Interaction Effects on Radiative Transitions in Nuclei*

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Electromagnetic transition processes in nuclei are considered on the assumption that the influence of an external electromagnetic field on a nucleus can be incorporated into a gauge invariant Schroedinger equation for the nucleons (phenomenological theory). From the results of the preceding paper it is concluded that equations for electric multipole transition probabilities have the usual form but that the f-sum rule is modified in a manner depending on the nuclear interactions. Explicit expressions for the generalization of the f-sum rule to all electric multipole orders are given for exchange and velocity dependent interactions. The magnetic multipole moments, and therefore the corresponding transition probabilities, depend markedly on the form of the interaction with the electromagnetic field. General formulas are given for all magnetic multipole moments of nuclei in which exchange forces and velocity dependent interactions play a role. In addition it is possible to incorporate into the theory the spin-antisymmetric addition to the magnetic dipole moment implied by the static moments of H^3 and He^3.

Detailed application is limited in this paper to magnetic dipole transitions. The theoretical cross section for the capture of thermal neutrons by protons is found to have about a 4 percent addition, due entirely to the spin-antisymmetric term. In heavier nuclei this spin term, exchange interactions, and the velocity interaction proposed to account for high energy nucleon-nucleon scattering contribute to the magnetic dipole transitions. The ratio of magnetic dipole to electric quadrupole transition probabilities is of the order of $(25/\hbar\omega)^2 A^{-4/3}$, where A is the nuclear mass number and $\hbar\omega$ is the photon energy in Mev. Similar emphasis of the magnetic over electric transitions at low energies is anticipated for higher multipole orders, so interpretations of isomeric transitions on the basis of lifetime require re-examination.

I. INTRODUCTION

THE structure of nucleons and the mechanism of interaction between nucleons seem to involve charge bearing quanta in some way. It should therefore be possible to investigate questions of either structure or interaction mechanism by utilizing the influence of an external electromagnetic field on a nuclear system. For example, Pais[1] and Villars[2] have shown that in meson theory nuclear radiative transition probabilities and nuclear magnetic moments may depend on the detailed nature of the meson field. Relationships of this type were first emphasized by Siegert.[3]

At present the theoretical investigation of such problems by means of meson theories has all the disadvantages of the tenuous nature of current theories. This, combined with the difficulty of carrying through the required calculations, discourages the study of electromagnetic effects in terms of a detailed meson theory. As a matter of fact, certain general properties of the electromagnetic effects are obscured by the complicated character of each meson calculation. To the extent that the description of a nucleus may be given by a gauge invariant Schroedinger equation involving only nucleon variables and electromagnetic field variables, such properties of electromagnetic effects can be associated with the interaction appearing in the Schroedinger equation.[4] While a theory of this type may not have a domain of validity much beyond that of the adiabatic approximation in meson theory, it does provide a simple phenomenological approach which yields quantitative results. It is the purpose of this paper to derive some of the results which such an approach yields for radiative transitions in nuclei.

Specific examples of the relationships to be considered here have already been discussed. Feenberg has shown[5] that space exchange forces cause a modification of the f-sum rule for electric dipole transition probabilities, and this fact has been applied[6–8] many times. Furthermore, certain phenomenological magnetic moments,[9] which are a direct consequence of the assumption of exchange forces, indicate a modification of magnetic dipole transition probabilities.

It has been customary to call all effects associated with the nuclear interactions *exchange effects*. In view of the fact that they are not necessarily related to an exchange process the term seems to be inappropriate. In this paper they will instead be termed *interaction effects*. For example, *interaction moment* designates any contribution to the magnetic moment which is associated with the nucleon-nucleon interaction mechanism. The term *exchange effect* will be reserved for the particular interaction effects which arise directly from the introduction of a space exchange operator in the nucleon-nucleon coupling. Since even this does not yield a unique definition,[9,10] because free choice of a divergenceless term in the current is possible, we choose to include only those effects which arise in the "natural"

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[1] A. Pais, Kgl. Danske Videnskab. Selskab Mat.-fys. Medd. 20, No. 17 (1943).

[2] F. Villars, Phys. Rev. 72, 257 (1947); Helv. Phys. Acta 20, 476 (1947).

[3] A. J. F. Siegert, Phys. Rev. 52, 787 (1937); C. Møller and L. Rosenfeld, Kgl. Danske Videnskab. Selskab Mat.-fys. Medd. 20, No. 12 (1943).

[4] See the preceding paper, hereinafter referred to as $\mathfrak{S}$. Numbered equations in $\mathfrak{S}$ will be referred to as Eq. ($\mathfrak{S}$-1), etc.

[5] E. Feenberg, Phys. Rev. 49, 328 (1936).

[6] K. Way, Phys. Rev. 51, 552 (1937).

[7] J. S. Levinger and H. A. Bethe, Phys. Rev. 78, 115 (1950).

[8] J. F. Marshall and E. Guth, Phys. Rev. 78, 738 (1950).

[9] R. G. Sachs, Phys. Rev. 74, 433 (1948); Erratum, Phys. Rev. 75, 1605 (1949).

[10] R. K. Osborne and L. L. Foldy, Phys. Rev. 79, 795 (1950).

way (i.e., according to a conventional prescription) demonstrated in reference 9.

II. NATURE OF THE RADIATIVE EFFECTS

The most convenient expression of the interaction with the electromagnetic field is for present purposes given by Eq. (𝔖-27), which exhibits all first-order radiative interactions of a system. The equation gives the effective interaction for the emission of a photon in a nuclear transition. It is apparent that, as has often been conjectured for the dipole term, the electric multipole terms do not depend explicitly on the nuclear interaction.[11] That this result is plausible may be seen by an extension of the argument of Siegert. Although a modification of the definition of current density may be forced[3,9] by the forms of the nuclear interactions, the definition of charge density need not be changed. Thus, one would expect the electric moment operators to be unchanged, but the magnetic moment operators to reflect the change in the current density.

The change in the magnetic moment operators is contained implicitly in the form of the $\mathbf{M}_l$ given by Eq. (𝔖-26). There are two distinctly different reasons for believing that the $\mathbf{M}_l$ contain terms in addition to the ordinary 2^l-pole magnetic moments. One is the evidecne that the nuclear interactions involve a space exchange factor, which is already known to lead to additional terms. The other is the relationship between the nuclear interaction mechanism and static magnetic moments which is so strongly suggested by the apparent[12] non-additivity of the spin and orbital moments in H^3 and He^3. Theories which have been devised to account for the phenomenon would appear to fit into the gauge invariant Schroedinger formalism treated in 𝔖. They involve the introduction into the nuclear hamiltonian of an interaction between the electromagnetic field and pairs of nucleons which is antisymmetric in the nucleon spins.[2,10,13]

Both contributions may lead to observable modifications of magnetic multipole transitions in nuclei.

For the particular case $l=1$, the operator $\mathbf{M}_1$, which provides magnetic dipole transitions, is identical with the static magnetic moment operator. This exhibits the close correspondence between static and dynamic effects and has the consequence that *any interaction effect on the magnetic moment should show up as an equivalent effect on the magnetic dipole transition probability.* Thus, the aforementioned interpretation of the H^3 and He^3 moments must have immediate consequences for magnetic dipole transition probabilities in all other nuclei as well as for their magnetic moments.

There appear to be several possibilities for the experimental investigation of the effects discussed here. One would be the interaction effect on the cross section for photo-disintegration of the deuteron or on the inverse process of neutron-proton capture. Another possibility concerns the interpretation of isomeric transitions. It would seem that those parity and angular momentum assignments made on the basis of lifetime may be in error wherever a magnetic multipole transition is a possibility. The probabilities for magnetic transitions may be sufficiently increased by interaction effects that they would be confused with electric transitions of the same multipole order.

Conclusions concerning the absorption of high energy gamma radiation by nuclei are also affected. Thus an extension of the calculation of Levinger and Bethe,[7] taking into account the magnetic dipole effects, would seem to be in order.

It is of interest that the treatment of electric dipole absorption by Levinger and Bethe offers a good example of an interaction effect on electric multipole transitions. They make use of the modification of the f-sum rule due to exchange forces. A similar modification for the quadrupole and higher multipole transitions is provided by Eq. (𝔖-35). From this equation it is seen that although interaction effects on electric multipole transitions are not immediately apparent, they do appear for the sum rules. However, here the *second-order* term, H_2, in the hamiltonian is involved, rather than the first-order term, H_1, which contains all the magnetic interaction effects.

Formulas are given in the Appendix for the magnetic multipole moment operators of nuclei and the generalized f-sum rules for ordinary interactions, exchange interactions, and the velocity dependent interactions linear in the momentum.

III. MAGNETIC DIPOLE EFFECTS

Applications of the phenomenological method will be limited in this paper to interaction effects on magnetic dipole transitions, and three cases are considered. These include the two effects whose existence was emphasized in the preceding section, and a third but much more tentative effect associated with a velocity dependent nuclear interaction. This interaction was introduced recently to account for high energy nucleon-nucleon scattering as well as spin-orbit coupling in heavy nuclei.[13,14]

The transition probability due to exchange effects can be computed from the exchange moment operator whose form, already known, is given by Eq. (A-16). However, as the exchange moment does not account[15] for the static moment anomaly of H^3 and He^3 the need for some other moment operator is seen. Fortunately, rather general statements can be made concerning the form of the required operator.

[11] This fact is convenient for calculation and has been used in the dipole case by several authors. Since the forms of the operators are not affected by the nuclear interactions, it is necessary only to guess at wave functions in order to estimate nuclear electric multipole transition probabilities.

[12] R. Avery and R. G. Sachs, Phys. Rev. **74**, 1320 (1948).

[13] Blanchard, Avery, and Sachs, Phys. Rev. **78**, 292 (1950).

[14] K. M. Case and A. Pais, Phys. Rev. **80**, 203 (1950).

[15] R. Avery and E. N. Adams, Phys. Rev. **75**, 1106 (1949). There is a misprint in the expression given for the three-body wave function. A factor ½ should be inserted in the exponent.

The interaction moment is presumed to arise in conjunction with two-body interactions. Then it must have an operational form satisfying the following conditions.[15a] (a) It is a sum of operators, each depending on one pair of particles. (b) It is symmetric for interchange of any pair of nucleons (in isotopic spin notation). (c) It can be written in terms of the internal variables, $\boldsymbol{\rho}_\alpha$, and other internal operators such as the spins and relative momenta. (d) It is a pseudovector which changes sign on time reversal.

The operator making up the sum in (a) will contain as a factor a symmetric scalar function, $\Phi(|\boldsymbol{\rho}_{\alpha\beta}|)$, which gives the intensity of the interaction effect as a function of the distance between the pair.

All proposals extant concerning interaction moments lead to a magnetic moment operator satisfying these conditions.[2, 10, 13] Of the operators satisfying (a), (b), (c), and (d), there is one particularly simple form which is capable of accounting for the H^3 and He^3 moments if the ground states of these nuclei are assumed to be predominantly S state. This is

$$\Delta\mathbf{M}_1=(e\hbar/2Mc)\tfrac{1}{2}\sum_{\alpha,\beta}(\boldsymbol{\sigma}_\alpha-\boldsymbol{\sigma}_\beta)(\tau_{\alpha\beta}/2)\Phi_{\alpha\beta}. \qquad (1)$$

For the sake of definiteness Eq. (1) will be used as the basis for the discussion of radiative transitions.[16] Analysis of the three-body problem assuming that the form of Φ is

$$\Phi(\rho)=(MJ_0/\hbar^2\mu^2)(e^{-\mu\rho}/\mu\rho), \qquad (2)$$

with $\mu^{-1}=1.18\times10^{-13}$ cm, leads to

$$J_0=-13\text{ Mev}, \qquad (3)$$

as the value required to account for the moment anomaly.

Turning to the velocity-dependent interaction of Case and Pais,[14] we note that they finally settled on the linear combination[15a] $\frac{1}{2}(\text{II}+\text{III}'+\text{IV})$, which is just (3) of reference 13. The additional interaction moment operator thereby produced is given by the same linear combination of the corresponding magnetic moments, given in the Appendix:

$$\Delta_v\mathbf{M}_1=(e/4\hbar c)\sum_{\alpha,\beta}\left\{[\boldsymbol{\rho}_\alpha\times(\mathbf{S}_{\alpha\beta}\times\boldsymbol{\rho}_{\alpha\beta})]\frac{1-\tau_\alpha}{2}\frac{3+\boldsymbol{\tau}_\alpha\cdot\boldsymbol{\tau}_\beta}{2}\right.$$
$$\left.-\frac{i}{2\hbar}(\boldsymbol{\rho}_{\alpha\beta}\times\mathbf{p}_{\alpha\beta}\cdot\mathbf{S}_{\alpha\beta})[\boldsymbol{\rho}_\alpha\times\boldsymbol{\rho}_\beta]\frac{\tau_{\alpha\beta}}{2}\frac{1+\boldsymbol{\tau}_\alpha\cdot\boldsymbol{\tau}_\beta}{2}\right\}J_{\alpha\beta}{}^{(v)}. \qquad (4)$$

The factor $J_{\alpha\beta}{}^{(v)}$ is taken to be[14]

$$J^{(v)}(\rho)=(J_v/\lambda\rho)\frac{d}{d(\lambda\rho)}(e^{-\lambda\rho}/\lambda\rho), \qquad (5)$$

with

$$J_v=3\text{ Mev}. \qquad (6)$$

IV. NEUTRON-PROTON CAPTURE; PHOTO-EFFECT ON THE DEUTERON

The capture of slow neutrons by protons provides the clearest nuclear example of a magnetic dipole transition. Theoretical determinations of the capture cross section have not taken into account the interaction effects; yet they are in quite good agreement with the experimental value.[17] A large interaction effect, and therefore any interaction which leads to such an effect, is to be excluded. However, it turns out that none of the interactions set forth in Sec. III is excluded in this way.

The velocity-dependent term, Eq. (4), makes no contribution to the deuteron problem, since it vanishes for all even states of the two-body system. Similarly, the exchange moment operator, Eq. (A-16), vanishes for the two-body system, since it is proportional to $\boldsymbol{\rho}\times\boldsymbol{\rho}$. Thus, the only interaction effect on the capture cross section arises from the operator, Eq. (1), whose introduction is an immediate consequence of the interpretation of the H^3 and He^3 moments. This suggests that the neutron-proton capture might provide a direct test of that interpretation.

The interaction moment acts only within a region comparable with the range of the forces, while the spin-orbital moment acts over the entire volume of the deuteron. Thus, for this ${}^1S\rightarrow{}^3S$ transition the ratio of the matrix element of the moment Eq. (1) to that of the ordinary moment can be estimated as

$$\langle\Delta\mathbf{M}\rangle/\langle\mathbf{M}\rangle\approx-(MJ_0/\hbar^2\mu^2)(2\gamma/\mu)^3 2/(\mu_P-\mu_N), \qquad (7)$$

where μ_P and μ_N are neutron and proton moments in nuclear magnetons. The quantity γ is related to the deuteron binding energy by

$$E_B=\hbar^2\gamma^2/M, \qquad (8)$$

so $1/2\,\gamma$ is the "radius" of the deuteron. For $\mu^{-1}=1.18\times10^{-13}$ cm we have $\gamma=0.28\mu$, and the value of (7) is

$$\langle\Delta\mathbf{M}\rangle/\langle\mathbf{M}\rangle\approx0.03. \qquad (9)$$

The transition probability is proportional to the square of the matrix element of the total moment, so the change in the cross section introduced by Eq. (9) amounts to about 6 percent.

An accurate determination of this interaction effect requires a detailed knowledge of the distribution function $\Phi(\rho)$ in Eq. (1). A convenient way to avoid too detailed assumptions is to treat the auxiliary problem of the effect produced by a thin shell distribution,

$$\Phi(\rho)=\Phi_0\delta(\rho-b), \qquad (10)$$

as a function of the radius, b, of the shell. From the results, it is possible to draw rather general conclusions

[15a] See Appendix for notation.

[16] All static terms which are capable of yielding the three-body moments are incorporated in Eq. (1) if the definition of $\Phi_{\alpha\beta}$ is extended to include terms linear in $\boldsymbol{\rho}_\alpha+\boldsymbol{\rho}_\beta$ as well as functions of $\boldsymbol{\rho}_{\alpha\beta}$. The only information concerning $\Phi_{\alpha\beta}$ is to be obtained from the three-body moments, and this one datum would not appear to justify taking too general a form for the function.

[17] For a detailed discussion of the comparison with experiment see H. A. Bethe and C. Longmire, Phys. Rev. 77, 647 (1950).

concerning the form of the distribution. This calculation was carried out as follows. The interaction moment of the triton was determined from Eqs. (1) and (10) by using the space-symmetric ^2S wave function given in reference 15, normalized to 100 percent. The value of $\Phi_0(b)$ was fixed by setting this moment equal to 0.27 nuclear magnetons, the presumed experimental value. Then the matrix element of the operator Eq. (1), for the same function Φ, was calculated for the $^3S \rightarrow {}^1S$ transition in the deuteron, using wave functions given by Bethe and Longmire.[17] The resulting relative contribution to this matrix element is shown in Fig. 1.

The exponential rise of the curve for large values of b is a direct consequence of the small size of the triton relative to the deuteron; since the triton function is small at large distances, a very large Φ_0 is required when b is large compared with the triton radius. It follows that if the actual distribution function Φ has appreciable magnitude beyond distances comparable to the triton radius, a large interaction effect would result. On the other hand, if the distribution of magnetization is limited to a small radius, the effect is very nearly independent of the shape of Φ and the change, ϵ_i, in the cross section has the minimum value, 3.5 percent. Thus,[18]

$$\epsilon_i \gtrsim 0.035. \tag{11}$$

The influence of variations in the triton wave function may be estimated by noting that in the region of interest ϵ_i is roughly proportional to the damping length of that function.

Since the interaction effects are expected to occur only within the range of the nuclear forces and since the triton wave function extends somewhat beyond this range, the value of ϵ_i is not likely to exceed greatly the lower limit given in Eq. (11). However, any excess may be of some use in determining the shape of Φ. A numerical example may serve to illustrate the point. For $\Phi \sim e^{-\mu\rho}/\mu\rho$, with $\mu^{-1} = 1.18 \times 10^{-13}$ cm one finds $\epsilon_i = 0.036$. On the other hand, for $\Phi \sim \rho^2 e^{-\mu\rho}/\mu\rho$ the distribution is moved out far enough to yield $\epsilon_i = 0.052$.

The conditions which must be met in order to obtain an experimental value of ϵ_i are best expressed in terms of the ratio of theoretical to experimental cross section. Referring to the paper by Bethe and Longmire,[17] modification of their expression to include the interaction effect yields

$$\sigma_{\text{th}}/\sigma_{\text{exp}} = 1.10[1 - 0.116(r_{os} - r_{ot}) + 2.8\epsilon_1 + 1.5\epsilon_2 - 0.15\epsilon_3 - \epsilon_4 + \epsilon_i]. \tag{12}$$

Here r_{os} and r_{ot} are the singlet and triplet effective ranges, expressed in units of 10^{-13} cm; ϵ_1, ϵ_2, ϵ_3, ϵ_4 are, in order, the experimental uncertainties in deuteron binding energy, slow neutron-proton total scattering cross section, slow neutron-proton coherent scattering cross section, and slow neutron-proton capture cross section. The values used by Bethe and Longmire are: H^2 binding energy, 2.235 Mev; thermal NP total scattering cross section, 20.36×10^{-24} cm^2; thermal NP coherent scattering cross section, 0.624×10^{-24} cm^2; thermal NP capture cross section, 0.310×10^{-24} cm^2 at 2200 m per sec neutron velocity.

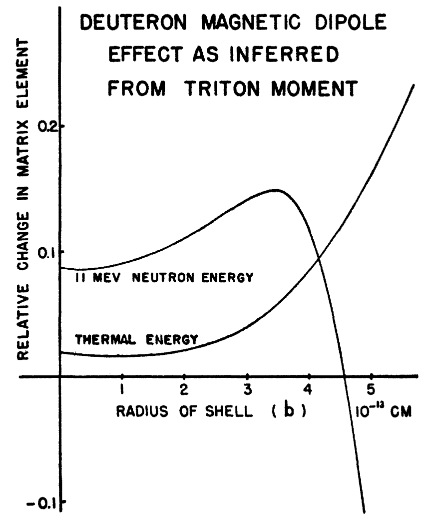

FIG. 1. Relative contribution of thin shell (δ-function) interaction moment operator to the matrix element for a radiative transition in the deuteron. The corresponding contribution for any other distribution of magnetization, $\Phi(\rho)$, is the harmonic mean of this curve with respect to a weight function given by multiplying $\Phi(\rho)$ by the product of the initial and final state radial wave functions of the deuteron.

We take the ϵ_2 and ϵ_4 values of Bethe and Longmire

$$\epsilon_2 = \pm 0.005, \quad \epsilon_4 = \pm 0.04. \tag{13}$$

The best value[19] of ϵ_1 appears to be

$$\epsilon_1 = -0.004 \pm 0.001. \tag{14}$$

The best value[20] of ϵ_3 is taken from the liquid mirror experiments:

$$\epsilon_3 = -0.08 \pm 0.02. \tag{15}$$

The value chosen for r_{ot} is[17,21]

$$r_{ot} = (1.59 + 2.7\epsilon_1 + 5.0\epsilon_2 - 1.5\epsilon_3). \tag{16}$$

For r_{os} we take the value[21] from the proton-proton system

$$r_{os} = 2.71 \pm 0.13. \tag{17}$$

The introduction of these into Eq. (12) gives

$$\sigma_{\text{th}}/\sigma_{\text{exp}} = 1.02 \pm 0.05 + \epsilon_i, \tag{18}$$

if the uncertainties are all treated as random.

Comparison of Eqs. (11) and (18) leads to no contradiction. It does suggest that ϵ_i has nearly its mini-

[18] Note that the term linear in $\varrho_\alpha + \varrho_\beta$, mentioned in reference 16, would contribute neither in the deuteron nor in the triton ^2S state.

[19] R. C. Mobley and R. A. Laubenstein, Phys. Rev. **80**, 309 (1950).

[20] Hughes, Burgy, and Ringo, Phys. Rev. **77**, 291(L) (1950).

[21] H. A. Bethe, Phys. Rev. **76**, 38 (1949).

TABLE I. Selection rules for magnetic dipole transitions.

Magnetic moment	J	Parity	L	S	Configuration
Spin orbital	$\Delta J=\pm 1,0$[a]	No change	$\Delta L=\pm 1,0$	$\Delta S=\pm 1,0$	No change.
Exchange	$\Delta J=\pm 1,0$[a]	No change	$\Delta L=\pm 1,0$[a]	$\Delta S=0$	At most two particles change state.
Interaction, Eq. (1)	$\Delta J=\pm 1,0$[a]	No change	$\Delta L=0$[b]	$\Delta S=\pm 1,0$	At most two particles change state.
Velocity, Eq. (4)	$\Delta J=\pm 1,0$[a]	No change	$\Delta L=\pm 2,\pm 1,0$	$\Delta S=\pm 1,0$	At most two particles change state.

[a] $0\rightarrow 0$ forbidden.
[b] $\Delta L=\pm 2,\pm 1$ would be allowed if a slightly more general form than Eq. (1) were assumed.

mum value, as expected. The result indicates that a more precise determination of the cross section for the capture of slow neutrons by protons would be very useful. In the measurement of that quantity lies the principal source of the uncertainty in Eq. (18).

Examination of the behavior of the interaction effect at somewhat higher energies shows that it should be of greater relative importance than at thermal energy. This occurs because the range of the interaction moment operator is small compared with the size of the deuteron. Therefore, the contribution to the matrix element by the interaction moment arises from a region in which energy changes of the free neutron state have very little influence on the ^1S wave function, at least until the neutron wave length is comparable to the range of the forces. On the other hand, the matrix element for the usual magnetic effect involves principally the outer region and decreases rapidly with energy.

Such behavior immediately suggests that a measurement at higher energy should yield more definite information concerning the interaction effect. Unfortunately, with increasing energy the electric dipole moment rapidly becomes the main contributor to photo-disintegration, so that a separation of magnetic and electric effects is required as a preliminary to the resolution of the interaction effect. The separation can be accomplished in principle by a measurement of the angular distribution of the photo-disintegration products, but this difficult experiment has not yet been carried out with an accuracy at all sufficient for our purposes. The relative contribution of the interaction moment to the photo-magnetic effect nevertheless is given in Fig. 1 for a photon energy of 13 Mev. Note that for small b this contribution is much larger than at low energies, but not yet so strikingly large as to be separable, for the photo-magnetic cross section at this energy is only about 2 percent of the photoelectric cross section.

V. MAGNETIC DIPOLE TRANSITIONS IN HEAVIER NUCLEI

The interaction effects on heavier nuclei might be expected to influence lifetimes in isomeric transitions as well as level widths in nuclear reactions. The former usually correspond to transitions of higher order than magnetic dipole. However, in order to obtain an early judgment concerning the order of magnitude of the effects, estimates will be made here only for magnetic dipole transitions.

Each of the three interaction moments described in Sec. III can be expected to play a role in the emission of quanta in magnetic dipole transitions. The selection rules for their contributions to the magnetic moment are given in Table I. J is the total angular momentum, L the orbital, and S the spin angular momentum.

The column labeled "configuration" refers to one nucleon wave functions and is useful if these functions provide a reasonable approximation. It is very often assumed that the configuration can be specified. Then the selection rules show that interconfigurational magnetic dipole transitions can take place only if interaction effects are present. The limitation on the number of particles that can change state is a consequence of the assumption of two-body forces.

In order to obtain an estimate of the order of magnitude of the magnetic dipole transition probability it will be assumed that the configuration can be specified and that S is given for each of the states involved. All the interaction effects have the property that a transition in which only one nucleon changes state is much more probable than a two-nucleon transition. The reason is that only the term in the moment involving both particles contributes to the two-particle transition, while the number of terms involved in the single particle transition is equal to the number of particles with which the particle interacts. The matrix element for the one-nucleon transition is about A times that for the two-nucleon transition, so the latter will be neglected.

The interactions act only over a range μ^{-1}, so a contribution to the matrix element arises only within the fraction $(\mu R)^{-3}$ of the total nuclear volume, $4\pi R^3/3$. Since

$$(\mu R)^3\approx A, \tag{19}$$

this factor very nearly compensates for the large number of terms that contribute to the one-particle transition. Thus, the order of magnitude of the matrix element for the one-particle transition would be $(e/\hbar c)(\bar{J}/\mu^2)$, where $\bar{J}$ is the average strength of those interaction moments which contribute to the particular transition. A convenient standard for comparison is provided by the quadrupole transitions, with which the magnetic dipole transitions can often compete. The order of magnitude of the matrix element of the quadrupole moment may be taken to be $e\omega R^2/c$. Then the ratio of the magnetic dipole transition probability to the quadrupole transition probability is roughly

$$w_m/w_q\approx(\bar{J}/\hbar\omega)^2(\mu R)^{-4}. \tag{20}$$

According to Eq. (19),

$$w_m/w_q\approx(\bar{J}/\hbar\omega)A^{-4/3}. \tag{21}$$

A reasonable estimate of $\bar{J}$ is

$$\bar{J} \approx 25 \text{ Mev}. \quad (22)$$

Then for light nuclei ($A \approx 27$), the magnetic transitions predominate for $\hbar\omega$ less than 2.5 Mev. For the heaviest nuclei ($A \approx 238$) the corresponding energy is 1 Mev. Thus, it seems to be very probable that magnetic dipole transitions due to interaction effects are of much greater importance than quadrupole transitions for the very low lying levels of nuclei. Of course, the intraconfigurational transitions should be included in the consideration of the very low levels so that the magnetic transition probability will be increased further by the spin-orbital moment, which is then also effective.

VI. CONCLUSION

On the basis of prevailing ideas about nuclear forces, all magnetic multipole transitions will show interaction effects, and it is possible to make reasonable estimates of their influence on magnetic dipole transition probabilities. These estimates show that for transitions between low lying nuclear levels, magnetic dipole radiation is strongly favored over electric quadrupole when the selection rules allow both. Similar results are to be expected for the higher multipole orders, so that previous identifications of isomeric transitions on the basis of lifetime may be in error.

The most compelling test of the theory is suggested by the 4 percent contribution to the neutron-proton capture cross section. More accurate measurement of this cross section would be very helpful. However, information by this means is limited to the spin-antisymmetric interaction presumed to be responsible for the anomaly in the H^3, He^3 magnetic moments. Information concerning exchange effects or the velocity dependent interaction of Case and Pais can only be obtained from nuclei heavier than the deuteron. No simple method of discrimination of the various effects in the heavier nuclei is suggested in view of the lack of knowledge concerning nuclear wave functions.

APPENDIX. SOME FORMULAS FOR SPECIFIC INTERACTIONS

For a given hamiltonian the interaction effects may be obtained directly from the equations given in 𝕾. Several important examples will be considered to illustrate the effects in question. Ignoring, for the sake of simplicity, the contributions of the spin magnetic moments, the internal hamiltonian takes the form

$$H=(\tfrac{1}{4}AM)\sum_{\alpha,\beta}[\mathbf{p}_{\alpha\beta}-(e_\alpha\mathbf{A}_\alpha/c)+(e_\beta\mathbf{A}_\beta/c)]^2+U, \quad \text{(A-1)}$$

where U is the gauge invariant nuclear interaction potential. $\mathbf{p}_{\alpha\beta}=\mathbf{p}_\alpha-\mathbf{p}_\beta$ is the relative momentum of the α and β nucleons. $\mathbf{A}_\alpha=\mathbf{A}(\mathbf{r}_\alpha)$ is the vector potential at the position of the designated particle. The integer A is the nuclear mass number, and M is the mass of a nucleon.

Ordinary Forces

For this case,

$$U=\tfrac{1}{2}\sum_{\alpha,\beta}{}' J_{\alpha\beta}^{(0)}, \quad \text{(A-2)}$$

which is the sum over nucleon pairs of ordinary, two-body potentials. The only nonvanishing H_n are H_1 and H_2. These are

$$H_1=-(1/4AMc)\sum_{\alpha,\beta}\{[\mathbf{p}_{\alpha\beta}\cdot(e_\alpha\mathbf{A}_\alpha-e_\beta\mathbf{A}_\beta)] +[(e_\alpha\mathbf{A}_\alpha-e_\beta\mathbf{A}_\beta)\cdot\mathbf{p}_{\alpha\beta}]\} \quad \text{(A-3)}$$

$$H_2=(1/2AMc^2)\sum_{\alpha,\beta}(e_\alpha\mathbf{A}_\alpha-e_\beta\mathbf{A}_\beta)^2. \quad \text{(A-4)}$$

Equations (𝕾-20) and (𝕾-26) provide the magnetic multipole moment operators

$$\mathbf{M}_l^{(0)}=\frac{el}{2Mc(l+1)!}\sum_\pi\{[\boldsymbol{\varrho}_\pi\times(\mathbf{p}_\pi-\mathbf{P}/A)](\boldsymbol{\kappa}\cdot\boldsymbol{\varrho}_\pi)^{l-1} +(\boldsymbol{\kappa}\cdot\boldsymbol{\varrho}_\pi)^{l-1}[\boldsymbol{\varrho}_\pi\times(\mathbf{p}_\pi-\mathbf{P}/A)]\}, \quad \text{(A-5)}$$

where $\boldsymbol{\kappa}$ is the unit propagation vector; the subscript π denotes a proton variable; $\boldsymbol{\varrho}_\alpha=\mathbf{r}_\alpha-\mathbf{R}$ is the position of the αth particle relative to the center of mass; $\mathbf{P}$ is the center-of-mass momentum.

Defining the electric 2^l-pole moment by Eq. (𝕾-25),

$$D_l=(e/l!)\sum_\pi(\mathbf{u}\cdot\boldsymbol{\varrho}_\pi)(\boldsymbol{\kappa}\cdot\boldsymbol{\varrho}_\pi)^{l-1}, \quad \text{(A-6)}$$

where $\mathbf{u}$ is the unit polarization vector, satisfying $(\mathbf{u}\cdot\boldsymbol{\kappa})=0$. Then the generalized oscillator strength for electric 2^l-pole radiation is Eq. (𝕾-32),

$$f_{jn}{}^l=(2M\omega_{nj}/\hbar e^2)|(D_l)_{nj}|^2, \quad \text{(A-7)}$$

for a transition from state n to state j of the nucleus. The sum rule may now be obtained from Eq. (𝕾-35) and Eq. (𝕾-19) by use of Eq. (A-4):

$$\Sigma_n f_{jn}{}^l=(1/2Ae^2(l!)^2)\sum_{\alpha,\beta}\langle[e_\alpha(\boldsymbol{\kappa}\cdot\boldsymbol{\varrho}_\alpha)^{l-1}-e_\beta(\boldsymbol{\kappa}\cdot\boldsymbol{\varrho}_\beta)^{l-1}]^2 +(l-1)^2[e_\alpha(\mathbf{u}\cdot\boldsymbol{\varrho}_\alpha)(\boldsymbol{\kappa}\cdot\boldsymbol{\varrho}_\alpha)^{l-2}-e_\beta(\mathbf{u}\cdot\boldsymbol{\varrho}_\beta)(\boldsymbol{\kappa}\cdot\boldsymbol{\varrho}_\beta)^{l-2}]^2\rangle_{jj}, \quad \text{(A-8)}$$

where the angular brackets indicate the expectation value for state j of the nucleus. In particular, for dipole transitions ($l=1$) one obtains the usual result in terms of Z, the number of protons, and N, the number of neutrons:

$$\Sigma_n f_{jn}{}^1=ZN/A. \quad \text{(A-9)}$$

Similarly, for quadrupole radiation:

$$\Sigma_n f_{jn}{}^2=(1/8Ae^2)\sum_{\alpha,\beta}\langle[\boldsymbol{\kappa}\cdot(e_\alpha\boldsymbol{\varrho}_\alpha-e_\beta\boldsymbol{\varrho}_\beta)]^2 +[\mathbf{u}\cdot(e_\alpha\boldsymbol{\varrho}_\alpha-e_\beta\boldsymbol{\varrho}_\beta)]^2\rangle_{jj}. \quad \text{(A-10)}$$

Exchange Interaction

In Eq. (A-1) U is taken to be[9]

$$U=\tfrac{1}{2}\sum_{\alpha,\beta}{}'\exp\left\{\frac{i}{\hbar c}(e_\alpha-e_\beta)\int_{\rho_\beta}^{\rho_\alpha}A_s ds\right\}J_{\alpha\beta}^{(x)}P_{\alpha\beta}. \quad \text{(A-11)}$$

$P_{\alpha\beta}$ is a space exchange operator, A_s is the component of $\mathbf{A}$ along the line joining $\boldsymbol{\varrho}_\alpha$ and $\boldsymbol{\varrho}_\beta$, and the line integral is taken along that straight line. Only the interaction, U, need be considered, as the kinetic energy terms are the same as for ordinary forces. Expansion of U in successive orders in $\mathbf{A}$, corresponding to the expansion Eq. (𝕾-2), gives terms of all orders, with

$$U_1=\frac{ie}{\hbar c}\sum_{\pi,\nu}\left(\int_{\rho_\nu}^{\rho_\pi}A_s ds\right)J_{\pi\nu}^{(x)}P_{\pi\nu}, \quad \text{(A-12)}$$

$$U_2=-\frac{e^2}{\hbar^2c^2}\sum_{\pi,\nu}\left(\int_{\rho_\nu}^{\rho_\pi}A_s ds\right)^2 J_{\pi\nu}^{(x)}P_{\pi\nu}, \quad \text{(A-13)}$$

where π, ν label proton and neutron variables.

As can be seen from Eq. (𝕾-26) the magnetic multipole moment is made up additively of a contribution from the kinetic energy, given by Eq. (A-5), and a contribution from U_1:

$$\mathbf{M}_l^{(x)}=\mathbf{M}_l^{(0)}+\Delta_x\mathbf{M}_l, \quad \text{(A-14)}$$

with the exchange multipole moment given by

$$\Delta_x\mathbf{M}_l=\frac{ie}{\hbar c(l+1)!}\sum_{\pi,\nu}\frac{(\boldsymbol{\varrho}_\pi\times\boldsymbol{\varrho}_\nu)}{(\boldsymbol{\kappa}\cdot\boldsymbol{\varrho}_{\pi\nu})}\{(\boldsymbol{\kappa}\cdot\boldsymbol{\varrho}_\pi)^l-(\boldsymbol{\kappa}\cdot\boldsymbol{\varrho}_\nu)^l\}J_{\pi\nu}^{(x)}P_{\pi\nu}. \quad \text{(A-15)}$$

For $l=1$ we obtain the expression for the exchange moment operator:[22]

$$\Delta_x \mathbf{M}_1=(ie/2\hbar c)\sum_{\pi,\nu}(\boldsymbol{\varrho}_\pi\times\boldsymbol{\varrho}_\nu)J_{\pi\nu}{}^{(x)}P_{\pi\nu}. \tag{A-16}$$

The sum rules are also changed from Eq. (A-8) by the addition of a term introduced by U_2 into Eq. (B-35). Since Eq. (B-35) is linear in H_2, the change is simply additive and can easily be shown to be

$$\Delta_x \Sigma_n f_{jn}{}^l=\frac{-M}{\hbar^2(l!)^2}\Big\langle\sum_{\pi,\nu}[(\mathbf{u}\cdot\boldsymbol{\varrho}_\pi)(\boldsymbol{\kappa}\cdot\boldsymbol{\varrho}_\pi)^{l-1} -(\mathbf{u}\cdot\boldsymbol{\varrho}_\nu)(\boldsymbol{\kappa}\cdot\boldsymbol{\varrho}_\nu)^{l-1}]^2 J_{\pi\nu}P_{\pi\nu}\Big\rangle_{jj}. \tag{A-17}$$

For $l=1$ this result is identical with that obtained by Feenberg[5] and recently applied in some detail by Levinger and Bethe.[7] The result has not been given before[23] for higher l.

Velocity-Dependent Interactions

Velocity-dependent interactions[13,14] provide a useful example of interaction effects other than those due to exchange. The two-body interactions, linear in the momentum, which are tabulated by Wigner and Eisenbud,[24] are considered here. Although one of the six interactions (IV) can be eliminated on the basis of experience,[13] it will be considered for the sake of completeness.

In Eq. (A-1) U is taken to be

$$U=\frac{1}{4\hbar}\sum_{\alpha,\beta}\left\{\left(\boldsymbol{\varrho}_{\alpha\beta}\times\left[\mathbf{p}_{\alpha\beta}-\frac{e_\alpha}{c}\mathbf{A}_\alpha+\frac{e_\beta}{c}\mathbf{A}_\beta\right]\cdot\mathbf{S}_{\alpha\beta}\right)T_{\alpha\beta} + \text{hermitian conjugate}\right\}J_{\alpha\beta}{}^{(v)}. \tag{A-18}$$

The operators appearing in U are most easily described in terms of the isotopic spin vector $\boldsymbol{\tau}_\alpha$ of the αth particle, with components $\tau_{\alpha1}$, $\tau_{\alpha2}$, $\tau_{\alpha3}$; $\tau_{\alpha3}=1$ for a neutron, $\tau_{\alpha3}=-1$ for a proton. For the sake of simplicity $\tau_{\alpha3}$ will be replaced by τ_α wherever possible and $(\tau_\alpha-\tau_\beta)$ by $\tau_{\alpha\beta}$. Then the e_α are the operators,

$$e_\alpha=\tfrac{1}{2}(1-\tau_\alpha)e, \tag{A-19}$$

and the operators $\mathbf{S}_{\alpha\beta}$ and $T_{\alpha\beta}$ are defined as follows for each of the six interactions:[25]

$$\mathbf{S}_{\alpha\beta}=(\boldsymbol{\sigma}_\alpha+\boldsymbol{\sigma}_\beta),\quad\begin{cases}T_{\alpha\beta}=\tfrac{1}{2}(\tau_\alpha+\tau_\beta) & \text{(I)}\\ T_{\alpha\beta}=\tfrac{1}{2}(1+\tau_\alpha\tau_\beta) & \text{(II)}\\ T_{\alpha\beta}=\tfrac{1}{2}(1+\boldsymbol{\tau}_\alpha\cdot\boldsymbol{\tau}_\beta)\times\exp\left\{\dfrac{-ie}{\hbar c}\dfrac{\tau_{\alpha\beta}}{2}\displaystyle\int_{\rho_\alpha}^{\rho_\beta}A_s ds\right\} & \text{(III}'\text{)}\\ T_{\alpha\beta}=\tfrac{1}{2}(\tau_\alpha-\tau_\beta) & \text{(IV)}\end{cases}$$

$$\mathbf{S}_{\alpha\beta}=(\boldsymbol{\sigma}_\alpha-\boldsymbol{\sigma}_\beta),\quad T_{\alpha\beta}=\tfrac{1}{2}(\tau_\alpha-\tau_\beta) \tag{V}$$

$$\mathbf{S}_{\alpha\beta}=[\boldsymbol{\sigma}_\alpha\times\boldsymbol{\sigma}_\beta],\quad T_{\alpha\beta}=\tfrac{1}{2}[\boldsymbol{\tau}_\alpha\times\boldsymbol{\tau}_\beta]_3\exp\left\{\frac{-ie}{\hbar c}\frac{\tau_{\alpha\beta}}{2}\int_{\rho_\alpha}^{\rho_\beta}A_s ds\right\}. \tag{VI}$$

Gauge invariance, as expressed by Eq. (B-9), can easily be established for these interactions by making use of the well-known commutation relations for the components of $\boldsymbol{\tau}_\alpha$. The first- and second-order contributions of A to U are

$$U_1=-(1/2\hbar c)\sum_{\alpha,\beta}(\boldsymbol{\varrho}_{\alpha\beta}\times[e_\alpha\mathbf{A}_\alpha-e_\beta\mathbf{A}_\beta]\cdot\mathbf{S}_{\alpha\beta})T_{\alpha\beta}J_{\alpha\beta}{}^{(v)}, \tag{A-20}$$

$$U_2=0, \tag{A-21}$$

for the non-exchange interactions (I), (II), (IV), and (V). For the exchange interactions (III′) and (VI)

$$U_1=\frac{-1}{4\hbar c}\sum_{\alpha,\beta}\left\{(\boldsymbol{\varrho}_{\alpha\beta}\times[e_\alpha\mathbf{A}_\alpha-e_\beta\mathbf{A}_\beta]\cdot\mathbf{S}_{\alpha\beta})T_{\alpha\beta}{}^{(0)} +\frac{ie}{\hbar}(\boldsymbol{\varrho}_{\alpha\beta}\times\mathbf{p}_{\alpha\beta}\cdot\mathbf{S}_{\alpha\beta})T_{\alpha\beta}{}^{(0)}\left(\frac{\tau_{\alpha\beta}}{2}\right)\times\int_{\rho_\alpha}^{\rho_\beta}A_s ds+\text{H.c.}\right\}J_{\alpha\beta}{}^{(v)} \tag{A-22}$$

$$U_2=\frac{e}{2\hbar^2c^2}\sum_{\alpha,\beta}\left\{i(\boldsymbol{\varrho}_{\alpha\beta}\times[e_\alpha\mathbf{A}_\alpha-e_\beta\mathbf{A}_\beta]\cdot\mathbf{S}_{\alpha\beta})T_{\alpha\beta}{}^{(0)}\left(\frac{\tau_{\alpha\beta}}{2}\right)\int_{\rho_\alpha}^{\rho_\beta}A_s ds -\frac{e}{2\hbar}(\boldsymbol{\varrho}_{\alpha\beta}\times\mathbf{p}_{\alpha\beta}\cdot\mathbf{S}_{\alpha\beta})T_{\alpha\beta}{}^{(0)}\left(\frac{\tau_{\alpha\beta}}{2}\right)^2\times\left(\int_{\rho_\alpha}^{\rho_\beta}A_s ds\right)^2+\text{H.c.}\right\}J_{\alpha\beta}{}^{(v)}, \tag{A-23}$$

where $T_{\alpha\beta}{}^{(0)}$ is obtained from $T_{\alpha\beta}$ by setting $\mathbf{A}=0$.

The 2^l-pole interaction magnetic moments are obtained in the usual way from Eq. (B-26). For the non-exchange interactions

$$\Delta_v\mathbf{M}_l=\frac{el}{\hbar c(l+1)!}\sum_{\alpha,\beta}(\boldsymbol{\kappa}\cdot\boldsymbol{\varrho}_\alpha)^{l-1}[\boldsymbol{\varrho}_\alpha\times(\mathbf{S}_{\alpha\beta}\times\boldsymbol{\varrho}_{\alpha\beta})]\frac{1-\tau_\alpha}{2}T_{\alpha\beta}J_{\alpha\beta}{}^{(v)}. \tag{A-24}$$

For the exchange interactions

$$\Delta_{vx}\mathbf{M}_l=\frac{el}{2\hbar c(l+1)!}\sum_{\alpha,\beta}\left\{[\boldsymbol{\varrho}_\alpha\times(\mathbf{S}_{\alpha\beta}\times\boldsymbol{\varrho}_{\alpha\beta})]\frac{1-\tau_\alpha}{2} -\frac{i}{l\hbar}(\boldsymbol{\varrho}_{\alpha\beta}\times\mathbf{p}_{\alpha\beta}\cdot\mathbf{S}_{\alpha\beta})\frac{(\boldsymbol{\varrho}_\alpha\times\boldsymbol{\varrho}_\beta)}{(\boldsymbol{\kappa}\cdot\boldsymbol{\varrho}_{\alpha\beta})}(\boldsymbol{\kappa}\cdot\boldsymbol{\varrho}_\alpha)\frac{\tau_{\alpha\beta}}{2}\right\}\times(\boldsymbol{\kappa}\cdot\boldsymbol{\varrho}_\alpha)^{l-1}T_{\alpha\beta}{}^{(0)}J_{\alpha\beta}{}^{(v)}+\text{hermitian conjugate}. \tag{A-25}$$

The change in the sum rule is again to be obtained from Eq. (B-35), with U_2 given by Eq. (A-21) or Eq. (A-23). For the non-exchange interactions:

$$\Delta_v\Sigma_n f_{jn}{}^l=0. \tag{A-26}$$

For the exchange interaction

$$\Delta_{vx}\Sigma_n f_{jn}{}^l=\frac{M}{\hbar^2(l!)^2}\Big\langle\sum_{\alpha\beta}\Big\{i(\boldsymbol{\varrho}_{\alpha\beta}\times[\mathbf{u}(\boldsymbol{\kappa}\cdot\boldsymbol{\varrho}_\alpha)^{l-1} +(l-1)\boldsymbol{\kappa}(\mathbf{u}\cdot\boldsymbol{\varrho}_\alpha)(\boldsymbol{\kappa}\cdot\boldsymbol{\varrho}_\alpha)^{l-2}]\cdot\mathbf{S}_{\alpha\beta})\frac{1-\tau_\alpha}{2} -\frac{1}{2\hbar}(\boldsymbol{\varrho}_{\alpha\beta}\times\mathbf{p}_{\alpha\beta}\cdot\mathbf{S}_{\alpha\beta})(\mathbf{u}\cdot\boldsymbol{\varrho}_\alpha)(\boldsymbol{\kappa}\cdot\boldsymbol{\varrho}_\alpha)^{l-1}\frac{\tau_{\alpha\beta}}{2}\Big\}\times\{(\mathbf{u}\cdot\boldsymbol{\varrho}_\alpha)(\boldsymbol{\kappa}\cdot\boldsymbol{\varrho}_\alpha)^{l-1} -(\mathbf{u}\cdot\boldsymbol{\varrho}_\beta)(\boldsymbol{\kappa}\cdot\boldsymbol{\varrho}_\beta)^{l-1}\}\frac{\tau_{\alpha\beta}}{2}T_{\alpha\beta}{}^{(0)}J_{\alpha\beta}{}^{(v)}+\text{H.c.}\Big\rangle_{jj}. \tag{A-27}$$

The determination of the hermitian conjugate operator, indicated in the foregoing equations, is greatly facilitated by the relationships

$$T_{\alpha\beta}{}^{(0)}\tau_\alpha=\tau_\beta T_{\alpha\beta}{}^{(0)}$$

and

$$\tau_{\alpha\beta}T_{\alpha\beta}{}^{(0)}=-T_{\alpha\beta}{}^{(0)}\tau_{\alpha\beta}.$$

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[22] Note that Eq. (A-16) differs from the expression given in reference 9 in that the vector $\boldsymbol{\varrho}_\pi\times\boldsymbol{\varrho}_\nu$ has replaced $\mathbf{r}_\pi\times\mathbf{r}_\nu$. However, reference 9 is concerned only with the expectation value of the magnetic moment operator and the expectation value of the difference of the two operators can easily be seen to vanish.

[23] The statement by Marshall and Guth (reference 8) that the correction vanishes for $l=2$ appears to be inconsistent with Eq. (A-17).

[24] L. Eisenbud and E. P. Wigner, Proc. Nat. Acad. Sci. 27, 281 (1941).

[25] These expressions are, with one exception, ordered in the same manner as in reference 13 and numbered accordingly. (III′) differs from (3) of that reference by the elimination of the non-exchange part.