Low Energy Electron Resulting from a Stopped μ-Meson*

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IN the course of experiments in this laboratory, using a cloud chamber expanded at random, we have incidentally observed in approximately 10,000 expansions a stopped μ-meson resulting in the emission of a negative electron with an energy of 70 kev. The electron track is unfortunately so faint in both stereoscopic views that it does not lend itself to reproduction. A total of three stopped mesons were observed, of which only one was accompanied by the emission of a charged particle. The cloud chamber was horizontal, filled with 0.6 atmos helium and 0.4 atmos argon, and had a diameter of 24 cm and an illuminated region 3 cm deep. The magnetic field strength was 350 gauss, so that no statement can be made concerning the charge of any one of the stopped mesons.

The emission of electrons of such a low energy accompanying the stoppage of mesons could not have been observed in cloud-chamber experiments designed to study μ-meson decay, since magnetic fields of the order of 10,000 gauss have been used in investigations of this kind.

It seems to us worth while to report our finding in view of the frequent appearance of electrons of energies between 10 and 60 kev at the end of μ-meson tracks in photographic emulsions reported, among others, by Frey,[1] who interpreted them as atomic electrons ejected from the heavy elements in the emulsions during the capture of the meson. In the case of the evidence obtained from photographic emulsions there might, however, exist an alternative explanation for the appearance of at least a part of these electrons; namely, that they are internal conversion electrons or beta-decay of radioactive isotopes formed by the capture of μ-mesons in bromine or silver.

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[1] W. F. Frey, Phys. Rev. 79, 893 (1950).

Production of Highly Polarized Neutron Beams by Bragg Reflection from Ferromagnetic Crystals

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THE theory of magnetic scattering of neutrons, as given for instance by Halpern and Johnson,[1] predicts a polarization in the neutrons scattered by a lattice containing aligned atomic magnetic moments. For the particular case where the magnetization vector is perpendicular to the plane of scattering, the intensity of scattering for the two neutron spin states, respectively parallel and antiparallel to the magnetic field, is given as

$$I_1=b(C-D)^2$$
$$I_2=b(C+D)^2,$$

where C is the nuclear scattering amplitude, D is the magnetic scattering amplitude, and b is a proportionality constant. The greatest difference between I_1 and I_2, and hence the highest polarization, will be obtained when $C=D$, for which there will be scattered intensity for only a single spin state. This situation of balanced nuclear and magnetic amplitudes has been found in the (220) reflection of Fe_3O_4 described earlier,[2] for which $C=0.95\cdot10^{-12}$ cm and $D=0.97\cdot10^{-12}$ cm, and hence a very high polarization would be expected in this reflection.

We have looked for this polarization and find it to be indeed very high. A thin slice of Fe_3O_4 approximately one square centimeter in area and 0.1 cm in thickness was cut along the (220) planes in a natural, single crystal of magnetite. This slice was magnetized in the gap of a permanent magnet ($H=4500$ oersteds), and a monochromatic beam of neutrons ($\lambda=1.204$A) was reflected from the (220) planes with the crystal slice set in transmission orientation and with the magnetization vector perpendicular to the plane of scattering. The degree of polarization in the reflected beam was determined by passage through an analyzing block of polycrystalline iron which could be magnetized with a field of 8000 oersteds in the gap of an electromagnet. Single transmission measurements were taken of the analyzing block with analyzing field off and on (with field always parallel to the polarizing field in order to avoid any depolarization of the neutron beam in the intervening space) for both the polarized beam from Fe_3O_4 and an unpolarized beam from a copper crystal. These measurements permit evaluation of the degree of polarization after allowance for depolarization effects in the analyzing block according to formulas of Halpern and Holstein.[3] Analysis of the data showed the polarization in the (220) Fe_3O_4 reflection to be 100 percent within the experimental uncertainty of perhaps 5 percent. This means that the relative intensities of the two neutron spin states are in ratio at least 40 to 1.

Other crystal reflections are of interest as possible polarizing reflections. Some reflections from Co have very favorable amplitudes for this purpose; but this material is difficult to magnetize, and there may result internal depolarization of the beam. The (110) reflection from an Fe crystal is not too favorable for polarization purposes, since by calculation the expected polarization is only about 60 percent. We have studied this reflection with an Fe crystal (5 percent silicon) in the same fashion as for Fe_3O_4 above and find it to be about 41 percent polarized. This value, which is lower than calculated, could be explained on the basis of extinction effects, depolarization, or silicon impurity within the crystal lattice.

The above Fe_3O_4 (220) reflection is also interesting because *the polarization direction is just reversed from that obtained in the (110) Fe reflection.* This shows up in the sign of the single transmission effect in the analyzing block and results because only Fe^{+++} ions at tetrahedral positions in the magnetic lattice contribute to the intensity in the (220) Fe_3O_4 reflection. The iron atoms at the tetrahedral positions are coupled antiferromagnetically to those at the octahedral positions; and since the latter are in the majority and consequently will be aligned in the applied field direction, the tetrahedral ions will be aligned antiparallel to the external field. The polarization observation constitutes direct proof of the antiferromagnetic nature of the Fe_3O_4 lattice. Other Fe_3O_4 reflections, also highly polarized, are normal with respect to showing polarization parallel to the applied field direction.

It is possible by this method to produce a collimated beam of monochromatic, completely polarized neutrons with an intensity of about 10^6 neutrons/sec. The beam can also be pulsed as suggested earlier.[2]

[1] O. Halpern and M. H. Johnson, Phys. Rev. 55, 898 (1939).
[2] Shull, Wollan, and Strauser, Phys. Rev. 81, 483 (1951).
[3] O. Halpern and T. Holstein, Phys. Rev. 59, 960 (1941).

Thermal Expansion at Low Temperatures

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IN the customary derivation of the Gruneisen[1] relation between the thermal expansion and the atomic heat of an isotopic solid, namely,

$$3\alpha V/\chi=\gamma(C_V)_D, \quad \text{where} \quad \gamma=-d(\ln\theta)/d(\ln V), \qquad (1)$$

one neglects the temperature dependence of the kinetic energy of the conduction electrons. It is fairly obvious that this is invalid at low temperatures as the electronic specific heat of the solid at these low temperatures becomes comparable with the Debye atomic heat.

We have

$$3\alpha/\chi=-\partial/\partial T[(\partial\Phi/\partial V)_T], \qquad (2)$$

where Φ is the free energy of the system. In order to take into account the contribution of the conduction electrons at low temperatures, we regard the electrons as a Fermi gas and include the free energy thereof in the free energy of the whole system. We regard the contributions from the ionic lattice and the electron gas as independent.

The total free energy of the system Φ is given by

$$\Phi(V, T)=\Phi_0(V)+\Phi_D(V, T)+\Phi_e(V, T) \tag{3}$$

and consequently

$$\frac{3\alpha}{\chi}=-\frac{\partial}{\partial T}\left(\frac{\partial\Phi_D}{\partial V}\right)_T-\frac{\partial}{\partial T}\left(\frac{\partial\Phi_e}{\partial V}\right)_T. \tag{4}$$

The first term on the right-hand side leads to the usual Debye term $\gamma(C_V)_D/V$. Now to evaluate the second term, we must know the free energy of the electron gas. The free energy Φ_e of the electron gas is given[2] by

$$\Phi_e(V, T)=NkT\mu-\Omega, \tag{5}$$

where

$$\Omega=-kTZ=-V\int_0^\infty g(\epsilon)\ln(1+e^{\mu-\epsilon/kT})d\epsilon. \tag{6}$$

Here $g(\epsilon)$ is the density of electron states per unit volume of the metal as a function of the energy and μ is essentially the thermodynamic potential in the Fermi-Dirac distribution function

$$f(\epsilon)=\frac{1}{\{\exp[(\epsilon/kT)-\mu]+1\}}, \quad \text{and} \quad N=\partial Z/\partial\mu. \tag{7}$$

From (6) and (7), we have

$$-\frac{\partial}{\partial T}\left(\frac{\partial\Phi_e}{\partial V}\right)_T=-\frac{\partial}{\partial T}(kTZ/V) \tag{8}$$

Defining $G(\epsilon)=\int_0^\epsilon g(\epsilon)d\epsilon$,

$$kTZ/V=\int_0^\infty g(\epsilon)\ln(1+e^{\mu-\epsilon/kT})d\epsilon=\int_0^\infty G(\epsilon)fd\epsilon. \tag{9}$$

Now[3]

$$\begin{aligned}\frac{\partial}{\partial T}\left(\frac{\partial\Phi_e}{\partial V}\right)_T&=-\frac{\partial}{\partial T}(kTZ/V)=-\frac{\partial}{\partial T}\int_0^\infty G(\epsilon)fd\epsilon\\&=G(\mu')(\pi^2k^2T/3)g'(\mu')/g(\mu').\end{aligned} \tag{10}$$

The electronic specific heat per gram atom of the metal

$$(C_V)_e=V\frac{\partial}{\partial T}\int_0^\infty \epsilon g(\epsilon)fd\epsilon=\tfrac{1}{3}\pi^2k^2Tg'(\mu')V\mu'. \tag{11}$$

Expressing (10) in terms of (11), we are led to

$$-\frac{\partial}{\partial T}\left(\frac{\partial\Phi_e}{\partial V}\right)_T=-\frac{\partial}{\partial T}(kTZ/V)=\frac{1}{V}\frac{d(\ln\mu')}{d[\ln G(\mu')]}(C_V)_e. \tag{12}$$

Finally we have the modified Gruneisen relation as

$$\frac{3\alpha V}{\chi}=\gamma(C_V)_D+\frac{d(\ln\mu')}{d[\ln G(\mu')]}(C_V)_e. \tag{13}$$

In the free electron case (alkali metals) $g(\epsilon)=A\epsilon^{\frac{1}{2}}$ and (13) reduces to

$$3\alpha V/\chi=\gamma(C_V)_D+\tfrac{2}{3}(C_V)_e. \tag{14}$$

Equation (13) can also be written as

$$3\alpha V/\chi=\gamma(C_V)_D+(aNT/\mu'), \tag{15}$$

where a is a numerical factor emerging from the reduction of the second term in (13).

The electronic specific heat at low temperatures is $(C_V)_e=\frac{1}{3}\pi^2k^2TVN(\mu_0)$; and so from (12) above, we expect a similar proportionality to T, also obvious from (14), for the thermal expansion provided we can neglect the temperature variation of χ. On account of the second term of (13) the thermal expansion of a metal at liquid helium temperatures falls more slowly than expected from the behavior at moderate temperatures; and, therefore, the measurement of α seems to be plausible even at very low temperatures. In the case of transition metals which show a large electronic specific heat or have a low thermodynamic potential (μ'), the effect of the conduction electrons on the thermal expansion should be considerable. Keesom and Kurrelmeyer[4] found from their experiments on α-iron, the relation

$$C_V=0.60\times10^{-3}RT+2.36\times10^{-6}RT^3. \tag{16}$$

We readily see that $(C_V)_e>(C_V)_D$ and the two become equal at about 15°K, so that the effect of the conduction electrons should be noticeable around this temperature.

It is a pleasure to thank Professor H. Brooks for his suggestions and illuminating discussions.

[1] Gruneisen, *Handbuch der Physik*, Vol. X.
[2] E. C. Stoner, Phil. Mag. **21**, 145 (1936).
[3] F. Seitz, *Modern Theory of Solids* (McGraw-Hill Book Company, Inc., New York, 1940).
[4] W. H. Keesom and Kurrelmeyer, Physica **6**, 364 (1939).

The Directional Distribution of Photo-Electrons Ejected by Polarized Quanta*

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AN earlier Letter[1] reported an azimuthal asymmetry in the directional distribution of photo-electrons ejected coincidentally by annihilation radiation. If the cross-polarization of the quanta is accepted on the basis of previous theoretical and experimental results,[2] then these data bear upon the directional distribution of photo-electrons produced by polarized quanta. Previous data[3] in this regard were for low energy photons (<40 kev) and involved a distinction between polarized and unpolarized photons by somewhat indirect means.

In order to interpret the results in the present case, it is necessary to carry through a computation of the expected coincidence asymmetry for cross-polarized quanta, employing some distribution function for photo-electrons ejected by polarized photons. For the nonrelativistic case this distribution is of the form,[4]

$$J_1(\theta, \phi)=A+B(\theta)\cos^2\phi, \tag{1}$$

where ϕ is the angle between the plane containing the beam of photons and the electric vector and the plane containing the beam and the direction of the photo-electron, and θ is the angle between the direction of the beam and that of the photo-electron. The second member, containing the factor $\cos^2\phi$, arises principally from photo-electrons ejected from the K and L_1 levels, and the first from those from higher levels. The asymmetry with respect to ϕ is involved in the experiment under consideration.

From such a distribution the coincidence rate expected for cross-polarized annihilation quanta and for ideal geometry can be obtained following the method of Snyder, Pasternack, and Hornbostel[5] in their consideration of the experiment where the scattered quanta are detected. By analysis of each beam into two orthogonal components of polarization, one finds for the total coincidence rate for fixed θ_1 and θ_2, and for relative azimuthal orientation of the counters, $\phi_2-\phi_1$,

$$J_2(\phi_2-\phi_1)=2(A^2+AB)+B^2\sin^2(\phi_2-\phi_1). \tag{2}$$

For the case of finite counter geometry, one must integrate with respect to θ_1, θ_2, ϕ_1, and ϕ_2 over the finite spans of the counters.

For annihilation radiation, however, relativistic expressions for A and B are required in Eq. (1). This calculation has been performed for K-shell electrons by Sauter[6] employing the Dirac theory. However, even without correction for finite geometry, his result cannot be reconciled with the data reported in reference 1, nor does it seem to be physically reasonable.

For $\theta=\pi/2$, Sauter's result for the K-shell electron distribution may be written,

$$\begin{aligned}J_1(\tfrac{1}{2}\pi, \phi)=\text{const}\Bigg[\cos^2\phi\Bigg\{(1-\beta^2)^{\frac{1}{2}}-\frac{1-(1-\beta^2)^{\frac{1}{2}}}{2(1-\beta^2)^{\frac{1}{2}}}\Bigg\}\\+\tfrac{1}{4}\left(\frac{1-(1-\beta^2)^{\frac{1}{2}}}{(1-\beta^2)^{\frac{1}{2}}}\right)^2\Bigg].\end{aligned} \tag{3}$$