On the Density Spectrum and Structure of Extensive Air Showers*†

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Experiments on the particle density spectrum and structure of extensive air showers were conducted in the Arctic regions by means of a G-M counter arrangement which allowed simultaneous measurements on showers of high and low densities. The mean densities investigated ranged from about 40 to 1000 particles/m^2. The integral density spectrum of shower particles at sea level was found to be $N(>\rho)=562\rho^{-1.4}$ hr^{-1} (ρ in particles/m^2). With this spectrum it was possible to satisfy all the experimental data obtained with arrangements of various counter areas and multiplicities of coincidences. The decoherence curve of two counters, where extensive showers were first selected by a master coincidence, was found to be flat for small separations down to 7 cm, in disagreement with the results of other workers who did not select extensive showers. This result provides evidence against the existence of a large number of "cores" distributed throughout the extent of the shower.

No latitude dependence of extensive showers was found between geomagnetic latitudes $\lambda=50°$ and $\lambda=80°$ to 90°.

I. INTRODUCTION

EXTENSIVE air showers (Auger showers) are ascribed to the very high energy component of the cosmic radiation (10^{13}–10^{17} ev), and manifest themselves by blanketing areas of the order of 100 m radius with thousands, and even millions, of time-correlated particles. Their discovery by Auger[1] and his collaborators first provided evidence for the existence of cosmic-ray primary particles possessing nearly macroscopic energies (10^{17} ev is about 4 millicalories). In their initial experiments, Auger and his collaborators obtained estimates of the density and energy of air showers, and also described their principal features: the large number of particles, predominantly electrons produced in a cascade process; the existence of a heavy concentration of particles in the center of the shower forming the shower core; the penetrating quality of some of the shower particles.

The high energies of the shower-originating primary particles make the subject of considerable interest from the astrophysical point of view. Their presence must be explained by any successful theory of the origin of cosmic rays. It is also of importance to establish whether there exists an *upper limit* to the energy of cosmic-ray particles. It has been suggested[2] that our galaxy has a magnetic field of the order of 10^{-12} to 10^{-9} gauss extending over a distance of 10,000 parsecs. If cosmic rays originate in the galaxy, then there exists a critical energy (10^{13}–10^{16} ev) below which recirculating cosmic-ray particles will have a finite chance of hitting the earth. Particles of energy higher than critical will escape from the galaxy and hence will not be detected.

In practically all cosmic-ray studies the question of the existence or absence of a *geo*magnetic effect is of fundamental interest since it provides a means of establishing the ratio of momentum to charge of the primary particles responsible for the phenomenon under study. The normal latitude effect displayed by the total cosmic radiation, which is caused by the geomagnetic cutoff at lower latitudes of particles with energies in the range of 10^{10} ev or less, is not expected to be observed for extensive air showers. However, the possibility has been pointed out[3] of observing a latitude effect at high latitudes produced by a cutoff (due to radiation loss in the earth's magnetic field) at the high energy end of the primary particle spectrum (10^{17} ev).

Of great physical interest are studies which bear on the mode of origin of the extensive showers in the upper portion of the atmosphere. The complete description of their initiation process constitutes perhaps the central problem in the study of showers. The application of experimental techniques to this problem involves the study of certain properties of showers at relatively low altitudes sufficiently far removed from their place of origin so they can be termed "extensive." To allow discrimination between various types of proposed initiation mechanisms, a rather high degree of precision is necessary in certain of these measurements in view of the extrapolation involved in applying data obtained at low altitudes to processes occurring near the top of the atmosphere.

To gain more information on some of the problems just mentioned, it was decided to carry out a shower experiment along with other investigations conducted in connection with a U. S. Navy expedition into the Arctic during the summer of 1947. The following objectives were to be accomplished:

(1) A very careful absolute redetermination of the particle density spectrum of extensive air showers at sea level, and the extension of these measurements to a wider range of densities.

* Supported by the Navy Bureau of Ordnance.

† Based on a dissertation submitted to Princeton University, June, 1948, in partial fulfillment of the requirements for a Ph.D. degree.

[1] Auger, Maze, and Grivet-Meyer, Compt. rend. **206**, 1721 (1938).

[2] H. Alfvén, Z. Physik **107**, 579 (1937); L. Spitzer, Jr., Phys. Rev. **70**, 777 (1946).

[3] I. Pomeranchuk, J. Phys. (U.S.S.R) **2**, 65 (1940). See also Sec. VI.

(2) Verification of the concept of shower density over a variety of experimental arrangements to help resolve discrepancies in earlier work of other investigators.

(3) A new determination of the decoherence curve of two counters for small separations in order to gain more information about the structure of showers.

(4) Investigation of the existence of a latitude effect; in particular the "inverse" latitude effect suggested by Pomeranchuk.[3]

Experimentally, showers can be detected by coincidence counts between Geiger-Müller counters, generally located in a horizontal plane and separated by several meters, but so disposed that they cannot all be set off by a single particle. Care must be taken to insure that there is a minimum of material above the counters to avoid the effects of "local showers" which may be produced in the material by single particles. In the present experiments these precautions were carefully observed, the efficiencies and effective areas of the G-M counters used were measured in separate experiments, and considerable effort was spent in making the whole system completely reliable in order to assure an accurate measurement of the particle density spectrum. An improvement in technique was obtained by designing the equipment to collect data on showers of high and low densities simultaneously through use of a system of master and subsidiary coincidences. Then, in reduction of the data, counters could be considered in various parallel or multifold coincidence arrangements. The results obtained in this manner, in agreement with the findings of other workers, indicate that the *density spectrum* of particles in extensive showers can be expressed quite well in the form of an *inverse power law*. Furthermore, the *same* spectrum satisfies all the data obtained with *different* experimental arrangements. The results of the decoherence experiment clearly favor the existence of a single *central* shower core as opposed to a large number of subsidiary cores *distributed* throughout the extent of the shower. Comparison of the results obtained at $\lambda=50°$ with those obtained in the Arctic regions ($\lambda=80°$–$90°$) show an *absence* of any *latitude dependence*. As far as we can determine there are no comparable results of other investigators in this range of latitudes. The Pasadena group[4] in recent experiments at lower latitudes has similarly found no evidence for geomagnetic effects of extensive showers; their results may be regarded as complementary to ours.

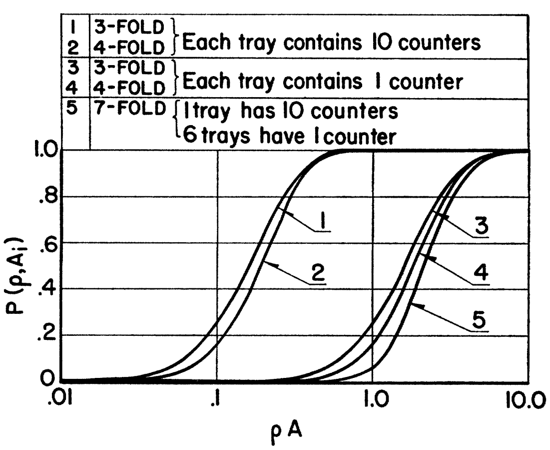

FIG. 1. Probability of obtaining a coincidence *vs* ρA (the number of particles per counter tube area A) for G-M counter arrangements of various areas and multiplicities of coincidence.

II. ANALYSIS OF COUNTER EXPERIMENTS ON SHOWER DENSITY[4a]

The utility of the concept of shower density (particles per unit surface area) is well known[5] and has been applied extensively to the analysis of counter experiments.[6–8] In analogy with the concept of intensity in telescope experiments, it enables one to express results in a manner which does not depend very strongly on the dimensions of the equipment, and thus provides a means of comparing the shower experiments of different workers. The concept is meaningful because (a) extensive showers contain a very large number of particles, and (b) the density does not vary appreciably over the dimensions of the usual experimental set-up (2 to 4 meters), i.e., distances which are a small fraction of the shower radius[9] (75 m at sea level), but large enough to assure that narrow showers which originate in the vicinity of the apparatus are not detected. Point (b) is perhaps subject to criticism if the equipment should be located near the core of the shower where a large density gradient exists; however, it has already been shown experimentally[6] that these events can be neglected. With the assumption of uniform density[9a] in the vicinity of the apparatus subject only to statistical fluctuations, the classical probability argument[5] can be applied to counter experiments. Assuming a shower density ρ particles/m^2, we find that the probability of missing a counter of area A m^2 is $\exp(-\rho A)$. The probability for the counter to be hit by one or more particles is

$$p(\rho, A)=1-\exp(-\rho A). \tag{1}$$

The probability for n counters, of areas A_i, to be set

[4] Biehl, Neher, and Roesch, Phys. Rev. **76**, 914 (1949).

[4a] More precisely: *Local* particle density of an extensive air shower. The expressions "high density shower" and "low density shower" are not strictly correct. It is well known that the concept of "density of a shower" is not meaningful, since particle densities ranging from very low to very high values may be observed in the same shower depending on the distance of the point of observation from the shower core (see Appendix).

[5] C. G. Montgomery and D. D. Montgomery, Phys. Rev. **48**, 786 (1935).

[6] Cocconi, Loverdo, and Tongiorgi, Nuovo cimento **2**, 14 (1944); Phys. Rev. **70**, 841 (1946).

[7] P. Auger and J. Daudin, J. phys. radium **6**, 233 (1945).

[8] G. Cocconi and V. Cocconi-Tongiorgi, Phys. Rev. **75**, 1058 (1949).

[9] Defined as $E_s X_0/\epsilon$, where $E_s=21$ Mev, ϵ=critical energy, X_I=radiation length. See B. Rossi and K. Griesen, Revs. Modern Phys. **13**, 240 (1941).

[9a] The requirement of uniform density can be relaxed if in order to determine the shower density spectrum only the effective areas of the counter trays are changed in a proportional manner. It can be shown that it is then only necessary to assume that all showers—irrespective of the number of particles in the shower—have the same lateral structure function (see Appendix).

off by a shower with average density ρ at the location of the counters is the product of the independent probabilities

$$P(\rho, A_i)=\prod_{i=1}^{n}[1-\exp(-\rho A_i)]. \quad (2)$$

This expression also gives the probability of a coincidence count for a certain counter arrangement due to a shower of density ρ; in Fig. 1 $P(\rho, A_i)$ is plotted as a function of ρ for various combinations of n and A_i.

In general, the equipment will experience particle densities ranging from very low to very high values and occurring with frequencies given by a certain distribution of densities $\nu(\rho)$. The quantity $\nu(\rho)d\rho$ represents the number of showers per unit time with densities between ρ and $\rho+d\rho$ at the point of observation. The coincidence rate is then given as

$$C_n=\int_0^\infty P(\rho, A_i)\nu(\rho)d\rho. \quad (3)$$

Since $P(\rho, A_i)$ is a function which rises from zero to unity over a narrow interval of ρ, while $\nu(\rho)$ is found to be a rapidly decreasing function of ρ, the integrand of (3) shows a rather sharp maximum at a particular value of ρ. Thus, a specific experimental arrangement records predominantly events occurring with a particular value of density.[6,7] An integrand of the form in (3), computed for a certain density spectrum and coincidence arrangement, is shown in Fig. 2.

Integral (3) can be evaluated in closed form in certain special cases. Let the experimental density spectrum be represented by a power law of the form

$$\nu(\rho)d\rho=K\rho^{-\gamma}d\rho. \quad (4)$$

On substitution into (3) we find

$$C_n=\int_0^\infty \prod_{i=1}^{n}[1-\exp(-\rho A_i)]K\rho^{-\gamma}d\rho. \quad (3')$$

Expanding integral (3′) we get

$$C_n=K\int_0^\infty \{1-[\exp(-\rho A_1)+\exp(-\rho A_2)+\cdots] +[\exp(-\rho A_1-\rho A_2)+\exp(-\rho A_1-\rho A_3) +\exp(-\rho A_2\rho-A_3)+\cdots] -[\exp(-\rho A_1-\rho A_2-\rho A_3)+\exp(-\rho A_1-\rho A_2 -\rho A_4)+\cdots]+[\cdots]\}\rho^{-\gamma}d\rho.$$

Even though individual terms of the integral diverge, we can make use of the formula

$$\int_0^\infty x^n \exp(-Ax)dx=A^{-(n+1)}\Gamma(n+1)$$

and evaluate the integral (3′) formally;[10] we find for the rate of coincidences

$$C_n=K\Gamma(1-\gamma)\{-[A_1^{\gamma-1}+A_2^{\gamma-1}+A_3^{\gamma-1}+\cdots] +[(A_1+A_2)^{\gamma-1}+(A_1+A_3)^{\gamma-1}+(A_2+A_3)^{\gamma-1}+\cdots] -[(A_1+A_2+A_3)^{\gamma-1}+(A_1+A_2+A_4)^{\gamma-1}+\cdots]\}. \quad (5)$$

We can apply expression (5) to our experimental results; the coincidence rates C_n are known for different experimental arrangements of counters, and we would like to determine whether the data can be correlated with a power law spectrum of particle densities $K\rho^{-\gamma}$. In other words, can we find a value for γ in (5) which will make the calculated coincidence rates fit the observed rates over the whole range of densities? After γ has been determined, the constant K can be normalized to the most precisely known experimental point.

Having obtained numerical values for K and γ, it is of some interest to determine the mean density of showers $\bar{\rho}$ as recorded by different arrangements of counters, and as defined by

$$\bar{\rho}\equiv C_n^{-1}\int_0^\infty \prod_{i=1}^{n}[1-\exp(-\rho A_i)]K\rho^{-(\gamma-1)}d\rho. \quad (6)$$

It is evident that (6) can be evaluated from expression (5) by substituting $(\gamma-1)$ for γ throughout.

Another arrangement of interest is that of n separate counters of equal area A which experience a shower of density ρ. The probability for particles striking any k counters and missing the remaining $(n-k)$ counters is from (2) and (1):

$$P_n(k)=\{n!/[k!(n-k)!]\} \times[1-\exp(-\rho A)]^k[\exp(-\rho A)]^{n-k}. \quad (7)$$

$P_n(k)$ has been computed for $n=4$ and is plotted as a function of k in Fig. 3 for different values of ρ. In the experiment the densities involved are given by the spectrum (4) selected by a coincidence arrangement

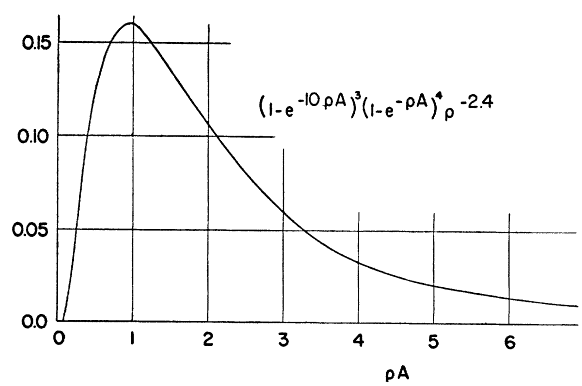

FIG. 2. Differential frequency distribution of particle densities in showers recorded with experimental arrangement ($ABCd^4$) (sevenfold coincidence between trays A, B, and C, each containing 10 counters, and 4 single counters in tray D). The "differential response function" of this particular experimental arrangement has been calculated using the density spectrum $\nu(\rho)\propto\rho^{-2.4}$. Note the sharp maximum corresponding to $\rho A\sim 1$, i.e., for a density of the order of one particle per counter area.

[10] The mathematical justification for this procedure is given by D. Broadbent and L. Jánossy, Proc. Roy. Soc. (London) 192, 364 (1948).

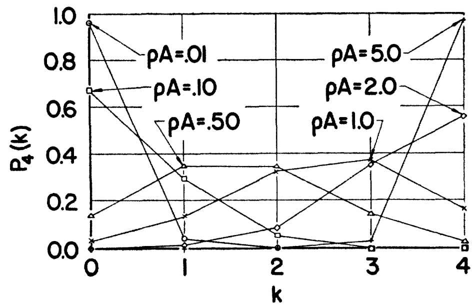

FIG. 3. Probability $P_4(k)$ for setting off a number k ($k=0$, 1, 2, 3, 4) out of 4 counters as a function of k for different average densities of shower particles.

and, therefore, modified by a probability function of the form (2): i.e.,

$$\nu'(\rho)d\rho = P(\rho, A_i)\nu(\rho)d\rho. \quad (8)$$

This expression might be termed the "differential response function" of the particular experimental arrangement described by (2); an example is shown in Fig. 2. If we select showers by means of a set of i counters of areas A_i in coincidence, the rate at which k counters out of n additional counters of equal area A are set off, is then given by the expression

$$C_n(k) = \int_0^\infty P_n(k)\nu'(\rho)d\rho. \quad (9)$$

These integrals can be evaluated by the procedure used to obtain Eq. (5).

The results of this section which are contained in Eqs. (3′), (6), and (9) will be used to interpret the experimental data in terms of a power law spectrum of particle densities and to establish the range of densities over which the power law is valid.

III. EXPERIMENTAL ARRANGEMENT

The arrangement which was used to detect the Auger showers, measure their densities, and gain some information concerning their structure was laid out on the signal deck of the USS Wyandot (AKA 92) as shown in Fig. 4. It consisted of four trays, each containing 10 G-M counters; they were designated, respectively, G-M trays A, B, C, and D, and located at the corners of a 2 m rhomboid. The equipment was designed to gather a large amount of data about individual showers, essentially by sampling each shower at twenty locations by means of twenty G-M counters. By analyzing the data so as to obtain the effects of various counter areas or multiplicities of coincidences, it was possible to determine the frequency distribution of densities $\nu(\rho)$.

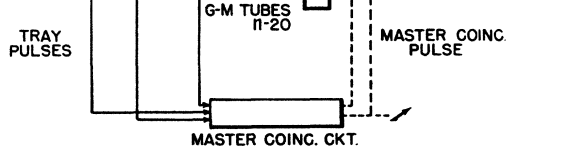

FIG. 4. Diagram of Auger shower experiment showing arrangement of G-M counter trays. Trays A, B, and C produce a master coincidence which is then used to establish subsidiary coincidences with the ten counters of tray A and the ten counters of tray D.

As has been shown by Cocconi, Loverdo, and Tongiorgi,[6] the counting rate of an arrangement of this type does not depend appreciably on the counter tray separation for distances of the order of two to eight meters. As can be seen from Fig. 4, the separation between trays B and C was about four meters. Furthermore, the trays were arranged so that no particle traveling in a straight line could set off more than two counter trays. The trays were housed in individual wooden boxes, whose temperatures were thermostatically controlled. Each tray was placed directly beneath the wooden cover which had a thickness of 0.6 g/cm². No other material was located above the trays.

The equipment operated in the following manner. In G-M trays A, B, and C, the outputs of the G-M counters were connected *in parallel* into the master coincidence circuit. An extensive shower which set off at least one counter in each of the three trays would operate the master coincidence circuit and produce a master coincidence pulse at the output of the circuit. Subsidiary coincidences were then formed between the master coincidence pulse and the twenty *individual* G-M counters of trays A and D. A 20-pen recorder indicated the counters in trays A and D which were set off by the Auger shower.

A detailed description of the experimental arrangement follows:

(a) Each G-M tray held ten self-quenching G-M counters, arranged side by side and touching, so as to form a large rectangular area. The counters were matched as to G-M threshold and flatness of plateau as determined by tests of counting rate *vs* voltage on exposure to a radium source. The threshold was around 960 volts; the counters were operated at 1100 volts. The counters were of the "all-metal" type with 0.8-mm thick copper walls, 0.10-mm tungsten wires, filled with a mixture of 10 cm argon and 1 cm absolute ethyl alcohol. In auxiliary experiments, the effective length of the counters was measured with an arrangement similar to that of Street and Woodward.[11] The active volume of the counters was found to be a cylinder 2.38 cm in diameter and 29.2 cm long, and effective area of 69.5 cm². The efficiency of these counters for cosmic-ray particles was tested in coincidence telescopes and found to be greater than 99.3 percent. The recovery time as measured on a cathode-ray oscilloscope was about 3×10^{-4} sec. No appreciable variation in the counting rate was found for temperatures between 10° and 100°C.

(b) The pulses of each G-M tube were fed into individual cathode followers, and then added into a common tray output by means of a crystal diode mixing circuit (Fig. 5). In the case of trays A and D, the outputs of the cathode followers were also connected to individual coincidence circuits to be described later.

[11] J. C. Street and R. H. Woodward, Phys. Rev. 46, 1029 (1934).

The tray counting rate due to radioactive background and cosmic rays was approximately 16 counts/sec for each of the four trays.

(c) In trays B and C provision was made for changing the tray counting area from its full value (10 counters) to one-tenth of the full value (one counter) by disconnecting nine counters from the high voltage supply.

(d) The master coincidence (MC) circuit (Fig. 6) was a Rossi-type triple coincidence circuit fed from trays A, B, and C. By measuring the accidental coincidence rate of two counter trays when they were exposed to radium sources, the resolving time of the MC circuit was determined to be 3×10^{-6} sec. The bias on the 6AQ5 output tube was set to give complete discrimination against double coincidences. The output (MC pulse) was a shaped negative 80 v square pulse, 8×10^{-6} sec wide, fed from a low impedance circuit to the twenty subsidiary coincidence circuits. To allow rapid testing of the G-M counters and coincidence and recorder circuits of trays A and D, a negative dc voltage ($-22\frac{1}{2}$ v) could be switched into the output of the MC circuit by the Operate-Test switch. It was effectively an artificial MC pulse, which was on continuously. The actual counting rates of the G-M counters of trays A and D were thereby recorded.

(e) Coincidence and recorder circuit trays were attached to G-M trays A and D (Fig. 7). Each had 10 independent sections—one for each G-M tube—consisting of a two-pentode coincidence

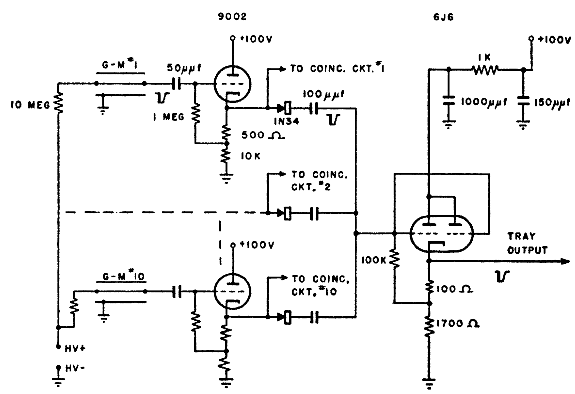

FIG. 5. Circuit for G-M trays A and D. In the circuits for trays B and C the outputs to the subsidiary coincidence circuits are omitted.

circuit and a thyratron driver for the recorder. The grid of the first pentode was connected to the corresponding G-M tube cathode follower in the G-M tray, the grid of the second pentode was fed from the MC circuit. These coincidence circuits had a resolving time of about 10×10^{-6} sec.

(f) An Esterline-Angus 20-pen operations recorder indicated by markings on paper tape the G-M counters set off by a shower. MC pulses were recorded directly through a thyratron driver circuit on the MC circuit chassis by a second tape recorder which was monitoring other experiments conducted on the same expedition.

(g) The resolving time in all coincidence circuits was sufficiently short to allow us to neglect all chance coincidences without further correction. At the same time, it was ample to allow for differences in the arrival time of pulses caused by spurious delays in G-M counters, fixed delays in circuits and cables, and possible differences in rise times of pulses.

For equipment to be used in the field which required stable operation over an extended period of time, reliability, coupled with a simple procedure for operational checks, was a most important requirement. Consequently, the following program was adopted:

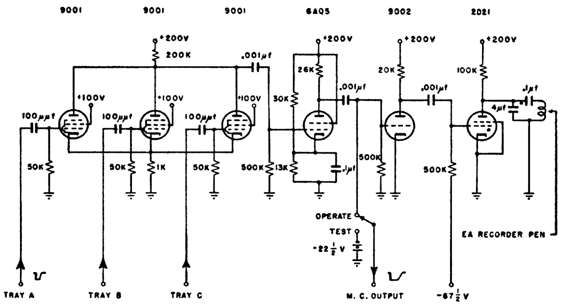

FIG. 6. Master coincidence circuit. With the switch in the TEST position the individual G-M counters and coincidence circuits of trays A and D are automatically checked for proper operation.

(a) In the original design of the circuits a large factor of safety was allowed for, so that variations in supply voltages, aging of vacuum tubes and components, temperature changes, etc., would not affect the proper operation of the experiment.

(b) Over-all checks were made on the system with an artificial pulser to establish the operating ranges of supply voltages and their interdependence. It was found, for example, that outside of this operating range, crosstalk caused by capacitive coupling between adjacent tray circuits would actuate recorder pens; or conversely, pens would fail to be actuated by genuine pulses.

(c) Detailed checks were made at every point of the system with artificial pulser and cathode-ray oscilloscope to establish critical levels of pulse height in the various circuits. With G-M counters active, the G-M pulses were later verified to be well above the critical levels.

(d) G-M counter rates were checked daily by means of a pulse scaling circuit and oscilloscope. A set of simple tests was evolved which were used for rapid trouble shooting but which were also applied daily to check the operation of all circuits and recorders.

(e) The problem of outside interference (electrical pick-up) was solved by proper electrical shielding and by making solid ground connections everywhere, particularly to the G-M counter shells and the negative side of the high voltage supply. Coaxial cable was used for all high voltage and pulse cables. To check absence of electrical pick-up, the high voltage on the G-M counters was reduced from 1100 v to 810 v (150 v below the G-M threshold). No coincidence counts were observed even when severe electrical interference was produced by operating a Tesla coil close by. Runs with reduced voltage were carried out periodically.

During the course of the experiments, all circuits were found to be quite stable in operation and their

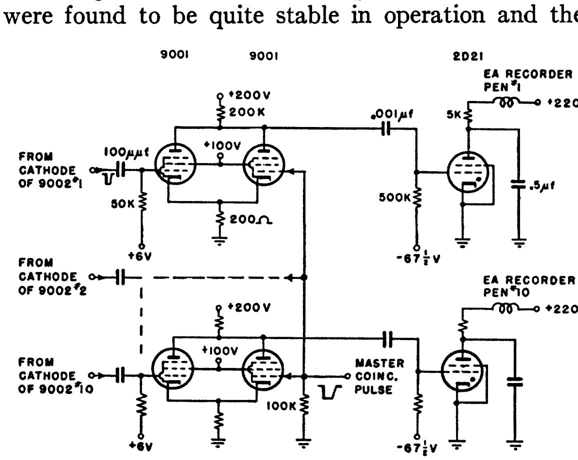

FIG. 7. Subsidiary coincidence and recorder circuits for trays A and D.

TABLE I. Coincidence rates for experimental runs during period July 16–September 24, 1947, at geomagnetic latitude $\lambda=80°–90°$.

1 Run	2 Coincidence arrangement	3 Time (hr)	4 No. of coinc.	5 Threefold coinc. rate (hr^{-1})	6 Fourfold coinc. rate (hr^{-1})
ABC	(*ABC*)	410	5362	13.08±0.18[a]	
	(*ABCD*)	382	3286		8.60 ±0.15
AbC	(*AbC*)	347.5	1072	3.09±0.09	
	(*AbCD*)	345.5	805		2.33 ±0.08
Abc	(*Abc*)	847	893	1.05±0.035	
	(*AbcD*)	847	793		0.936±0.033

[a] All rates are corrected for accidental coincidences, counter efficiency, and counter dead time. All errors quoted are standard deviations assigned only on the basis of the number of counts, i.e., square root of number of counts divided by time.

characteristics did not have to be readjusted. It was not necessary to replace any of the G-M counters.

As a further check on the reasonableness of the results, a plot of the differential frequency distribution of time intervals between showers was prepared for certain portions of the experimental runs. As anticipated, it gave the exponential relation expected from random events.

IV. DENSITY SPECTRUM OF PARTICLES IN EXTENSIVE AIR SHOWERS

The experimental data will now be summarized and applied to the computation of a density spectrum. In 70 days of data collecting, about 7500 master coincidences produced by trays *A*, *B*, and *C* were observed, each accompanied by up to twenty subsidiary coincidences with the counters of trays *A* and *D*. Three types of experimental runs were carried out:[12]

(i) *ABC* Run: Trays *A*, *B*, and *C* each contain 10 counters.

(ii) *AbC* Run: Trays *A* and *C* each contain 10 counters, tray *B* only one active counter.

(iii) *Abc* Run: Tray *A* contains 10 counters, trays *B* and *C* only one active counter each.

Columns 5 and 6 of Table I show the striking change in counting rate when the area of a counter tray is changed, which reflects the rapid fall-off with density of the number of events of a given density. This decrease may be deduced from the consideration that the *ABC* arrangement is quite sensitive to low density events, which would thus appear to be very numerous; it also records all high density events, but their number is comparatively small. For *Abc* runs, however, it is only this small number of high density events which gives rise to the observed coincidences, since the sensitivity of the *Abc* arrangement to the low density events is practically zero (see Fig. 1).

In Table I the threefold coincidences are, of course, the master coincidences. The fourfold coincidences correspond to cases in which at least one counter was struck in tray *D*. A more detailed analysis of the subsidiary coincidences is possible. For this purpose, we select four counters in tray *A* (counters #1, #4, #7, and #10) and four counters in tray *D* (counters #11, #14, #17, and #20). These counters are "spaced," i.e., separated sufficiently, so that the chance for a single particle to set off two of the counters is very small.[13] We can now present some of the subsidiary coincidence data by giving the rates at which k out of 4 spaced counters in trays *A* and *D* are set off: $k=0$, 1, 2, 3, 4.

It is of interest to note the rapid decrease—in passing from *ABC* to *AbC* and then to *Abc* runs—in the counting rate for events in which none of 4 counters are hit. At the same time we notice the very slight decrease in the rate for all out of 4 counters going off in the event of a shower. Another interesting fact is apparent

TABLE II. Rates with which k out of 4 spaced counters are set off in trays *A* and *D* for *ABC*, *AbC*, and *Abc* runs.

1 Run	2 Tray	3 Time (hr)	4 Number of counts $k=0$	 1	 2	 3	 4	5 Rates (hr^{-1}) $k=0$	 1	 2	 3	 4
ABC	*A*	317	1554	1769	519	232	140	4.87 ±0.12	5.58 ±0.13	1.64 ±0.07	0.73 ±0.05	0.44 ±0.04
	D	317	2331	1138	446	185	108	7.35 ±0.15	3.59 ±0.11	1.41 ±0.07	0.58 ±0.04	0.34 ±0.03
AbC	*A*	317	267	341	174	96	106	0.84 ±0.05	1.08 ±0.06	0.55 ±0.04	0.30 ±0.03	0.33 ±0.03
	D	317	403	255	161	86	78	1.27 ±0.06	0.80 ±0.05	0.51 ±0.04	0.27 ±0.03	0.25 ±0.03
Abc	*A*	831.5	152	222	146	154	207	0.183 ±0.015	0.267 ±0.018	0.176 ±0.015	0.185 ±0.015	0.249 ±0.018
	D	831.5	204	214	152	137	174	0.245 ±0.017	0.257 ±0.018	0.183 ±0.015	0.165 ±0.014	0.210 ±0.016

[12] Lower case letters indicate *one* active counter in the corresponding tray, upper case letters *ten* active counters.
[13] It will be shown in Sec. V that this assertion is indeed correct.

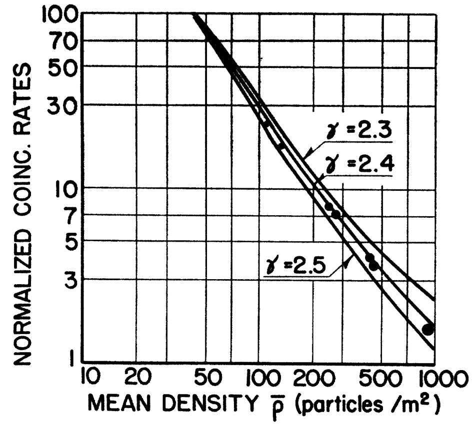

FIG. 8. Normalized coincidence rates *vs* mean density. The coincidence rates of nine experimental arrangements are plotted as circles. The size of the circle indicates the experimental error. These values are compared with the expected coincidence rates computed under the assumption of a differential density spectrum of the form $\nu(\rho)d\rho = K\rho^{-\gamma}d\rho$. It is seen in Table III that the exponent $\gamma = 2.4$ gives the best fit to the experimental points. The mean densities corresponding to the various experimental arrangements have been computed using $\gamma = 2.4$; their use as abscissa serves only the purpose of graphical presentation of the comparison in Table III.

from the data in Table II: the rate for all out of 4 counters is about 70 percent lower than the rate for 3 out of 4 counters in the case of *ABC* runs, about the same in the case of *AbC* runs, and about 20 percent *higher* in the case of *Abc* runs. The explanation, of course, rests on the fact that *Abc* runs, and to a lesser degree *AbC* runs, are sensitive only to high particle densities. The difference in the rates between trays *A* and *D* arises because at least one counter (out of 10) had to fire in tray *A* to produce a shower count, whereas tray *D* did not contribute to the master coincidence.

One can now select from the data coincidence combinations which are sensitive to even higher densities:

(i) (*abc*): Coincidence rate observed with one active counter in trays *A*, *B*, and *C*. This requirement is partially satisfied by *Abc* runs (*Abc*); to obtain the effect of a single counter in tray *A*, we fix our attention on one particular counter of tray *A* out of the four spaced counters and determine the rate at which it is hit by showers selected in *Abc* runs. (We have, of course, the data to get this rate directly; however, the procedure given below is more convenient to apply.) Clearly, if all 4 spaced counters of tray *A* are set off, our chosen counter is fired with a probability of 1; if 3 out of 4 counters fire, the probability for our counter is 3/4; for 2 counters the probability is 1/2, for 1 counter going out of 4, the chance for our counter is 1/4. Applying these weighting factors to the data of Table II, we get for the coincidence rate of three single counters

$$(abc) = 0.547 \pm 0.026/\text{hr}.$$

(ii) (*Abcd*): Using the procedure described above, we consider *Abc* runs and select one counter in tray *D*.

$$(Abcd) = 0.488 \pm 0.024/\text{hr}.$$

(iii) ($Abcd^4$): We consider *Abc* runs and require all four spaced counters in tray *D* to be struck.

$$(Abcd^4) = 0.210 \pm 0.016/\text{hr}.$$

TABLE III. Observed coincidence rates, and coincidence rates calculated using a power law density spectrum of the form $K\rho^{-\gamma}d\rho$. Mean densities $\bar{\rho}$ calculated using spectrum $787\rho^{-2.4}d\rho$ hr^{-1}.

1 Counter combination	2 Obs. coinc. rate (hr^{-1})	3 Obs. coinc. rate (normalized to $(ABC)=100$)	4 Calc. coinc. rate (normalized) $\gamma=2.3$	 $\gamma=2.4$	 $\gamma=2.5$	5 Mean density (No. of particles/m²)
(*ABC*)	13.08 ±0.18	100 ±1.4	100	100	100	45.7
(*ABCD*)	8.60 ±0.15	65.7 ±1.1	70.5	67.5	64.1	63
(*AbC*)	3.09 ±0.09	23.6 ±0.7	24.2	22.6	21.1	118
(*AbCD*)	2.33 ±0.08	17.8 ±0.6	20.2	17.6	16.3	144
(*Abc*)	1.05 ±0.035	8.0 ±0.27	9.0	7.7	6.6	268
(*AbcD*)	0.936±0.033	7.15±0.25	8.4	7.1	5.8	291
(*abc*)	0.547±0.026	4.18±0.20	5.0	4.0	3.2	457
(*Abcd*)	0.488±0.024	3.73±0.18	4.9	3.9	3.1	478
($Abcd^4$)	0.210±0.016	1.60±0.12	2.44	1.76	1.3	975

It is now possible to determine (a) whether the observed coincidence rates can be checked by calculation on the assumption of a power law density spectrum of the form $K\rho^{-\gamma}$, and (b) if such a spectrum exists, what are the values for K and γ.

It can be seen from Table III and Fig. 8, which incorporates the data of Table III, that the differential density spectrum can indeed be represented by a power law expression of the form $K\rho^{-\gamma}$. The calculated coincidence rates C_n were obtained from Eq. (3′) using the following values for the differential spectrum exponent: $\gamma = 2.3$, $\gamma = 2.4$, $\gamma = 2.5$. It is seen that the experimental values (column 3) agree very closely with the $\gamma = 2.4$ results. They differ considerably from the $\gamma = 2.3$ and the $\gamma = 2.5$ results. This comparison indicates the sensitivity of the agreement between experimental and calculated coincidence rates to the value of the spectrum exponent γ. On comparison of the absolute observed coincidence rate per hour with the value of the integral (3′) calculated using $\gamma = 2.4$, we find K to be 787 hr^{-1}, so that finally we obtain as the differential density spectrum of extensive air showers at sea level

$$\nu(\rho)d\rho = 787\rho^{-2.4}d\rho \ \text{hr}^{-1}, \tag{10}$$

where ρ is given in particles/m².

It is now of some interest to calculate with the help of our experimentally determined density spectrum and Eq. (6) the mean particle densities which were recorded with the different experimental arrangements. The results of these calculations are listed in column 5 of Table III.

We next want to make use of a group of data obtained in this experiment, but not employed for the determination of the density spectrum, in order to check the spectrum law independently. Let us consider *ABC* runs, i.e., cases in which at least one counter has been set off in trays *A*, *B*, and *C*. If we fix our attention on the four spaced counters of tray *D* (#11, #14, #17, and #20), we notice that combinations of them (i.e., (i) none, (ii) one, (iii) any two, (iv) any three, (v) all four) fire at different rates. These rates are tabulated in column 5 of Table II, where it is seen that they decrease by a factor of more than 20 in going from (i) no counters firing to (v) all counters firing. It is now possible to

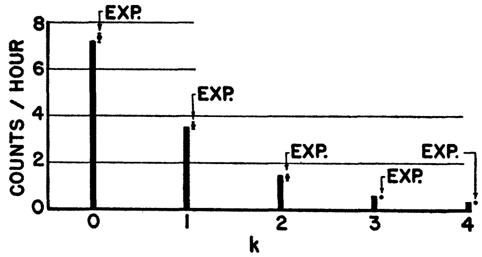

FIG. 9. Rates for setting off k out of four spaced counters (counters #11, #14, #17, and #20 of tray D), when extensive showers are selected by coincidences between trays A, B, and C, each containing 10 counters. The experimental points are as indicated. The calculated rates are shown by the heights of the solid bars; they were computed with the use of the density spectrum $\nu(\rho)d\rho=787\rho^{-2.4}d\rho$, obtained independently through other data of this experiment.

check these rates by calculation. We have earlier derived an expression (9) giving the rate at which k out of n counters go off. It remains now to substitute $n=4$ and to put our observed density spectrum $787\rho^{-2.4}$ hr^{-1} inside the integral. The excellent agreement between observed and calculated values is evident from Fig. 9.

Our result (10) for the particle density spectrum of extensive air showers at sea level and geomagnetic latitude[14] 80° to 90° N can also be expressed in the integral form

$$N(>\rho)=562\rho^{-1.4}\ \text{hr}^{-1}. \qquad (11)$$

Very roughly, this spectrum corresponds to:

~20 events per hour with density greater than 10 particles/m^2;
~ 1 event per hour with density greater than 100 particles/m^2;
~ 1 event per day with density greater than 1000 particles/m^2.

The experimental results can be stated in a different manner by giving the twofold, threefold, and fourfold coincidence rates of an arrangement of counters, each of area A m^2:

$$C_2(A)=1330A^{1.4}\ \text{hr}^{-1},$$
$$C_3(A)=\ 546A^{1.4}\ \text{hr}^{-1},$$
$$C_4(A)=\ 360A^{1.4}\ \text{hr}^{-1}.$$

These expressions are applicable to counters located in a horizontal plane at sea level, separated about 2 to 4 m, and with areas of the order of 0.0010 to 0.0700 m^2.

The results given in this section provide corroboration for the validity of an analysis in terms of a density spectrum of showers over a wide range of densities. Such confirmation of the validity of the density concept tends to increase our confidence in the general picture of an air shower as we think of it now.

[14] And, hence, at any latitude between 50° and 90° N, as the latitude effect is negligible between these regions according to results given in Sec. VI.

V. THE DECOHERENCE CURVE FOR SMALL SEPARATIONS

From the data obtained in the present experiment it is possible to determine the coincidence rate between two counters as a function of their separation. However, we *first select* extensive showers (by means of a master coincidence) and *then* carry out a decoherence experiment.

We again fix our attention on the four spaced counters of tray D (#11, #14, #17, and #20) and consider only cases in which two out of these four counters go off. The twofold coincidence rate of two counters is now determined as a function of their separation in the tray:

(i) 7.6-cm separation (for counters #11 and #14, #14 and #17, #17 and #20).
(ii) 15.2-cm separation (for counters #11 and #17, #14 and #20).
(iii) 22.8-cm separation (for counters #11 and #20).

The results are listed in column 5 of Table IV after applying proper weight factors to the data of column 4.

We can obtain one additional point on the decoherence curve by computing the rate which corresponds to a separation of the order of the extension of the experimental set-up, about 2 to 4 meters. We consider the case in which the three master coincidence trays A, B, and C each have ten active counters (ABC). There are also four single counters connected to subsidiary coincidence circuits (D); the rate with which two particular counters (and only these two) are set off, is just given by:

$$(ABCd_1d_2)=\int_0^\infty\{[1-\exp(-\rho A)]^2[\exp(-\rho A)]^2\}\times\{[1-\exp(-10\rho A)]^3K\rho^{-\gamma}\}d\rho. \qquad (12)$$

On evaluating this expression using $A=0.00695$ m^2 and our experimental spectrum (10) we obtain for $(ABCd_1d_2)$0.24 hr^{-1} for a separation of 2 to 4 m. This is very closely the same as the experimental rate of 0.22±0.02 hr^{-1} for small separations of the order of 10 to 20 cm.

We find then that when extensive showers are *first selected* by a group of counters, the coincidence rate between two additional counters located in the vicinity of the selector—within our experimental error—is independent of their separation for distances of the order of a few meters down to 7 cm. From the observations of other workers[1] in a different region of separations, the coincidence rate can be expected to decrease

TABLE IV. Coincidence rate as a function of counter separation of spaced counters in tray D.

1	2	3	4 No. of coinc. for counter separation of			5 Coinc. rate/hr for counter separation of		
Run	Coinc. arrangement	Time (hr)	(i) 7.6 cm	(ii) 15.2 cm	(iii) 22.8 cm	(i) 7.6 cm	(ii) 15.2 cm	(iii) 22.8 cm
ABC	$(ABCd_1d_2)$	233	155	94	47	0.22 ±0.02	0.20 ±0.02	0.20 ±0.03
AbC	$(AbCd_1d_2)$	227	54	41	21	0.08 ±0.01	0.09 ±0.01	0.09 ±0.02
Abc	$(Abcd_1d_2)$	565	53	30	20	0.031 ±0.004	0.027 ±0.005	0.035 ±0.008
All runs			262	165	88	1.00 ±0.06[a]	0.95 ±0.07[a]	1.01 ±0.11[a]

[a] The coincidence rates averaged over all runs are normalized by a factor 3/262.

for separations large enough so as to constitute an appreciable fraction of a shower radius.

VI. LATITUDE DEPENDENCE

Pomeranchuk[3] has pointed out that primary electrons will lose energy by radiation on being accelerated in the earth's magnetic field. The effect becomes important for electrons coming in vertically in the plane of the geomagnetic equator at energies greater than 10^{17} ev. The radiation loss is zero at all energies for electrons entering along the geomagnetic axis. A variation with geomagnetic latitude could, therefore, exist for showers provided two conditions are satisfied:

(i) There exist in the primary radiation electrons of energy 10^{18} ev and higher. However, it has been pointed out recently[15,16] that electrons traversing intergalactic space cannot maintain an energy greater than about 10^{12} ev because of Compton collisions with photons; primary particles of higher mass would not be latitude-sensitive even at the extremely high energy of 10^{18} ev.

(ii) But even assuming the presence of electrons of energies as high as 10^{18} ev in the primary radiation, the experimental consequences of the radiation loss may not be easily observable. Tzu[17] has shown that the excess energy of these electrons would be radiated in the form of some 600 high energy photons, all of comparable energy, which would give rise to extensive showers at sea level. These showers will be practically superimposed on the original shower and will, therefore, be experimentally indistinguishable from it. The main effect would consist in shifting the shower maximum to a higher altitude. This shift might be difficult to demonstrate experimentally.

On the basis of the foregoing considerations we do not expect to find a latitude dependence, particularly so in view of the fact that only a very small fraction of the events observed in the present experiment are caused by primaries of energy greater than 10^{17} ev. Experimentally, no evidence of a latitude effect was found between 50° and 90°, as shown in Table V, which compares data at 50° with the data of Table I which were obtained almost entirely at geomagnetic latitudes between 80° and 90°. Every effort was made to keep other experimental conditions as nearly identical as possible. Comparison of coincidence rates shows the absence of a dependence on latitude within the limits of experimental error.

VII. DISCUSSION

This section is devoted to relating the results of the present experiment on the particle density spectrum and the structure of showers to the general experimental picture of showers, in particular to the question of what type of process in the upper atmosphere is responsible for the initiation of extensive showers.

Origin of Extensive Showers

It has now been established that 98 to 99 percent of the ionizing particles in extensive showers at low altitudes consist of electrons which have been produced by cascade multiplication. The remainder, the so-called "penetrating component" of showers, is presumably made up of μ-mesons;[18] there is also a small admixture of nucleonic component.[19]

TABLE V. Coincidence rates for experimental runs at Silver Spring, Maryland ($\lambda = 50°$).

1 Run	2 Coincidence arrangement	3 Time (hr)	4 Number of coincidences	5 Coincidence rate (hr^{-1})
ABC	(*ABC*)	145	1881	13.0 ±0.3
Abc	(*Abc*)	339	338	1.00±0.05
From Table I: Coincidence rates at $\lambda = 80°$–$90°$				
ABC	(*ABC*)	410	5362	13.08±0.18
Abc	(*Abc*)	847	893	1.05±0.035

Initially it was believed that Auger showers were originated by high energy primary electrons, the presence of penetrating particles (mesons) being explained by assuming them to be produced by energetic γ-rays. At present, however, there exists direct experimental evidence from cloud chamber measurements,[20] specific ionization,[21] and burst measurements[22] at very high altitudes against the presence of an appreciable fraction of primary electrons in the 10-Bev energy range. The lowest upper limit given is about 1 percent.[20] There is evidence against the existence of primary electrons with energies of 10^{13} to 10^{17} ev, which might lead to production of extensive showers, but the conclusion is indirect[15,16] and the possibility of having a few electrons of such high energies in the primary radiation cannot be completely excluded.

Nevertheless, it is now generally supposed that the electronic component of showers originates in the nuclear interactions of primary protons or heavier nuclei with air molecules in the upper atmosphere. The mesons present in showers can then be explained quite naturally as being produced in the same interaction. There is a good deal of experimental evidence from experiments in the lower atmosphere[23] for the production of electrons and/or photons with mesons.

The interactions observed so far in the experiments cited involve energies very much lower than those encountered in the initiation of air showers; however, they are suggestive of the higher energy processes. The detailed mechanism for the production of the electronic

[15] J. W. Follin, Jr., Phys. Rev. **72**, 743 (1947).
[16] E. Feenberg and H. Primakoff, Phys. Rev. **73**, 449 (1948).
[17] H. Y. Tzu, Proc. Roy. Soc. (London) **192**, 231 (1948).
[18] Cocconi, Cocconi-Tongiorgi, and Greisen, Phys. Rev. **75**, 1063 (1949); W. W. Brown and A. S. McKay, Phys. Rev. **76**, 1034 (1949).
[19] Cocconi, Cocconi-Tongiorgi, and Greisen, Phys. Rev. **74**, 1867 (1948).
[20] Critchfield, Ney, and Oleksa, Phys. Rev. **79**, 402 (1950).
[21] S. F. Singer, Phys. Rev. **76**, 701 (1949).
[22] Schein, Jesse, and Wollan, Phys. Rev. **59**, 615 (1941); R. I. Hulsizer, Echo Lake Conference (June, 1949); J. A. Van Allen, Echo Lake Conference (June, 1949).
[23] B. Rossi, Revs. Modern Phys. **20**, 537 (1948); Bridge, Hazen, Rossi, and Williams, Phys. Rev. **74**, 1083 (1948); C. Y. Chao, Phys. Rev. **75**, 581 (1949); W. B. Fretter, Phys. Rev. **76**, 511 (1949); B. P. Gregory and J. H. Tinlot, Phys. Rev. **79**, 205 (1950).

component is not known, and constitutes one of the outstanding problems of shower theory. Some of the mechanisms considered have been of the electromagnetic type, e.g. γ-rays produced in the deceleration of primary protons near an air nucleus[24] analogous to the bremsstrahlung of electrons, also γ-rays emitted in the acceleration of charged mesons[25] as they are generated in a nuclear interaction. In distinction, processes have been considered which are of the nuclear type. Lewis, Oppenheimer, and Wouthuysen[26] have proposed production of neutral mesons along with charged mesons. The neutral meson is assumed to be extremely unstable, decaying into two γ-rays which then initiate electronic cascades. There appears to be now firm experimental evidence for the existence of neutral mesons from experiments at Berkeley.[27] Independently, a very remarkable star[28] showing the interaction of a primary alpha-particle of about 10^{13} ev with an Ag or Br nucleus in a photographic emulsion, lends much support to this hypothesis for the origin of Auger showers. Bethe[29] has recently put forth a qualitative but very suggestive picture of the development of an extensive shower to fit the present experimental situation; it is based on the neutral meson hypothesis, and incorporates ideas on cascade multiplication of the N-component (nucleons and π-mesons).

Experimental Approach and Difficulties

The present experimental evidence on extensive showers is not sufficient to give a description of the initiating process or even to discriminate between various possibilities. One promising approach seems to be the study of some of the gross properties of extensive showers, the particle densities and structure of showers, as functions of the altitude. For all practical purposes this approach entails a study of the electronic component, which should behave in accordance with cascade theory. A complete analysis of such data on the basis of cascade theory gives in principle some information about the initial multiplicity and angular divergence of the cascade-initiating γ-rays or electrons. However, many factors enter in to complicate the simple picture.

One of the principal difficulties in the interpretation of high altitude data on showers arises from the contribution of nonvertical showers; at sea level, however, the showers detected arrive predominantly from the vertical direction if—as in the present experiment—the counter separations are not very large.[30]

[24] J. L. Powell, Phys. Rev. **75**, 32 (1949).
[25] L. I. Schiff, Phys. Rev. **76**, 89 (1949).
[26] Lewis, Oppenheimer, and Wouthuysen, Phys. Rev. **73**, 127 (1948).
[27] Bjorklund, Crandall, Moyer, and York, Phys. Rev. **77**, 213 (1950).
[28] Kaplon, Peters, and Bradt, Phys. Rev. **76**, 1735 (1949); see also discussion by R. E. Marshak, Phys. Rev. **76**, 1736 (1949).
[29] H. A. Bethe, Echo Lake Conference (June, 1949).
[30] G. Cocconi, Phys. Rev. **72**, 350 (1947); S. F. Singer, Phys. Rev. **74**, 844 (1948). A very strong zenith angle dependence (as great as $\cos^8\theta$) has been deduced for air showers by various observers: J. Daudin, J. phys. radium **6**, 302 (1945); W. W. Brown and A. S. McKay, Phys. Rev. **76**, 1034 (1949); H. L. Kraybill, Phys. Rev. **77**, 410 (1949).

In order to compare experimental data obtained at different altitudes, the results should be expressed in units which are independent of the local air density. Accordingly, the integral density spectrum (11) obtained in the present experiment should be stated as

$$N(>\rho)=1.0\times10^{8}\rho^{-1.4}\ \mathrm{hr}^{-1}, \tag{13}$$

where ρ is now expressed in particles per shower radius[9] squared.

From the experimental density spectrum, we can compute immediately an expression for the frequency distribution of showers as a function of the number of particles in the shower, under the assumption that the particles are distributed in every shower according to a certain structure function (see Appendix). Using this result together with certain expressions from cascade theory,[9] we have carried out a computation for the primary spectrum based on the early model of primary electrons initiating extensive showers at the top of the atmosphere. The results of this computation[31] give a primary spectrum about thirty times larger than that obtained by other workers;[32] the discrepancy arises mainly from the fact that we used a corrected value for the radiation length in air in order to take into account the bremsstrahlung and pair production effects of the atomic electrons.[33]

Comparison of Experimental Results on the Density Spectrum

Figure 10 shows a comparison of some integral density spectra determined from G-M counter and ionization chamber measurements near sea level. The three sets of counter data[6,8] are seen to be in good agreement with one another. Our spectrum is in excellent accord with Cocconi's recent measurements.[8] In agreement with his result, a trend toward a slightly higher value of γ is discernible in our data (in Table III and Fig. 8) for the point corresponding to our highest mean density (about 1000 particles per square meter). This increase in γ is very pronounced at even higher mean densities, which are more accessible to measurement by means of ion chambers. The results of Montgomery and Lapp shown in Fig. 10 were taken from Montgomery's[34] summary paper. The use of high pressure chambers in their burst measurements tends to minimize the effects of stars and local showers, and therefore leads to quite a reasonable value for γ. Nevertheless, a large discrepancy exists between G-M counter and ion chamber results at intermediate shower densities, possibly due to the fact—as pointed out by Cocconi[35]—that ion chambers do not measure particle densities directly but rather total ionization of the shower particles, which includes secondary electrons and possibly nuclear events.

[31] S. F. Singer, thesis, Princeton University, 1948.
[32] A. Migdal, J. Phys. (U.S.S.R.) **9**, 183 (1945); Cocconi, Loverdo, and Tongiorgi, Phys. Rev. **70**, 846 (1946).
[33] J. A. Wheeler and W. E. Lamb, Phys. Rev. **55**, 858 (1939).
[34] C. G. Montgomery and D. D. Montgomery, Phys. Rev. **72**, 131 (1947).
[35] G. Cocconi, Phys. Rev. **70**, 975 (1946).

Significance of the Density Spectrum Exponent γ

It is evident from Table III and Fig. 9 that the same spectrum can be used to compute the coincidence rates of all types of counter arrangements used in our experiments; in other words, the value of γ is the *same* in our experiment whether determined by variation of counter areas or by varying the multiplicity of coincidences. This, we feel, should certainly be the case if our concept of shower density (Sec. II) is correct; it is the result anticipated with the symmetrical arrangement of counter trays used in the present experiment.

Other experimenters who have not found such agreement have accounted for the discrepancy in various ways: (1) The applicability of Poisson's law to the analysis of counter experiments on showers may be doubtful in view of correlations existing between shower particles, according to Daudin.[36] We believe that while a correlation effect certainly exists,[37] it is not of great importance experimentally. This conclusion is drawn from our data on the decoherence curve at small separations (Sec. V). It is rather difficult to draw unambiguous conclusions from the experiments presented by Daudin, since in some cases showers were detected by only a twofold coincidence. (2) Treat and Greisen[38] have expressed the opinion that the discrepancy arises from extraneous causes, such as changes in barometric pressure, etc. Effects of this sort would not have affected our results appreciably since in our master coincidence arrangement essentially both methods for determining γ were carried on simultaneously. (3) Recently, Ise[39] has pointed out that in general one should not obtain a unique value for γ, since even in a counter arrangement of moderate extension most showers detected have their cores located close enough to give an appreciable density gradient over the extension of the equipment. This gradient should not affect the value of γ as determined by variation of counter areas (see Appendix), but would increase the value of γ obtained by adding additional counters for multifold coincidences. No estimate of the magnitude of the effect is given; in any case only an estimate of an upper limit of the increase may be meaningful since the assumptions on which the calculations are based are open to some question (see Appendix). We cannot check this effect by means of our experimental results since we used the additional tray (tray D) in a very symmetrical arrangement and quite close to tray A (Fig. 4). Because of this symmetrical set-up, one would expect that the concept of a distribution-in-density (which neglects the density gradients which actually exist) would be a good first approximation. This is what we find experimentally. On the other hand, Ise's arrangement—consisting of three counter trays in a straight line, with a fourth one at right angles—is much less symmetric, and the assumption of constant density over this arrangement is much less adequate.

In any case, we feel reasonably certain that the concept of shower density can now be considered well confirmed, and that it provides an adequate basis for the expression and comparison of results obtained with different experimental arrangements.

Structure of Extensive Showers

The existence of a large number of shower cores distributed throughout the extent of the main shower has been suggested by some authors,[40] but no clearcut experimental evidence has been brought forth on this point. Recently, this question has assumed added importance because of a theory concerning the origin of extensive showers involving neutral mesons of very short lifetime.[26] According to the authors, the neutral mesons are produced with an angular divergence of $(2M/E_0)^{\frac{1}{2}}$, a divergence sufficiently great to effect appearance of a multiple core structure in the shower, with a core separation—near sea level—of the order of 10 m. Furthermore, the possibility cannot be ruled out *a priori*, that individual penetrating particles have associated with them clusters of electrons (genetically related to them) so as to give a type of "fine structure"

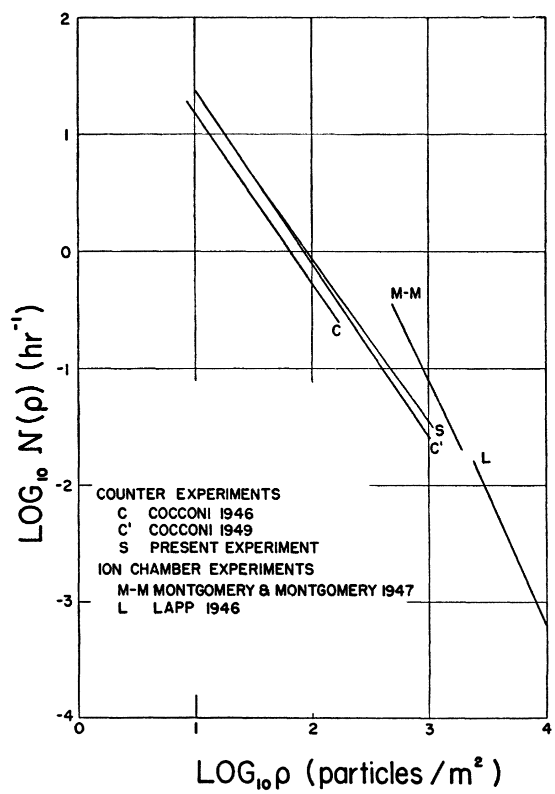

FIG. 10. Integral density spectrum of extensive air showers at sea level. A comparison of results obtained with counters and with thin-walled high pressure ionization chambers.

[36] J. Daudin, J. phys. radium **10**, 65 (1949).
[37] V. Berestetzky, J. Phys. (U.S.S.R.) **9**, 197 (1945).
[38] J. E. Treat and K. I. Greisen, Phys. Rev. **74**, 414 (1948).
[39] J. Ise, Jr., and W. B. Fretter, Phys. Rev. **76**, 933 (1949).
[40] J. Clay, Physica **9**, 897 (1942).

to the well-known distribution function; this structure would be revealed only upon detailed examination.

It is possible to determine how our result of Sec. V, namely *no rise in the decoherence curve for small separations when extensive showers are first selected*, bears on this question of the existence of local condensations of particles or "cores." We will first examine the behavior of the decoherence curve of two counters for small separations where no selection of extensive showers is involved. Experimentally, the coincidence rate has been found to rise very rapidly as the counters are brought together.[41] This increase can be ascribed to a number of causes:

(i) The coincidence rate may rise just due to the effect of the core of extensive showers, as has been pointed out by Blatt.[42] However, the same basic assumptions which occur in Ise's calculations are involved here to an even greater degree and are open to some question (compare Appendix).

(ii) At small separations, the coincidence rate will also be affected by "narrow showers," knock-on processes, and possibly other phenomena not directly connected with extensive showers. This is thought to be the principal cause for the rise in the coincidence rate.[41]

(iii) Furthermore, if a large number of cores exist in a shower, the coincidence rate will rise rapidly as the counter separations decrease to distances of the order of magnitude of the dimension of the cores.

In applying the foregoing considerations to the present experiment, we note first that our results might seem to be in contradiction with point (i) above. Actually they give no information on point (i), since the master coincidence arrangement becomes quite insensitive to showers of very low particle densities; as shown in the Appendix, the cores of the low density showers are the ones which might have been responsible for a rise in the coincidence rate. The requirement of an extensive shower in our decoherence experiment clearly eliminates the possibilities mentioned under (ii). The absence of a rise in our decoherence curve, therefore, points to the absence of a large number of sharp density gradients extending over distances of a few centimeters and *distributed throughout the extent of the shower*.

Ion chamber experiments by Williams[43] have shown the absence of multiple shower cores with separations from a meter to about 30 m. Cocconi, Tongiorgi-Cocconi, and Greisen[44] by means of a "shower-core selector" experiment have found no evidence for subsidiary, multiple cores with separation greater than 10 meters, but rather a monotonic decrease in density with distance from the main shower core. This absence of widely separated cores is in disagreement with the anticipated results of the neutral meson theory as quoted in reference 26. However, it has been pointed out by one of its authors[45] and others that the essential features of the theory would not be affected by assuming that the neutral mesons are produced with a much smaller angular divergence, possibly of the order of M/E_0, analogous to photon pair production.

Taken together, all experimental evidence tends to exclude the presence of local condensations or cores *distributed throughout the extent of the shower*. We cannot, however, exclude the presence of a number of multiple cores—separated by a few centimeters—*located within the main core* of the shower; indeed their existence is to be expected in view of some of the initiation mechanisms proposed.

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APPENDIX

We are interested in computing from our experimental particle density spectrum $\nu(\rho)d\rho$ the distribution $f(\Pi)d\Pi$; $f(\Pi)d\Pi$ can be interpreted as the number of showers per hour, whose centers strike within a horizontal area of one square meter at the place of observation, and which contain a total number of particles between Π and $\Pi+d\Pi$. We will assume that the particles in showers of all sizes are distributed according to the same lateral structure function $\phi(r)$, computed by Molière[46] for the lateral distribution of particles at the plane of the cascade maximum. An analytical approximation to the Molière function has been given by Bethe (and quoted by Williams[43]):

$$\phi(r)=[2.85/(2\pi r_0 r)](1+4r/r_0)\exp[-4(r/r_0)^{\frac{2}{3}}], \qquad (14)$$

where r_0 is the shower radius defined earlier.[9] $2\pi\phi(r)dr$ can be interpreted as the probability for a shower particle to fall between distance r and $r+dr$ from the shower center. From this definition it follows that

$$\rho(r)=\phi(r)\cdot\Pi. \qquad (15)$$

Let us consider now the apparatus which measures shower densities located at the origin of a polar coordinate system. A shower containing Π-particles and centering at a distance r from the apparatus will produce at the origin a particle density given by (15) and (14). A distribution $f(\Pi)$ will, therefore, lead to a distribution of densities

$$\nu(\rho)\Delta\rho=2\pi\int_0^\infty f(\Pi)(\Delta\Pi)r dr, \qquad (16)$$

[41] Alichanian, Asatiani, and Alexandrian, J. Phys. (U.S.S.R.) **9**, 175 (1945); **10**, 296, 518 (1946); **11**, 16 (1947); J. Wei and C. G. Montgomery, Phys. Rev. **76**, 1488 (1949); Keck and Greisen, Echo Lake Conference (June, 1949).

[42] J. Blatt, Phys. Rev. **75**, 1584 (1949).

[43] R. W. Williams, Phys. Rev. **74**, 1689 (1948).

[44] Cocconi, Tongiorgi-Cocconi, and Greisen, Phys. Rev. **76**, 1020 (1949).

[45] J. R. Oppenheimer, private communication; see also p. 136 of reference 26.

[46] G. Molière, *Cosmic Radiation*, edited by W. Heisenberg (Dover Publications, New York, 1946).

where Π must satisfy (15), so that

$$\nu(\rho)\Delta\rho=2\pi\int_0^\infty f[\rho/\phi(r)][\Delta\rho/\phi(r)]rdr. \qquad (16')$$

If we assume a power law distribution for $f(\Pi)$

$$f(\Pi)d\Pi=k\Pi^{-\kappa}d\Pi,$$

we obtain

$$\nu(\rho)\Delta\rho=2\pi k\int_0^\infty [\rho/\phi(r)]^{-\kappa}[\Delta\rho/\phi(r)]rdr$$
$$=2\pi k\rho^{-\kappa}\Delta\rho\int_0^\infty [\phi(r)]^{\kappa-1}rdr. \qquad (17)$$

We can now compare this result with our experimental density distribution $\nu(\rho)d\rho=K\rho^{-\gamma}d\rho$. For agreement κ must equal γ, a result which is seen to be independent of the form of the structure function $\phi(r)$ and depends only on the assumption that the structure function is the same for all showers. The final result for $f(\Pi)$ based on our experimental spectrum $787\rho^{-2.4}$ hr^{-1} is

$$f(\Pi)d\Pi=2.5\times10^4\Pi^{-2.4}d\Pi \text{ hr}^{-1}\text{ m}^{-2}. \qquad (18)$$

When the shower radius[9] is used as the unit of length, Eq. (18) becomes:

$$f(\Pi)d\Pi=1.4\times10^8\Pi^{-2.4}d\Pi \text{ hr}^{-1}(r_0)^{-2}. \qquad (18')$$

In the calculation above it has been assumed implicitly that the dimensions of the experimental arrangement are small enough so that the particle density may be considered as uniform over the extension of the apparatus. In reality, however, the counter trays are separated by an appreciable fraction of a shower radius. The coincidence rate C_n of a set of n Geiger counters of areas A_i and located at distances r_i from the shower core is given by

$$C_n=\int_0^\infty\int_0^\infty\int_0^{2\pi}k\Pi^{-\gamma}d\Pi rdrd\theta\cdot\prod_i\{1-\exp[-\Pi A_i\phi(r_i)]\}. \qquad (19)$$

By making the substitution $\Pi'=\Pi A$, and taking $A_i=a_iA$, we can see immediately that $C_n\propto A^{\gamma-1}$; i.e., the counter areas need not be equal, but if they are changed in a proportional manner, the exponent of the distribution-in-number $f(\Pi)$ can be obtained directly from the coincidence rates C_n.

A numerical integration of expression (19) has been carried out by Ise[39] for a very useful experimental case, namely that of three counter trays of equal area A, located on a straight line with a spacing $a=0.05r_0$. The integral then becomes[39]

$$C_3(A)=\int_0^\infty\int_0^\infty\int_0^{\pi/2}k\Pi^{-\gamma}d\Pi\cdot2\pi rdr\cdot2d\theta/\pi\{1-\exp[-\Pi A\phi(r)]\}$$
$$\times\{1-\exp[-\Pi A\phi(r_1)]\}\{1-\exp[-\Pi A\phi(r_2)]\}, \qquad (19')$$

where $r_{1,2}=(a^2+r^2\pm2ar\cos\theta)^{\frac{1}{2}}$; all distances are expressed in units of r_0.

It is a rather trivial problem to extrapolate these calculations to the case of $a=0$, but comparison of the case $a=0$ with the case $a=0.05r_0$ gives considerable insight into the assumptions underlying these calculations, and is also of importance in the interpretation of decoherence experiments involving small separations. We first note that the probability of a coincidence is a maximum when the center of the shower coincides with the center tray.[47] Ise shows that (for $a=0.05$) this maximum probability is 1.0 for values of ΠA down to about 1.0, and then falls off, the probability being about 0.22 for $\Pi A=0.1$. In our case ($a=0$) the probability remains 1.0 for all values[48] of ΠA. This difference accounts for the discrepancy between the two sets of curves in Fig. 11 for small values of ΠA. There we have plotted the function $I(r)$, obtained through one integration (over θ) of a portion of the integrand of Eq. (19′):

$$I(r)=r\int_0^{\pi/2}\{1-\exp[-\Pi A\phi(r)]\}\{1-\exp[-\Pi A\phi(r_1)]\}$$
$$\{1-\exp[-\Pi A\phi(r_2)]\}d\theta. \qquad (20)$$

[47] The dimensions of the counter trays are neglected in this discussion. This will tend to overemphasize the difference between $a=0$ and the $a=0.05$ plots in Figs. 11 to 13.

[48] This no longer holds true for very small values of Π, as will be discussed later.

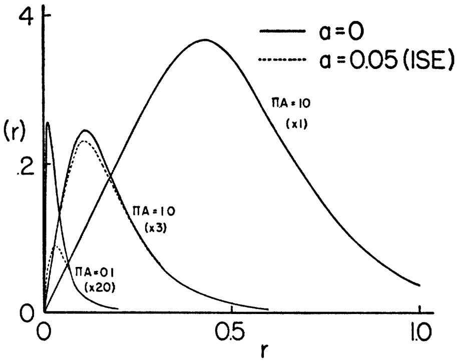

FIG. 11. Probability $I(r)$ of a threefold coincidence due to all showers striking at a distance r from the central counter tray for various values of ΠA.

This function represents the probability of a threefold coincidence due to all showers which strike at a distance r from the central counter for various values of πA.

The next step in the evaluation of Eq. (19′) can be the integration over r, yielding the function

$$R'(\Pi A)=2\pi\Pi^{-\gamma}\int_0^\infty I(r)dr. \qquad (21)$$

A graph of this function *vs* the logarithm of ΠA is shown in Fig. 12 for the two values of a and for $\gamma=2.5$. From the relation

$$R'(\Pi A)=dC_3(A)/d\Pi \qquad (22)$$

it is possible to interpret $R'(\Pi A)$ as the differential distribution with respect to Π of showers which contribute to the coincidence rate (19′). The function $R'(\Pi A)$ can be plotted *vs* ΠA on a linear basis (Fig. 12). The areas under the curves represent the coincidence rates (19′) for $a=0.05$ and $a=0$, respectively.

Instead of integrating over r, the second step in the evaluation of (19′) could also be an integration over ΠA, yielding the function

$$P(r)=\int_0^\infty(\Pi A)^{-\gamma}I(r,\Pi A)d(\Pi A). \qquad (23)$$

A graph of this function *vs* r is shown in Fig. 13. Again the areas under the curves represent the coincidence rates (19′) for $a=0.05$ and $a=0$, respectively. $P(r)$ can be interpreted as the differential distribution with respect to r of showers which contribute to the coincidence rate (19′).

Several interesting conclusions may be drawn from the comparisons shown[49] in Figs. 12 and 13, subject to certain reservations as stated below which involve the assumptions on which these calculations are based:

(i) In going from a tray separation of $0.05r_0$ to zero separation, the coincidence rate increases by not more than about 50 percent. To evaluate this increase precisely, it is definitely necessary to take into account the finite size of the counter trays.[50]

(ii) The counter arrangement with separation $a=0.05$ shows a sharp maximum at $\Pi A=0.04$ according to Ise[39] (Fig. 12), but a very *flat* maximum (Fig. 13) around $r=0.1$ (about 7.5 m at sea level). One is, therefore, not justified in assuming that the centers of most showers detected by counters with moderate separations strike within about 10 meters of the apparatus.

(iii) As anticipated for the case of zero separation, the maximum contribution to the coincidence rate occurs at a much smaller value of r and a very much lower value of ΠA. In other words,

[49] No great accuracy is implied in the curves presented in Figs. 11 through 13.

[50] Qualitatively, the finite size of the counters serves to exclude the extremely small showers, which lead to the very steep rise in the $a=0$ curves, e.g., in Figs. 12 and 13. The effect of the finite dimension of a detector has been considered in the case of ion chambers by Blatt (reference 42).

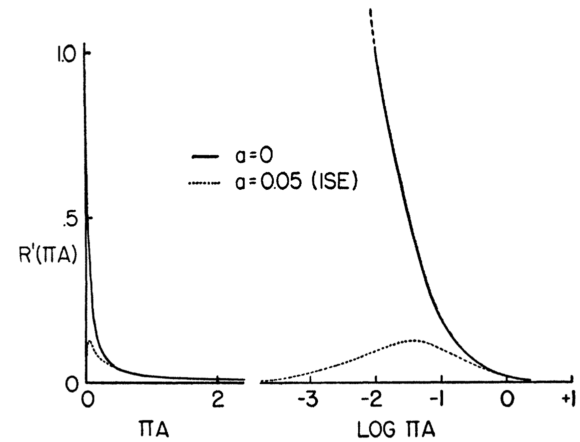

FIG. 12. Contribution to threefold coincidence rate by showers of various total numbers ΠA of electrons striking at all distances from the central counter.

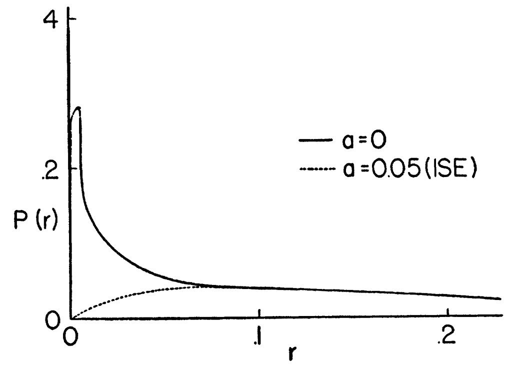

FIG. 13. Contribution to threefold coincidence rate by all showers (taking account of their distribution-in-number) at various distances r of shower core to central tray.

with very small counter separations a large fraction of the coincidence rate is due to showers, containing relatively few particles, whose cores strike a short distance away from the counter arrangement.

(iv) These considerations lead to ideas on the construction of a core selector based purely on the structure of showers. It would consist of two identical sets of counters. The first set of, say, three counters would be very closely spaced, the second set moderately spaced and in anticoincidence with the first set. A combination, such as shown in Fig. 13, would select showers whose cores are about one meter from the center of the counter arrangement. The area A of the counters would determine the mean size $\bar{\Pi}$ of the showers selected.

We finally wish to discuss briefly some of the assumptions on which the above considerations have been based.

(i) With a counter tray area of about $10^{-5}r_0^2$, most of the showers registered by our equipment contain in the neighborhood of $0.04\times10^5\sim4000$ particles (see Fig. 12). A shower with $\Pi\sim4000$ particles at sea-level can be expected to be an "old" shower,[9] i.e., to have passed through its stage of maximum development at a much higher altitude. Its structure *near the core* would not be as steep as given by the Molière function.[51] Furthermore, if Auger showers originate in a multiple process, as seems likely, the structure function would again be much flatter near the core of the shower.

(ii) In addition, we suspect that the distribution-in-number of showers does not follow a pure power law, but becomes much flatter for smaller values of Π (as it certainly must, in order not to exceed the total cosmic-ray intensity). Some evidence for this is already available from shower experiments.[8]

(iii) Finally, for very small values of Π the probability expressions we have been using become inaccurate and break down. This arises from the fact that the probability of a coincidence is no longer the product of the separate probabilities. Effectively, when one counter is hit by a particle, it "robs" the other counters of that particle, decreasing Π and, therefore, their probabilities to be hit.

These points have various degrees of importance, depending on the exact nature of the experimental arrangement (areas, separation, etc.). Broadly speaking, their effect will be to minimize the influence of the shower core; e.g., on the rise of the decoherence curve at small separations,[42] and on the discrepancy of the determination of the density spectrum exponent γ by different methods.[39] The above considerations, we believe, tend to bring experimental results into harmony with theory.

[51] The recent experiments of the Cornell group (reference 44) give information on the structure function only at some distance from the core.