver chloride, and argon, respectively is at best a very rare event.

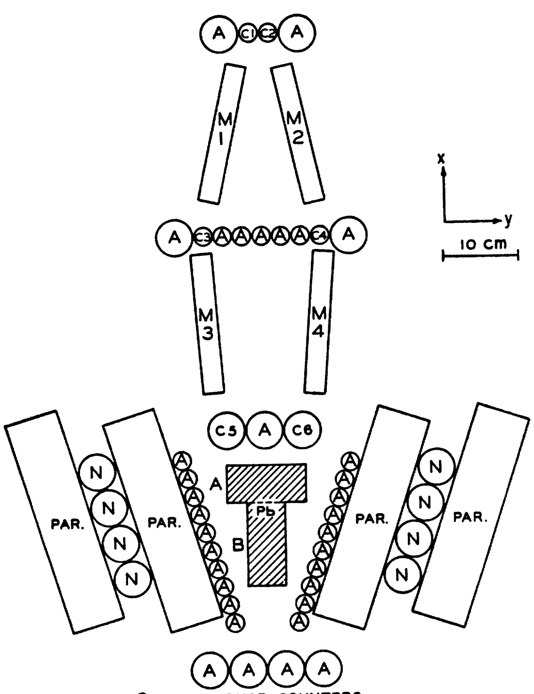

FIG. 1. Schematic diagram of the apparatus.

[12] K. J. Le Couteur, Proc. Phys. Soc. (London) **63A**, 259 (1950).
[13] D. H. Perkins, Phil. Mag. **40**, 601 (1949).

II. EXPERIMENTAL PROCEDURE

(A) Description of the Equipment

A schematic diagram of the apparatus is shown in Fig. 1. The upper part consists of a magnetic double spectrometer able to select mesons of either charge. The two top counters C_1 and C_2 are connected in parallel and form one spectrometer in coincidence with the counters C_3 and C_5, and a second spectrometer in coincidence with C_4 and C_6. The selection of the mesons with respect to charge (and energy) is provided by their deflection in the magnetized iron plates M_1, M_2, M_3, and M_4, which have a height of 20 cm, a thickness of 3 cm, and a length of 60 cm. The plates M_1 and M_2, and the plates M_3 and M_4, respectively, represent parts of two separate closed magnetic circuits, the field strengths in the plates M_1 to M_4 being approximately 18,000 gauss. The counters A are in anticoincidence with counters C_1 to C_6. The anticoincidence counters have the purpose of reducing the number of counts due to mesons not stopped in the lead absorber (Pb). All of the counters have a diameter of one or two inches as indicated in the figure, and an effective length of 50 cm. This is also the length of the lead absorber. Neutrons are recorded by a counting system consisting of two sections arranged symmetrically with respect to the magnetic spectrometer. Each section contains four proportional counters with a two-inch diameter and an effective length of 50 cm, filled with 0.39 atmos of enriched boron trifluoride (96 percent B^{10}).[17] The counters are sandwiched between layers of paraffin 8 cm thick, which extend in length a few inches beyond the counters.

Coincidences corresponding to the traversal of either a positive or a negative meson (depending on the direction of the magnetic field) through one of the spectrometers which was not accompanied by the discharge of one of the anticoincidence counters were recorded in the course of the experiment as well as discharges of the neutron counters from 5 to 135 μsec after these events.

(B) Determination of the Number of Stopped Mesons

For the determination of the number of mesons stopped in the lead block (Pb) the neutron counting

[14] C. F. Powell, *Cosmic Radiation* (Interscience Publishers, New York, 1949).
[15] W. Y. Chang, Revs. Modern Phys. **21**, 166 (1949).
[16] H. G. Voorhies and J. C. Street, Phys. Rev. **76**, 1100 (1949).
[17] The enriched BF_3 was obtained from the Isotope Division of the AEC. The counters were made by Mrs. N. Wood.

arrangement was removed and a second layer of anticoincidence counters installed around the lead absorber to make the anticoincidence arrangement more efficient. Counting rates were determined with and without the lead absorber in the position indicated in the figure. After making a minor correction to account for the fact that the solid angle subtended by the spectrometer was not entirely covered by anticoincidence counters, the number of mesons stopped during the experiment (2557 hours) was found to be 5500. The error in this determination is negligible compared with that of the number of neutrons connected with the stoppage of these mesons.

(C) Determination of the Number of Neutrons

The number of discharges of the BF_3 counters observed in 2557 hours with the telescope collecting negative mesons was 67. The expected number of chance counts during this period was 7.4. With the magnetic field reversed one count was observed during a period of 705 hours, while the expected number of chance counts was 2.3. It is thus apparent that the telescope was properly selective. The number of recorded neutrons caused by the stoppage of negative mesons was therefore 59.6 ($\pm$9.7 percent probable error).

To determine from this number the number of neutrons originating in the lead absorber due to the stopping of negative mesons, it is necessary to estimate the efficiency of neutron counting. This calibration involves the determination of (i) the efficiency $\epsilon(x, y, z)$ of counting with an appropriate neutron source placed at various positions from which neutrons due to μ-meson capture can originate, (ii) the spatial distribution $f(x, y, z)$ of the sources of the neutrons in the course of the experiment (stopped μ-mesons), and (iii) the mean life T of the neutrons in the counting arrangement, which must be known to take into account the fact that neutrons are counted only during the period from 5 to 135 μsec after the stoppage of a meson. The over-all efficiency, η, can then be expressed, in terms of these quantities, as follows:

$$\eta=[\exp(-5/T)-\exp(-135/T)]\times\int\epsilon(x, y, z)f(x, y, z)dV. \quad (1)$$

The integration is to be carried out over the region occupied by the lead absorber.

(i) The spectrometer has already been used in another investigation, in the course of which the probability of the transmission of a meson was determined as a function of the energy.[18] Although the meson spectrum in the energy range in question is rather flat, the transmission probability is such that the majority of the mesons coming to a stop in the lead absorber will do so in its lower part (part B in Fig. 1). With the lead absorber removed, a calibrated Na-Be photoneutron source was placed at several locations inside the space occupied by Part B of the absorber in the course of the experiments. The source was of known strength to within $\pm$20 percent and yielded monoenergetic neutrons of about 1 Mev energy.[19]

Numerous additional measurements were made with a Ra-Be neutron source in various positions. It was found that in spite of the different distribution of neutron energies, the dependence of the counting efficiency on the position of the source was approximately the same as for the Na-Be source. Thus, most of the relative efficiency measurements (and also periodic checks of the efficiency during the course of the experiment) could be made with the Ra-Be source, which was a great convenience, because the Na-Be photoneutron source decayed rapidly and gave off intense gamma-radiation. The description of the measurements is facilitated by the introduction of a cartesian coordinate system, with its (x-y) plane as indicated in Fig. 1. The variations of the counting efficiency in the x- and y-directions, in the region occupied during the experiment by part B of the lead absorber, were found to be quite small; and the only significant variations occurred in the z-direction, along which $\bar{\epsilon}(z)$ (averaged with respect to x and y), was determined at intervals of 5 cm.

(ii) The x- and y-dimensions of part B of the absorber are small compared with the path length of a meson through the telescope. Moreover, the counting efficiency $\epsilon(x, y, z)$ is essentially a function of z only. It is thus sufficient to consider the spatial distribution $f(x, y, z)$ of the stopped μ-mesons as a function of z only. The probability $f(z)$ can be obtained by simple geometric considerations. Let a ($=50$ cm) be the extension of the lead absorber (and of the effective length of the coincidence counters) in the z-direction, and let b be the length of the projection on the (x-y) plane of the path of a meson through the spectrometer, the length being measured from one of the top counters to a point P inside part B of the lead absorber at which the meson comes to a stop. Actually, there is a probability distribution of the length b; but, as has been pointed out before, the relative change in b due to variations of P in B, etc., is small, so that it is permissible to take a mean value for b ($=70$ cm). In Fig. 2, the rectangle $ABCD$ represents the approximate surface on which a meson traverses the spectrometer, the surface here being developed onto a plane. In this plane the path is rectilinear, if one neglects scattering. A particle incident at Q, at a distance ζ from A, and with an angle of incidence ϕ, will reach the line $\overline{CD}$ at P, at a distance z from C. We note that $\overline{AB}=a$, $\overline{AC}=b$, and $z=\zeta+b\tan\phi$. The spectrometer records essentially only mesons whose paths, before entry into the spectrometer,

[18] G. Groetzinger and G. W. McClure, Phys. Rev. 77, 777 (1950).

[19] This energy is somewhat lower than that of the neutrons to be expected in the evaporation following the capture of a meson in lead (Table I).

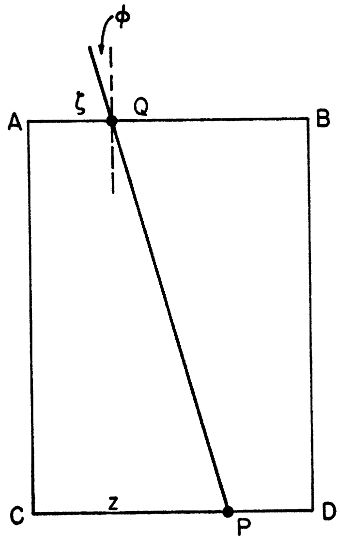

FIG. 2. Diagram illustrating the derivation of Eq. (3).

lie in two planes inclined 17° to the vertical (x-z) plane. Thus the distribution of the angles of incidence will be not much different from the zenith angle distribution, and we take it here to be proportional to $\cos^2\phi$. Since each point of incidence Q between A and B is equally likely, the joint frequency distribution of ϕ and ζ is $\text{const}\cdot\cos^2\phi d\phi d\zeta$. Let $F(z, \zeta)dzd\zeta$ be the joint frequency distribution of z and ζ. Then

$$F(z,\zeta)dzd\zeta = \text{const}\cdot\cos^2[\tan^{-1}\{(z-\zeta)/b\}]\frac{\partial(\phi,\zeta)}{\partial(z,\zeta)}dzd\zeta. \quad (2)$$

Evaluating the jacobian and integrating with respect to ζ from zero to a, one obtains

$$f(z)dz=\left[\int_0^a F(z,\zeta)d\zeta\right]dz = \text{const}dz\int_0^a[1+(z-\zeta)^2/b^2]^{-1} \times\cos^2[\tan^{-1}\{(z-\zeta)/b\}]d\zeta. \quad (3)$$

(iii) An approximate estimate was made of the mean life T of the neutrons in the counting system. With a Ra-Be source placed at the position of the lead absorber, it was found that the removal of either section of the counting system reduced the number of counts recorded by the other section by only three percent. Thus there was scarcely any reflection of neutrons from one section to the other, and it was permissible to base the estimate of the mean life on the consideration of one section only. Fifty percent of the volume of the gap between the two paraffin blocks of each section was occupied by neutron counters, so that the mean neutron absorption cross section of the entire gap could be assumed to be approximately one-half of that inside the counters. With[20] $\sigma v=1.6\times10^{-16}$, σ being the absorption cross section and v the thermal neutron velocity, one finds $T\approx230$ μsec. This is not very different from the value of T in an infinite medium of paraffin (180–200 μsec). We felt justified, therefore, in considering for the purpose of this approximate calculation a solid paraffin block 21 cm thick instead of two slabs 8 cm thick separated by a 5-cm gap containing the BF_3 counters. Rainwater and Havens,[21] who used neutrons which impinged on paraffin blocks with energies of a few Mev, give the mean life of these neutrons to be 45 and 100 μsec for paraffin thickness of 3.2 cm and 6.8 cm, respectively. These data can be fitted to the empirical expression

$$T(s)=190(1-e^{-0.095s}) \quad (4)$$

from which it follows that $T(21\text{ cm})\approx164$ μsec, and $A=[\exp(-5/T)-\exp(-135/T)]=0.53$. A variation of 15 percent in $T(21\text{ cm})$ will entail a variation of 10 percent in A.

Putting into expression (1) the empirical function $\bar{\epsilon}(z)$, and the function $f(z)$ (calculated by numerical integration from (3) and normalized to unity), and using the value $A=0.53$, one obtains for the over-all efficiency η a value of 5.5×10^{-3}. Thus, the total number of neutrons emitted was 10,800. The most important error in the calibration arises from the uncertainty of the strength of the Na-Be photo-neutron source ($\approx$20 percent). Altogether we would estimate the total probable error in the number of neutrons to be about 35 percent.

It should be pointed out that our equipment had a relatively very low efficiency for counting neutrons with energies in excess of, say, 25 Mev, so that very fast neutrons, which might be expected to emerge from the nucleus prior to the evaporation process, if the Marshak-Bethe type of interaction were the correct one, would not have been observed.

III. RESULTS AND DISCUSSION

According to the results of Sec. II, 5500 μ-mesons produced 10,800±4100 neutrons, i.e., the multiplicity was 1.96±0.72. This is consistent with the Sakata-interaction (assuming that the energy transfer to the nucleus takes place as calculated by Tiomno and Wheeler, and by Rosenbluth). Agreement with the Marshak-Bethe theory, on the other hand, can perhaps only be brought about by making the somewhat unlikely assumption that at least three-quarters of the rest energy of the μ-meson is carried off by fast nucleons.

We are indebted to Mr. Jack Aron for his help with the construction and operation of the equipment, and to Dr. Wattenberg and Mr. Eggler of the Argonne National Laboratory for providing us with the calibrated photo-neutron source.

[20] *The Science and Engineering of Nuclear Power* (Addison-Wesley Press, Inc., Cambridge, 1947), Vol. 1, Appendix C, p. 387.
[21] J. Rainwater and W. W. Havens, Phys. Rev. **70**, 136 (1946).