The Capture of π-Mesons in Deuterium

K. BRUECKNER, R. SERBER, AND K. WATSON
Radiation Laboratory, Department of Physics, University of California, Berkeley, California

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Panofsky, Aamodt, and Hadley find that the capture of π-mesons in deuterium leads to two neutron emission in 70 percent of the cases, and to two neutrons and a γ-ray in the other 30 percent. The former process is forbidden for capture of scalar mesons from the K shell. A detailed balancing argument, based on the observed cross section for $2p\rightarrow D+\pi^+$, permits calculation of the non-radiative capture rate from orbits of higher angular momentum. It is shown that this is too small to account for the observation, and the possibility that the meson is a scalar is thus ruled out. This conclusion could only be avoided by postulating the existence of an interaction between meson and nucleons sufficiently large to alter radically the magnitude of the p-wave functions near the nucleons. For pseudoscalar mesons, non-radiative capture from the K shell is permitted, and the ratio of radiative to non-radiative transitions can be related to the ratio of meson photoproduction cross section to the cross section for $2p\rightarrow D+\pi^+$. The result is compatible with the type of meson-nucleon coupling which might be expected for pseudoscalar mesons, but uncertainties in the measured quantities are too large to permit a sharper conclusion.

I. INTRODUCTION

In the preceding paper Panofsky, Aamodt, and Hadley[1] report that the capture of negative mesons by deuterons occurs in either of two ways, leading to the emission of two neutrons in 70 percent of the cases, and to two neutrons and a γ-ray in the other 30 percent. The first point of interest is that the two neutron process occurs at all. For if the π^- meson satisfied a scalar wave equation, and if the capture of the π^- took place from the K shell of the π^-+D system, this process would be forbidden, as Ferretti[2] has pointed out. The reason is that an s-state of the π^-+D system would have a total angular momentum $J=1$, and be an even state, whereas the only state of the two neutron system with $J=1$ is a ^3P-state, which is odd. Thus, the emission of two neutrons in about 70 percent of the captures proves either that the meson is not a scalar, or that the capture takes place from a higher Bohr orbit than the K shell, an orbit with $l\neq 0$.

To decide between these two possibilities we must estimate the rate, from the higher meson states, of capture with two neutron emission. We shall attempt to do this without recourse to an explicit meson theory, since we have learned by experience that meson theory cannot always be relied on even for orders of magnitude. In fact, we know of no meson theory which is not in gross disagreement with at least one of the presently believed experimental facts. The possibility of estimating the capture rate depends on the principle of detailed balancing. Although the cross section for the inverse of $\pi^-+D\rightarrow 2n$ has not been measured, it may reasonably be supposed that the reactions $2n\rightarrow D+\pi^-$ and $2p\rightarrow D+\pi^+$ have the same cross section. The latter reaction has been studied by Cartwright, Richman, Wilcox, and Whitehead,[3] who obtained a cross section $\sigma(2p\rightarrow D+\pi^+)=2\times 10^{-28}$ cm^2 for 340-Mev protons bombarding hydrogen. The most direct experimental support for the supposition of charge symmetry comes from work of Bradner, Bowker and Rankin,[4] who found a ratio of negative to positive mesons of 15 from carbon bombarded by 260-Mev neutrons, and the reciprocal ratio from carbon bombarded by protons of the same energy.

In the 340-Mev $2p\rightarrow D+\pi^+$ experiments the kinetic energy available in the center of mass system is 22 Mev. The principle of detailed balancing will give us the rate of $D+\pi^-\rightarrow 2n$ for 22-Mev relative energy of meson and deuteron, and we must extrapolate to lower energy to make the desired connection. The question is thus the dependence of the transition matrix element on meson momentum. Since the wave function, $\phi(\mathbf{r})$, of a meson in a p-state vanishes at the origin, we can write, for small $\mathbf{r}$,

$$\phi(\mathbf{r})=\mathbf{r}\cdot\mathrm{grad}\phi(0). \tag{1}$$

Here $\mathbf{r}$ is the coordinate of the meson relative to the center of mass of the deuteron. Thus $\phi(\mathbf{r})$, and the matrix element, will be proportional to the momentum of the meson near the deuteron, as long as (1) is adequate, which will be true as long as the wavelength of the meson is large compared to the distances, $\mathbf{r}$, which contribute to the process. The values of $\mathbf{r}$ in question are certainly not larger than the radius of the deuteron (which is about the same as the wavelength of a 20 Mev meson), but in fact will be considerably smaller because of the high relative energy (μc^2) which must be transferred to the nucleons. Such high energy transfers will occur only in intimate collisions of the nucleons; if the

[1] Panofsky, Aamodt, and Hadley, Phys. Rev. **81**, 565 (1951).

[2] B. Ferretti, *Report on the International Conference on Low Temperatures and Fundamental Particles* (Cambridge, 1946), p. 75. S. Tamor, Phys. Rev. **78**, 221 (A) (1950).

[3] Cartwright, Richman, Wilcox, and Whitehead (private communication). Although the experimental energy resolution is not good enough to establish conclusively the high energy meson peak as due to $2p\rightarrow D+\pi^+$ rather than $2p\rightarrow p+n+\pi^+$, we shall clearly overestimate the rate of capture in deuterium if we ascribe the observed cross section to the former process.

[4] H. Bradner (private communication).

transfer takes place in a single collision of the nucleons the distance would be about the wavelength of the neutrons, $r\sim\hbar/(M\mu)^{\frac{1}{2}}c=4\times10^{-14}$ cm. Since the wavelength of a 20 Mev meson is about five times larger, the proportionality of the matrix element to meson momentum would be expected to hold over the energy range in question.[5]

In addition to its dependence on the relative momentum of meson and deuteron, the matrix element will also depend on the neutron momenta. However, as the meson energy changes from 20 Mev to zero, the neutron momenta change by only 7 percent. We shall therefore neglect this dependence. In fact, quite apart from questions of the trustworthiness of meson theory, one of the advantages of using the detailed balancing argument is that in the calculation of the capture rate one avoids the question of the magnitude of high frequency Fourier components of the deuteron wave function, since nearly the same Fourier components are involved in the production process.

Finally, it should be pointed out that the foregoing argument assumes that there does not exist an interaction between meson and nucleons sufficiently strong to radically change the form of the p wave-function near the nucleons. Were such an interaction present, the argument would, of course, be invalidated.

II. THEORY

Let us denote by $R(\pi^-+D\rightarrow2n)$ the rate of capture from a bound p-state, and by $\sigma(\pi^-+D\rightarrow2n)$ the cross section of this process for an unbound meson. In the latter case let v_π and p_π be the relative velocity and momentum of meson and deuteron, v_n and p_n those of the two neutrons. According to the argument following (1), the ratio of the capture rate, $R(\pi^-+D\rightarrow2n)$, from the bound state to the capture rate, $v_\pi\sigma(\pi^-+D\rightarrow2n)$, from a plane wave of unit amplitude should be the ratio of the square of the gradient of the bound state function, $|\nabla\phi(0)|^2$, to the square of the same quantity for the plane wave, $p^2/\hbar^2$. Thus

$$R(\pi^-+D\rightarrow2n)=\hbar^2v_\pi\sigma(\pi^-+D\rightarrow2n)|\nabla\phi(0)|^2/p_\pi^2. \quad (2)$$

TABLE I. A comparison is given of the transition rates for meson absorption and radiation to the ground state from the lower p state Bohr orbits.

Transition from the state	Transition rates in units of 10^{10} sec^{-1} Decay by fast neutron emission	Radiation to ground state
$2p$	0.94	17.2
$3p$	0.33	5.1
$4p$	0.15	2.2

[5] For states with $l>1$, the matrix element will fall off even more rapidly as the momentum is reduced. Use of a linear momentum dependence, which is tantamount to assuming the entire production process takes place from p-states, can therefore only overestimate the rate of meson capture.

The principle of detailed balancing gives

$$\sigma(\pi^-+D\rightarrow2n)=\tfrac{2}{3}p_n^2\sigma(2n\rightarrow D+\pi^-)/p_\pi^2. \quad (3)$$

The factor $\frac{2}{3}$ is the product of 4/3, for the statistical weight of the spin states, and $\frac{1}{2}$, to allow for the indistinguishability of the two neutrons. Combining (2) and (3) we find

$$R(\pi^-+D\rightarrow2n)=\tfrac{2}{3}\hbar^2p_n^2\sigma(2n\rightarrow D+\pi^-)|\nabla\phi(0)|^2/\mu p_\pi^3,$$

where μ is the reduced meson mass, or using the approximate energy relation, $p_n^2/M=\mu c^2$, with M the neutron mass, and the assumption of charge symmetry,

$$R(\pi^-+D\rightarrow2n)=\tfrac{2}{3}\hbar^2Mc^2\sigma(2p\rightarrow D+\pi^+)|\nabla\phi(0)|^2/p_\pi^3. \quad (4)$$

If we put in the value of $|\nabla\phi(0)|^2$ for a p-state of principal quantum number, n, we find

$$R(\pi^-+D\rightarrow2n)=(2/9\pi)\hbar^2Mc^2(n^2-1)\sigma(2p\rightarrow D+\pi^+)/n^5a^5p_\pi^3, \quad (5)$$

where a is the mesonic Bohr radius, $a=\hbar^2/\mu e^2$. With the measured value of $\sigma(2p\rightarrow D+\pi^+)$, (5) gives

$$R(\pi^-+D\rightarrow2n)=1.0\times10^{11}(n^2-1)/n^5\ \text{sec}^{-1}.$$

In Table I this transition rate is compared with that for the radiative transition of the meson from the np-state to the $1s$-state. For $n>3$ it is estimated that the transition rate by molecular collisions should predominate over that for radiation. The table shows that the rate of the two neutron process is at least thirty times too small to account for the observed ratio of $\pi^-+D\rightarrow2n$ to $\pi^-+D\rightarrow2n+\gamma$.

The supposition that the meson is a scalar can, thus, be excluded, unless, indeed, the alternative of a large meson-nucleon interaction is the true one. If we imagine such an interaction to be representable by a short range potential well, it is evident that, in order to greatly change the p-wave function near the nucleon, the strength of the interaction must be nearly great enough to contain a bound p-state in the well; that is, nearly great enough to produce an isobaric state of the nucleon.

If the π-meson is a pseudoscalar particle, the angular momentum and parity selection rules allow non-radiative capture from the K shell of the π^-+D system. The question then is whether the observed ratio of the radiative and non-radiative processes can be easily understood.

If we suppose that s-states alone, rather than p-states, are involved in the reactions $\pi^-+D\rightarrow2n$ and $2p\rightarrow D+\pi^+$, the relation (4) would have the factor $\hbar^2|\nabla\phi(0)|^2/p_\pi^2$ replaced by $|\phi(0)|^2$. There is, however, no reason to exclude p-states from the production process, and we shall write

$$R(\pi^-+D\rightarrow2n)=\frac{2}{3}\frac{Mc^2\sigma(2p\rightarrow D+\pi^+)|\phi(0)|^2}{p_\pi(1+bp_\pi^2/\mu^2c^2)}, \quad (6)$$

where the term in the denominator proportional to p_π^2

represents the p-state contribution (relative to that of the s-state) to $\sigma(2p\rightarrow D+\pi^+)$. Indeed, it is characteristic of pseudoscalar meson theory that it describes the meson-nucleon interaction as primarily taking place in the p-state, the s-state coupling arising only from "small" relativistic correction terms. The smallness of these terms is characterized by the mass ratio μ/M, which is not, in truth, a very small parameter. According to the theory, the constant b would be of order M/μ.

To obtain the rate of the $\pi^-+D\rightarrow 2n+\gamma$ reaction, we shall again appeal to the principle of detailed balancing, making use of the information available on the photoproduction process, $\gamma+n\rightarrow p+\pi^-$. This can be regarded as the inverse of the radiative capture in deuterium to the extent that the presence of the additional neutron does not affect the capture rate. We have calculated the effect to be expected because of the exclusion principle and the existence of n-n forces in the final state, and conclude that the capture rate in deuterium will be reduced by a factor, f, of approximately two-thirds.[6] In analogy with (2), we now have

$$R(\pi^-+D\rightarrow 2n+\gamma)=fv_\pi'\sigma(\pi^-+p\rightarrow n+\gamma)|\phi(0)|^2,$$

where $\sigma(\pi^-+p\rightarrow n+\gamma)$ is the cross section for this reaction for very slow mesons, of velocity v_π'. From detailed balancing,

$$\sigma(\pi^-+p\rightarrow n+\gamma)=2\mu c^2\sigma(n+\gamma\rightarrow p+\pi^-)/p_\pi'^2,$$

so

$$R(\pi^-+D\rightarrow 2n+\gamma)\\=2f\mu c^2\sigma(n+\gamma\rightarrow p+\pi^-)|\phi(0)|^2/p_\pi'. \quad (7)$$

In this equation, $\sigma(n+\gamma\rightarrow p+\pi^-)/p_\pi'$ is to be evaluated just above threshold.

The experiments of Steinberger and Bishop[7] on photoproduction in hydrogen give an approximate value of the cross section $\sigma(\gamma+p\rightarrow n+\pi^+)=7\times10^{-29}$ cm^2 for a γ-ray of 180 Mev, corresponding to 23-Mev relative energy of meson and neutron in the center of mass system. According to an argument of Brueckner and Goldberger,[8] the cross section for $\gamma+n\rightarrow p+\pi^-$ would be expected to be larger, for non-relativistic meson energies, in the ratio $1/(1-\mu/M)^2=1.3$. The existence of a negative excess in the photoproduction from carbon has been demonstrated experimentally by McMillan, Peterson, and White[9] and Peterson.[10] While there is some doubt as to how well the observations agree in detail with the predictions of Brueckner and Goldberger, there is no doubt that the negative to positive ratio is at least as large as is indicated above. We shall accordingly take $\sigma(\gamma+n\rightarrow p+\pi^-)=10^{-28}$ cm^2. Since the lowest energy for which experimental data is available is that just quoted for production of 23 Mev mesons, while (7) requires the cross section just above threshold, an assumption about the energy dependence of the photoproduction cross section is necessary. If the matrix element for the process is a constant (i.e., if the variation of the cross section is determined solely by the available phase space), over this energy range, $\sigma(n+\gamma\rightarrow p+\pi^-)p_\pi'$ will be independent of energy, and the 23 Mev value can be used in (7). While this assumed energy dependence is the one to be expected on the basis of pseudoscalar meson theory, it should be emphasized that it is not based on any direct experimental information: our present knowledge of the excitation function for the photoproduction of mesons is not sufficiently accurate to permit an unambiguous extrapolation to zero energy.

The ratio of (7) to (6) gives

$$\frac{R(\pi^-+D\rightarrow 2n+\gamma)}{R(\pi^-+D\rightarrow 2n)}\\=3f\frac{\mu p_\pi'\sigma(n+\gamma\rightarrow p+\pi^-)}{Mp_\pi\sigma(2p\rightarrow D+\pi^+)}(1+bp_\pi^2/\mu^2c^2). \quad (8)$$

Inserting 3/7 for the ratio of the R's, $f=\frac{2}{3}$, $\mu/M=1/7$, $p_\pi'/p_\pi=1$, $\frac{1}{2}$ for the ratio of the σ's, we find $(1+bp_\pi^2/\mu^2c^2)=3$. This evaluation, in view of the uncertainties in the measurements of the various quantities appearing in (8), can hardly be trusted to better than a factor of two. However, the result is quite compatible with the notions one has as to the proper behaviour of a pseudoscalar field; since $p_\pi/\mu c=1/2$, a value of b of the order M/μ is indeed permissible. Quite apart from the uncertainties in the measured quantities, it should also be pointed out that the value found here is in the nature of an upper limit for the s-state contribution, since if the photoproduction cross section decreases more rapidly with decreasing meson momentum than we have assumed, the s-wave coupling will be correspondingly reduced. Cartwright, Richman, Whitehead, and Wilcox[3] and Peterson[11] have found that mesons from the $2p\rightarrow D+\pi^+$ reaction have a markedly non-uniform angular distribution in the center of mass system, which shows quite definitely the existence of coupling to other than s states.

The absolute value of the capture rate given by (7), found by inserting $|\phi(0)|^2=1/\pi a^3$ and the previously quoted value of the cross section, is

$$R(\pi^-+D\rightarrow 2n+\gamma)=2.7\times10^{14}\ \text{sec}^{-1}.$$

The capture of π^- mesons in elements heavier than deuterium leads characteristically to stars, rather than high energy γ-emission. Panofsky, Aamodt and Hadley[12] give as upper limits for the fraction of high energy γ-emissions 10 percent in helium and 0.5 percent in carbon. As we have previously remarked, the non-

[6] The effect of n-n forces, while modifying the γ-ray spectrum, does not appreciably influence the capture rate.
[7] J. Steinberger and A. S. Bishop, Phys. Rev. **78**, 494 (1950).
[8] K. Brueckner and M. Goldberger, Phys. Rev. **76**, 1725 (1949).
[9] McMillan, Peterson, and White, Science **110**, 579 (1949).
[10] J. Peterson, Dissertation, University of California (1950).
[11] V. Z. Peterson (private communication).
[12] Panofsky, Aamodt, and Hadley (private communication).

radiative capture in deuterium depends on the probability of finding a high relative momentum of the nucleons in the deuteron. In more tightly bound systems this probability will be considerably larger. Crude estimates, based on a semi-empirical momentum distribution for the nucleons in carbon given by Chew and Goldberger,[13] indicate that an increase in the non-radiative capture rate by a factor as large as 30 may be expected. On the other hand, the rate of emission of high energy γ-rays will be reduced in the more tightly bound systems relative to deuterium, because of the greater inhibiting effect of the exclusion principle in the final state. The experiments of Peterson and White[14] and of Mozley,[15] on the photoproduction of mesons show that a proton bound in carbon has only one-third as large a photoproduction cross section, at 260 Mev photon energy, as a free proton. Calculations by Bruno[16] indicate that a similar reduction by a factor of three is to be expected in the radiative capture rate in carbon. The γ-ray emission rate can thus be expected to be reduced, relative to deuterium, by a factor two. The combination of these two effects leads to the expectation that the 30 percent of γ-emission found in deuterium will be reduced to something of the order of 0.5 percent in heavier elements.

Finally, it may be mentioned that if it be supposed that the π^- meson has spin one (despite the fact that the π^0 certainly has not, since it decays to two γ-rays), an explanation of the observed ratio of the $\pi^-+D\rightarrow 2n$ and $\pi^-+D\rightarrow 2n+\gamma$ processes can be based solely on selection rules. A vector meson in the K-shell would have odd parity, and $J=0$, 1, or 2. It could thus decay to the ${}^3P_{0,1,2}$ states of the two neutron system. However, a pseudovector meson would give states of even parity; those with $J=0$ and 2 could decay to the ${}^1S_0$ and ${}^1D_2$ two neutron states, but decay from the $J=1$ state would be forbidden. If we suppose the states are occupied in proportion to their statistical weights, a ratio of radiative to non-radiative transitions of 1/2 is thus to be expected.

[13] G. F. Chew and M. L. Goldberger, Phys. Rev. **77**, 470 (1950).
[14] J. M. Peterson and S. R. White, Phys. Rev. **78**, 84 (A) (1950).
[15] R. Mozley, Phys. Rev. **80**, 493 (1950).
[16] B. Bruno, private communication.