Reflection of Neutrons from Magnetized Mirrors*

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The index of refraction for neutrons in ferromagnetic materials has been studied by total reflection from mirrors. It is shown that Bloch's constant, C, is unity; i.e., that the effective field for neutrons in a ferromagnet is $\mathbf{B}$. For the case of total reflection, it is found that the magnetic part of the index is given by the field averaged over a region large compared to the size of the magnetic domains. Completely polarized thermal neutrons have been produced, without the necessity of monochromatization, by reflection from magnetized cobalt. Measurement of the polarization, performed by reflection at a second cobalt mirror, requires careful elimination of reorientation effects in the region between the mirrors.

I. INTRODUCTION

THE neutron reflection experiments reported here are an outgrowth of the study of magnetic refraction of neutrons at domain boundaries already described by Hughes, Burgy, Heller, and Wallace.[1] The small angle deviations observed by them when neutrons were transmitted through unmagnetized iron were explained in terms of the variation of the index of refraction from domain to domain, which variation is caused by changes in the direction of magnetization. It was felt that the multiple refraction of neutrons in iron could be exhibited in a much more striking way by utilizing critical reflection from mirrors, a technique already used by Fermi and Zinn,[2] and Fermi and Marshall.[3] For each index of refraction there should be a corresponding critical angle which could be measured rather accurately. Because of the variation of critical angle with neutron spin orientation, the neutrons reflected from a magnetized mirror should be partly polarized;[1,4,5] and an additional purpose of the mirror experiments was to investigate the feasibility of reflection as a means of production of polarized neutrons.

After the experiments were begun, several theoretical papers[6,7] appeared concerning reflection from magnetized mirrors, and it became clear that the experimental methods in use were valuable for elucidating some of the fundamental features of the neutron-electron magnetic interaction. Additional ramifications developed when the correctness of the fundamental equation then in use for the index of refraction was questioned by Ekstein.[8] As a result of the theoretical uncertainties, the experiments were extended in scope until all the doubtful points could be resolved experimentally. In the present report the theory will be outlined briefly and the experimental results presented in some detail.[8a]

II. NEUTRON REFLECTION THEORY

The index of refraction for neutrons, and hence the critical angle, depends only on the average potential which the neutron experiences in a medium and is independent of molecular and crystalline structure. For a mirror of a single element with small neutron absorption the index of refraction, n, and the critical glancing angle θ_c, are given by:

$$n^2 = 1 - \lambda^2 Na/\pi \tag{1}$$

$$\theta_c = \lambda(Na/\pi)^{\frac{1}{2}}. \tag{2}$$

Here λ is the neutron wavelength, N is the density of nuclei, and a is the average coherent scattering amplitude. The variation of scattering amplitude from nucleus to nucleus, which may be caused by the presence of isotopes of different scattering amplitude, or of spin-dependent scattering, will produce a slight diffuse scattering and will have essentially no effect on the critical angle. The sign convention is such that a positive value of a corresponds to a phase shift of 180°, an index less than unity, and the occurrence of total reflection. It is noteworthy that θ_c depends on the coherent amplitude in a very direct way, there being no corrections for form factors, temperature diffuse scattering or crystal effects. As pointed out by Hamermesh,[6,9] this simplicity, which follows from the fact that the scattering is essentially forward, implies that the measurement of critical angles is a method of obtaining accurate coherent amplitudes in such important cases as n-p scattering[10] and the neutron-electron spin-independent interaction.

* Research carried out under contract with AEC.

[1] Hughes, Burgy, Heller, and Wallace, Phys. Rev. 75, 565 (1949).
[2] E. Fermi and W. Zinn, Phys. Rev. 70, 103 (1946).
[3] E. Fermi and L. Marshall, Phys. Rev. 71, 666 (1947).
[4] A. Achieser and J. Pomeranchuck, J. Exper. Theor. Phys. U.S.S.R. 18, 475 (1948).
[5] O. Halpern, Phys. Rev. 75, 343 (1949).
[6] M. Hamermesh, Argonne Laboratory Report ANL 4298 (May, 1949).
[7] O. Halpern, Phys. Rev. 76, 1130 (1949).
[8] H. Ekstein, Phys. Rev. 76, 1328 (1949); 78, 731 (1950).
[8a] A preliminary account of some of the results has already been given in Phys. Rev. 76, 1413 (1949).
[9] M. Hamermesh, Phys. Rev. 77, 140 (1950).
[10] Hughes, Burgy, and Ringo, Phys. Rev. 77, 291 (1950).

For the case of a material containing several elements the correct coherent amplitude is simply the average value, due account being taken of the algebraic sign of the individual coherent amplitudes. For a ferromagnetic material, the question of the correct coherent amplitude becomes very difficult to answer and has been the source of divergent opinions during the course of these experiments. The viewpoint adopted in the paper of Hughes, Burgy, Heller, and Wallace[1] was that the neutrons in a domain (in which the magnetic induction always has the saturated value, $\mathbf{B}_s$) experience a two-valued potential, $\pm\mu B_s$. It follows that there are just two distinct indices of refraction for neutrons in iron, whether the iron is magnetized or not in the macroscopic sense

$$n^2=1-\lambda^2\mathrm{Na}/\pi\pm\mu B/E, \qquad (3)$$

where E is the neutron energy.

The index of refraction as given in Eq. (3) had been obtained by Halpern, Hamermesh, and Johnson[11] for the special case of neutron propagation direction perpendicular to $\mathbf{B}_s$. Achieser and Pomeranchuck[4] derived the same formula, under the assumption that the neutron wavelength was greater than the lattice spacing (no Bragg reflections), but with no specific orientation of neutron direction relative to $\mathbf{B}_s$. Hamermesh[6] extended the treatment of Halpern, *et al.*[11] and found an index containing a term depending on the angle between the direction of neutron motion and $\mathbf{B}_s$. Ekstein[8] and Halpern,[7] however, concluded that there are only two indices as given in Eq. (3), regardless of the orientation of $\mathbf{B}$ and of wavelength. In view of these differences in results it was quite desirable to obtain experimental data on the index in magnetized iron as a function of magnetization direction.

In addition to the possible variation of index with magnetization direction, there was an uncertainty concerning the form of the neutron-electron magnetic interaction. The original form of the neutron-electron magnetic interaction used by Bloch[12] (dipole-dipole) in his discussion of magnetic scattering leads to a different angular distribution of scattered neutrons than the Dirac interaction used by Schwinger[13] and Halpern, *et al.*[11] Ekstein[8] showed that the different forms of interaction, when applied to the case of refraction, resulted in a simple interpretation: the Dirac interaction led to Eq. (3), whereas the Bloch interaction led to the same formula with $\mathbf{B}$ replaced by $\mathbf{H}$. An experimental verification of Eq. (3) by means of a measurement of the critical angle would thus demonstrate that the field affecting the neutron in its passage through iron is $\mathbf{B}$ rather than $\mathbf{H}$.

In order to produce polarized neutrons by reflection from magnetized iron, it would be necessary to use an angle of incidence which is greater than the critical angle for one spin state, for which the reflectivity is practically zero, and less than the critical angle for the other spin state. Because the critical angles depend on wavelength it would be necessary also to use monoenergetic neutrons to prevent overlapping of the critical angles. Hamermesh[9] suggested that all wavelengths might be used, however, if the mirror should be made of cobalt instead of iron. The magnetic term in Eq. (3) for cobalt is larger than the nuclear term so one index will be less than, and the other greater than, unity for all wavelengths. The neutrons for which the index is greater than unity will reflect to a negligible amount while the others will reflect totally and hence will be completely polarized.

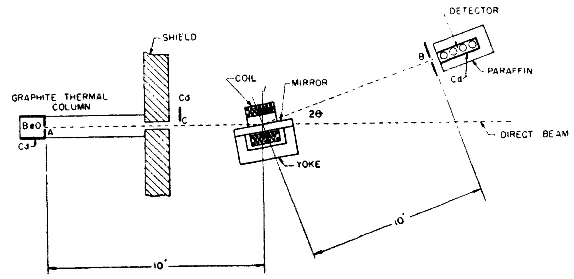

FIG. 1. Plan view of apparatus for reflection of neutrons from magnetized mirrors.

III. EXPERIMENTAL METHOD

Slow neutrons, from the thermal column of the Argonne deuterium-moderated pile, were used for the mirror experiments in the manner shown in Fig. 1. Because of the dependence of the critical angle on the wavelength as well as the amplitude a, the most direct method of measurement of a would consist of a determination of the critical angle for monochromatic neutrons. However, selection of monochromatic neutrons from the thermal column beam, as by Bragg reflection at a single crystal, involves a great loss in intensity. The BeO filter of Fig. 1 was chosen instead of a monochromator to obtain higher intensity. The scattering of polycrystalline BeO is exceedingly small for neutrons of wavelength greater than 4.4A (twice the largest lattice spacing in BeO) hence a block of BeO surrounded by Cd, as shown in Fig. 1, acts as a neutron filter. The neutrons of wavelength less than 4.4A are scattered in the block and captured by Cd while those of longer wavelength are transmitted almost completely. Although a monoenergetic beam is not obtained by this method, a distribution with high intensity and an exceedingly sharp cutoff on the short wavelength side results. The combination of sharp cut-off wavelength and high intensity is especially desirable for the determination of critical angles.

The filtered neutrons strike the mirror, usually 4 in. high by 10 in. in length, and are detected by a set of enriched BF_3 proportional counters connected in paral-

[11] Halpern, Hamermesh, and Johnson, Phys. Rev. **59**, 981 (1941).
[12] F. Bloch, Phys. Rev. **50**, 259 (1936); **51**, 994 (1937).
[13] J. Schwinger, Phys. Rev. **51**, 544 (1937).

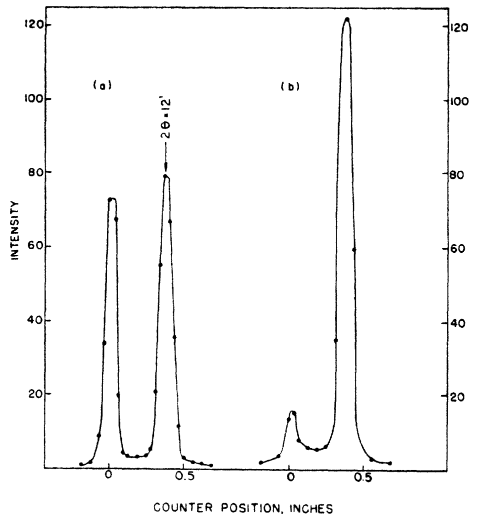

FIG. 2. Neutron beams observed with the apparatus of Fig. 1. The mirror intercepts about half the incident beam in (a) and almost the entire beam in (b). The direct beam is at the zero counter position while the reflected beam is at a position corresponding to a mirror angle, θ, of 6 minutes.

lel, which are placed in a shield of cadmium surrounded by paraffin (this shielding arrangement being effective for the fast neutron background). The collimating slits at A and B are 0.1 in. wide and 4 in. high while the width of the mirror normal to the beam is also 0.1 in. at an incident angle of 30 minutes. As 0.1 in. subtends an angle of 3 minutes as a distance of 10 ft there will be a spread in incident angles of about 3 minutes for a particular setting of the mirror. The result of the angular spread is that the sharp drop in intensity which would occur at the critical angle for monochromatic neutrons is smoothed out over a range of several minutes.

The incident angle, θ, is measured in terms of the angle, 2θ, between the reflected and the direct beam. The direct beam is located by moving the mirror sidewise slightly so that some of the incident neutrons miss the mirror and are detected by the counter with the counter in the position marked "direct beam" in Fig. 1. The counter and the slit B are mounted together on a platform, which is moved by a calibrated screw, and the distance between the direct and the reflected beam can be measured accurately enough so that θ can be determined to about 0.1 minute. Examples of the direct and reflected beams are shown in Fig. 2, for a case (a) in which the direct beam is about equal in intensity to the reflected beam and (b) in which the mirror is moved to reduce the direct beam and to increase the reflected beam. It was found to be simpler to line up the mirrors and counters and to measure the incident angles by use of the neutron beams than by use of auxiliary optical methods. The counting rates, shown on an arbitrary scale in Fig. 2, are actually of the order of several thousand counts per minute.

The intensity of the reflected beam is measured as θ is increased by moving the detector so as to keep it at an angle of 2θ to the direct beam. The intensity measured in this way will at first increase linearly with θ, because the mirror intercepts more of the incident beam as θ increases, and the reflectivity is complete provided θ is less than θ_c, the critical angle for 4.4A. After θ_c is passed the intensity will drop as neutrons of wavelength greater than 4.4A are successively eliminated from the reflected beam. The intensity will actually decrease as θ^{-3} because the wavelength distribution at low velocity, as seen by an opaque ("black") detector is proportional to λ^{-3}, its integral (the mirror reflects everything from λ_c to ∞) to λ^{-4}, or θ^{-4} [Eq. (1)], and the recorded intensity, finally, to θ^{-3}, as the factor, θ, for the increase in beam intercepted is included.

The intensity expected from the previous description is shown by the "ideal" solid line of Fig. 3, which is calculated for a Be mirror with θ_o taken to be 25.8 min. As already mentioned, the actual intensity curve will be rounded off because of the finite resolution in angle (about ± 1.5 minutes), and this effect can easily be calculated by graphical integration of a series of slightly displaced ideal intensity curves. Another refinement in the calculated curve is necessitated by the fact that the reflectivity[14] above the critical angle does not go to zero immediately but rather decreases as

$$R=\left[\frac{1-(1-\theta_c^2/\theta^2)^{\frac{1}{2}}}{1+(1-\theta_c^2/\theta^2)^{\frac{1}{2}}}\right]^2. \tag{4}$$

The dotted curve of Fig. 3 is the result of correction of the ideal curve for both the finite resolution and the finite reflectivity effects. It is seen that the sharp peak at the critical angle disappears but that it should nevertheless be a simple matter to correlate an observed intensity curve with the appropriate critical angle.

The experimental points of Fig. 3 are those obtained for a Be mirror during some exploratory runs. The

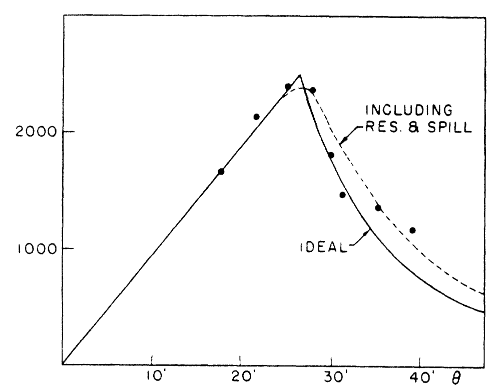

FIG. 3. Intensity of filtered neutrons reflected from a Be mirror as a function of incident angle, θ. The solid line is a theoretical curve for infinite resolution and zero reflectivity above the critical angle, while the dashed curve includes corrections for finite resolution and partial reflectivity above the critical angle ("spillover").

[14] M. L. Goldberger and F. Seitz, Phys. Rev. 71, 294 (1947).

mirror which was plane to better than half a fringe (NaD line) per inch when measured with an optical flat, was made in the machine and optical shops of the Argonne Laboratory. The dotted curve, with $\theta_c = 25.8$ min, was calculated for an amplitude of 2.42×10^{-13} cm to correspond with a scattering cross section ("free atom" value, measured at a neutron energy of several electron volts) for Be of 6.0 barns. In the calculation, the assumption was made that the 6.0 barn cross section was entirely coherent and hence that the coherent amplitude, a, could be obtained from the relation $\sigma_b = 4\pi a^2$ where σ_b is the bound atom cross section [$\sigma_b = 6\times(10/9)^2 = 7.2$ barns]. The agreement between the calculated curve and the point supports the correctness of the calculated intensity curve and the supposition that the scattering in Be is largely coherent. Of course, only spin dependent scattering could invalidate this supposition, for Be has no isotopes (no isotope disorder scattering), and inelastic scattering does not affect mirror reflection (because of unit form factor as already discussed). The results for Be show that the method described is a reliable one for critical angle and coherent amplitude measurements. Because of the preliminary nature of the experiments at the time, no effort was made to obtain careful quantitative results for the Be mirror.

The absolute reflectivity for the Be mirror for angles less than critical was also measured by comparing the reflected beam intensity with that of the direct beam, using a collimating slit at C (Fig. 1) small enough so that the whole mirror was not illuminated with neutrons. The intensity of the reflected beam was found to be within three percent of the direct beam intensity, where the latter intensity was measured with the mirror moved out of the beam.

IV. CRITICAL ANGLE MEASUREMENTS WITH IRON AND COBALT MIRRORS

As the results with the beryllium mirror had shown that the method of critical angle measurements using BeO filtered neutrons was satisfactory, the more complicated case of neutron reflection from iron was next studied. Corresponding to the two indices of refraction of Eq. (3) for neutrons in iron there will be two critical angles,

$$\theta_c = \left[\frac{\lambda^2 \mathrm{Na}}{\pi} \pm \frac{\mu(H+4\pi CI)}{E} f(\phi)\right]^{\frac{1}{2}}. \quad (5)$$

Equation (5) is written in the most general way, to include all of the theoretical uncertainties already mentioned which were to be investigated experimentally. The constant C (called "Bloch's constant" by Ekstein[8]) would be zero for the dipole-dipole neutron-electron interaction first used by Bloch,[12] and unity for the Dirac formulation used by the later theorists.[11,13] The function $f(\phi)$ is that of Hamermesh,[6] which function causes the critical angle to depend on the angle, ϕ,

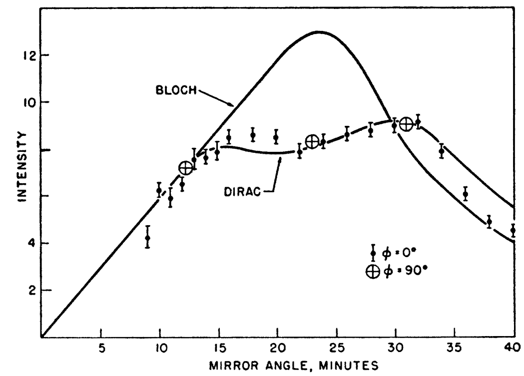

FIG. 4. Intensity of filtered neutrons reflected from a magnetized iron mirror. The theoretical curve labeled "Bloch" corresponds to Bloch's constant, C equal to zero, while "Dirac" corresponds to $C=1$. Experimental points for two directions of magnetization, $\phi=0°$ and 90°, are shown.

between the neutron propagation direction and the magnetic field, in a way which will not be described in detail. All of the theorists considered mainly the practical case in which the magnetization is in the plane of the mirror, with the somewhat artificial assumption that the field in the region outside the mirror is zero.

The first measurements with magnetized iron mirrors were designed to demonstrate the doubly refracting nature of iron and to determine Bloch's constant, C. The apparatus was as shown in Fig. 1 and as can be seen in that figure, the angle ϕ was essentially zero (to eliminate any effect of $f(\phi)$ which is unity for $\phi=0$). Under these conditions it is seen that if $C=0$ there will be no magnetic term, as **H** is continuous across the boundary, and only a single critical angle, corresponding to the nuclear scattering alone, should result. On the other hand, if $C=1$ there should be two well separated values of θ_c, because the field discontinuity at the boundary is $4\pi I$ (or essentially **B**, because of the small value of **H**).

The results of the experimental determination of Bloch's constant are seen in Fig. 4. The solid lines are the intensity distributions expected for $C=0$ (labeled "Bloch") and $C=1$ ("Dirac"), assuming a nuclear amplitude based on a coherent scattering cross section[15] of 10.3 and a B of 22,500 gauss. The curves have been corrected for "spill-over" (finite reflectivity above the critical angle) and angular resolution. The $C=1$ curve starts to drop as the lower critical angle is passed because neutrons of one spin orientation begin to disappear from the reflected beam, then rises as the mirror angle is increased and finally drops again after the mirror angle exceeds critical for the other spin orientation. The points agree quite well with the $C=1$ curve, thus proving that the Dirac interaction is correct and that the effective field in the iron is **B** rather than **H**. The present results do not constitute a precise determi-

[15] C. Shull and E. Wollan, private communication.

nation of C but show that its value is at least 0.9. The fact that the observed critical angles agree quantitatively with those calculated shows that both the magnetic interaction (given by the difference of the critical angles) and the nuclear interaction (given by their mean value) used in the calculation, 22,500 gauss and 10.3 b, respectively, were correct.†

With the value of the constant C definitely established by the work described, it was decided to investigate the reality of $f(\phi)$, the variation of the critical angles with direction of magnetization. The function $f(\phi)$, as derived originally by Hamermesh,[6] is unity at $\phi=0°$ and zero at $\phi-90°$, where ϕ is the angle between the field $\mathbf{B}$ (assumed in the plane of the mirror) and the direction of motion of the neutrons. For $\phi=0°$ the expected intensity curve is the same as the one marked "Dirac" in Fig. 4 and hence $f(\phi)$ is in agreement with the experimental results for $\phi=0°$. For $\phi=90°$, however, the $f(\phi)$ function leads to a single critical angle, thus a curve identical with the one labeled "C" = 0" in Fig. 4, which is the curve expected for the nuclear scattering alone.

The study of the angular dependence was made essentially as shown in Fig. 1, with the mirror now 4×4 in. in size, and the coil moved back on the iron yoke so that the mirror and yoke could be rotated 90° about a horizontal line perpendicular to the neutron direction. With this arrangement, a few points were run with $\phi=0°$ to check the curve already obtained, then three points were run carefully with $\phi=90°$. The 90° results, shown on Fig. 4 fall exactly on the 0° line and thus indicate that there is no variation of the critical angle with ϕ. The finding that the critical angle, and hence the index of refraction, has only two constant values, is in agreement with the results of Ekstein.[8] Ekstein has recently shown (second paper of reference 8) that the $f(\phi)$ function arises from an ambiguity in the usual expression for the scattered wave function in the limit of forward scattering. The correct form of Eq. (5) is finally seen to be:

$$\theta_c=\left[\frac{\lambda^2\mathrm{Na}}{\pi}\pm\frac{\mu B_s}{E}\right]^{\frac{1}{2}}. \tag{6}$$

Strictly speaking, $\mathbf{B}_s-\mathbf{H}$ should be used rather than $\mathbf{B}$ because of the continuity of $\mathbf{H}$ across the boundary (fields parallel to surface), but $\mathbf{H}$ is usually negligible compared to $\mathbf{B}_s$.

Before Eq. (6) was established some results had been obtained which seemed at the time to lend support to the idea that θ_c varied with direction of magnetization. The results concerned the reflected neutron intensity at a fixed incident angle as a function of current in the magnetizing coil. For these measurements the BeO filter was not used and the entire Maxwell velocity distribution was incident on the mirror. In general, without a filter, only those neutrons of wavelength greater than the limiting wavelength value given by Eq. (2) will reflect from a mirror. The particular value of θ used will of course determine two limiting wavelengths for a magnetized mirror, one for each neutron spin state, and these wavelengths will be given by Eq. (6) which can be rewritten as

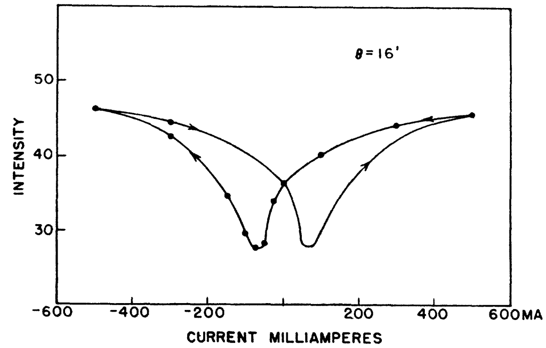

FIG. 5. Intensity of unfiltered neutrons reflected from a cobalt mirror as a function of magnetization.

$$\lambda=\theta_c\left[\frac{\mathrm{Na}}{\pi}\pm\frac{2m\mu B_s}{h^2}\right]^{\frac{1}{2}}. \tag{7}$$

Because of the larger range of reflected wavelengths, there will be more neutrons of the spin state corresponding to the shorter limiting wavelength than for the other spin state. If Eq. (6) is correct for an unmagnetized as well as for a magnetized mirror (the individual domains of both of course are magnetized to saturation) then one might expect the same two limiting wavelengths as for a magnetized mirror. It would then follow the reflected intensity would be independent of the state of magnetization of the mirror.

Equation (6) was checked as a function of magnetization by measuring the intensity of unfiltered neutrons reflected from a cobalt mirror at a fixed incident angle while the mirror magnetization was carried through a complete cycle by varying the magnetizing current. Cobalt was used instead of iron because, as will be shown later, the ratio of magnetic to nuclear scattering is larger than for iron. The results for an incident angle of 16 min are given in Fig. 5. They show that the reflected intensity is certainly not constant, and instead is a function of magnetization as is evidenced by the hysteresis-like form of the curve. It is thus clear that the limiting wavelength (which determines the intensity) for a certain incident angle varies with magnetization. Such behavior was thought at the time to support the $f(\phi)$ function because the variation of ϕ with magnetization would cause a variation in the limiting wavelength. However, after the investigations for $\phi=0°$ and 90° had shown no change of critical angle

† *Note added in proof:* Hamermesh and Eisner, Phys. Rev. **79**, 888 (1950), have pointed out that the data of Fig. 4 also show directly that the neutron spin is 1/2. The results constitute the equivalent of a Stern-Gerlach experiment for the neutron as a spin of 3/2 would have resulted in four values of the index of refraction rather than the two observed.

with magnetization it was necessary to seek some other explanation of the intensity results of Fig. 5.

The measurements of Fig. 4 has shown that Eq. (6) was certainly correct for a mirror magnetized to saturation, and those of Fig. 5 just as certainly that the critical angle for an unsaturated mirror varied with magnetization. Snyder[16] suggested that if the neutron scattering is coherent over an appreciable number of domains (even though the domain size is approximately 10^{-3} cm and the neutron wavelength about 10^{-8} cm) then the field to be used in Eq. (6) is the average **B** which could be anything from zero to the saturated value. As the magnetic small angle scattering results of Hughes, Burgy, Heller, and Wallace[1] had shown that when neutrons are transmitted through unmagnetized iron each domain has its discrete index of refraction, it had been expected that individual domains on the mirror surface would have discrete critical angles, hence constant limiting wavelengths. The suggestion that the average field was effective in determining the critical angle furnished a qualitative explanation of the intensity changes of Fig. 5 which did not necessitate invoking $f(\phi)$, and hence was consistent with Eq. (6).

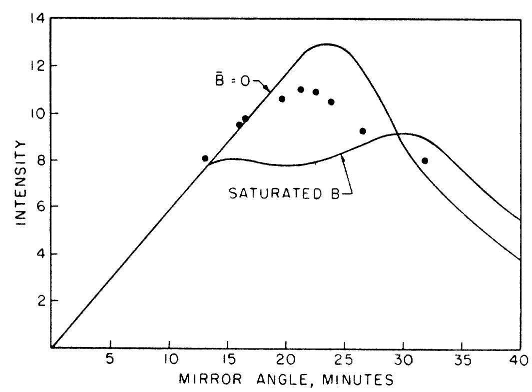

FIG. 6. Intensity of filtered neutrons reflected from an unmagnetized iron mirror as a function of incident angle, θ. The curves are the expected intensities for two values of the effective magnetic field in the iron.

In order to check Snyder's suggestion, the effect of the average field on the critical angle was determined from an intensity curve of the type of Fig. 3, using the BeO filter for an unmagnetized iron mirror. The results, shown in Fig. 6 are distinctly different from those for the $\phi-0°$ and 90° magnetized cases and hence cannot be considered as a result of random values of ϕ. Comparing Fig. 6 with Fig. 4, one sees that the points have moved toward the curve expected for $\mathbf{B}=0$ but are in a position to indicate a finite value of **B**, less than the saturated value. It thus seems certain that the coherent averaging of scattering amplitude over domains takes place but that the averaging does not result in an exactly zero effective field. The failure to reach exact zero field may be caused by incomplete demagnetization, although the mirror was carefully demagnetized by reversing the current as its value decreased. It is also possible that the coherent averaging does not extend over a sufficient number of domains to produce a zero average.

Some complex intensity *vs* magnetizing current curves were obtained with the cobalt mirror at a small incident angle, results which were puzzling for a time. The curve shown in Fig. 7, for example, obtained with unfiltered neutrons incident at an angle of 6′, is very similar to that of Fig. 5 with the exception of the sudden rise in intensity where the field **B** is expected to be small. These intensity changes can be explained in terms of the averaging process, keeping in mind the fact that the magnetic scattering in cobalt is larger than the nuclear scattering. When the cobalt is saturated, only half the neutrons, those in one spin state, will reflect as the others have an index greater than unity. As the average **B** is reduced the limiting wavelength for the particular incident angle used will increase [Eq. (7) with the negative sign] and the intensity therefore will decrease. At a certain value of the average **B**, however, the previously nonreflecting neutrons [positive sign in Eq. (7)] will begin to reflect, the long wavelengths first, as their index drops below unity. As the field is reduced further the two limiting wavelengths will approach each other and the reflected intensity will change in a manner that depends on the shape of the neutron distribution in the neighborhood of the limiting wavelengths. If the shorter wavelength is initially at a part of the distribution where there are few neutrons (extremely short wavelength), then neutrons of the other spin state will increase rapidly in intensity as the corresponding limiting wavelength, initially very long, approaches the wavelength of maximum intensity in the neutron distribution. The two limiting wavelengths will coincide when the average **B** is zero, and the intensity changes will then repeat in reverse order as the mirror is magnetized in the reverse direction. This explanation of the complex intensity curve of Fig. 7 requires that the initial value of the limiting wavelength be quite small, which object is attained by use of the small incident angle of 6 min. The curve of Fig. 5 has a much simpler shape because of the

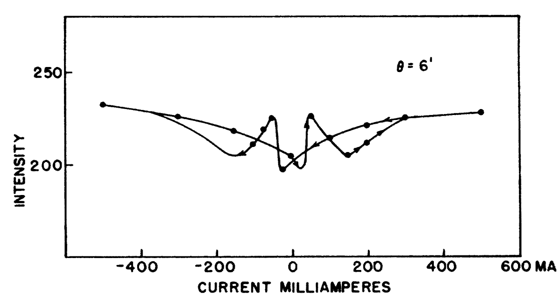

FIG. 7. Intensity curves similar to those of Fig. 5 but for a smaller value of θ.

[16] H. S. Snyder, private communication.

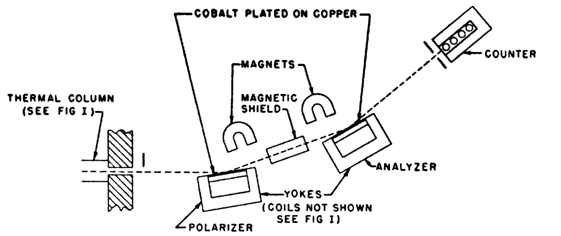

FIG. 8. Apparatus for the production of complete neutron polarization, and for measurement of polarization by the double reflection effect.

longer limiting wavelength, corresponding to the incident angle of 16 min which was used in that case.

All the measurements described thus far supported a simple interpretation of reflection of neutrons from magnetic mirrors.

(1) Equation (6) gives the correct values for the critical angles, in other words, Bloch's constant is unity and there is no angular dependence, $f(\phi)$.

(2) The value of **B** in Eq. (6) is the average over a region at least as large as several domains (domain size $\sim 10^{-3}$ cm). In addition, the results of Fig. 7 made it seem possible that mirror reflection from cobalt could be used to obtain completely polarized neutrons without monochromatization, as suggested by Hamermesh.[9] The neutrons reflected from an iron mirror without monochromatization are of course partially polarized because a larger range of wavelengths of one spin state are reflected than of the other. The polarization of the beam reflected from iron, for an incident Maxwell distribution, increases to an asymptotic value of about 90 percent as the incident angle is increased. On the other hand, the polarization would be 100 percent (neglecting spill-over) for monochromatic neutrons incident on an iron mirror at an angle between the two critical angles of Eq. (6). With saturated cobalt, of course, only one spin state reflects, regardless of wavelength (neglecting the small reflectivity for the index greater than unity), and the polarization should be complete without the loss of intensity involved in monochromatization.

V. NEUTRON POLARIZATION BY REFLECTION FROM COBALT

The nuclear and the saturated magnetic scattering amplitudes of cobalt[6] are $+3.78\times 10^{-13}$ cm and $\pm 4.61\times 10^{-13}$ cm, respectively. If a cobalt mirror is saturated (in the plane of the mirror so that **B** outside the mirror is small) then one neutron spin state, with an amplitude of 8.39×10^{-13} cm, will show total reflection if the incident angle is below critical, while the other, with -0.83×10^{-13} cm amplitude, will reflect to a negligible extent, regardless of incident angle. For a sufficiently small incident angle, even the very short wavelengths will reflect totally and hence be completely polarized after reflection. If the mirror is unsaturated, as is quite likely because of the difficulty of saturating polycrystalline cobalt, then the average **B** will be effective, and the magnetic amplitude correspondingly reduced. The separation of the spin states will result only if the magnetic amplitude remains larger than the nuclear, that is if **B** is greater than 82 percent of saturation. For values of **B** between 82 and 100 percent of saturation, the polarization will be complete but the minimum wavelength reflected for a fixed incident angle will be less than for higher **B**.

Before the cobalt mirrors were used it was feared that complete polarization might be unattainable because of the difficulty of magnetizing cobalt[17] as well as the mechanical problem of producing a cobalt mirror surface. It was felt that, with the magnetic yoke and coils shown in Fig. 1, higher magnetization would result if a film of cobalt on a nonmagnetic backing were used, rather than a solid cobalt mirror. H. Ross of the Argonne machine shop succeeded, after considerable research, in producing a rugged 10-mil electroplated layer of cobalt on a copper plate. It was found that if the cobalt were annealed quickly after deposition it could be polished successfully, but if not then stresses would develop and the coating would soon separate from the copper backing. It is possible that deposition or annealing at appropriate temperatures might produce a cobalt mirror of increased magnetizability, and magnetic measurements of such mirrors are now being performed.

The obvious method, and the one first used, to demonstrate and measure the polarization of the reflected beam is by use of a second cobalt mirror as an analyzer. The experimental arrangement, very similar to that of Fig. 1, appears in Fig. 8. The cobalt mirrors, 4×10 in. long, were each held in iron yokes, in turn mounted on wooden frames. Because of the number of degrees of freedom involved in aligning the magnets, it proved simpler to adjust them by sliding and rotating the frames on the table top, than by use of adjustable mirror mounts. A mirror could be centered in the neutron beam and the incident angle determined by making a few counting rate measurements. The polarization of the beam leaving the first, or polarizer mirror, is detected and measured by noting the change in reflectivity of the analyzer mirror which results when the magnetization in the latter is reversed. If the analyzer is magnetized in the same direction as is the polarizer, it is expected that a completely polarized beam will reflect with no loss of intensity at the analyzer (provided of course that the incident angle at the analyzer is no larger than that at the polarizer). For antiparallel magnetization of the analyzer, however, zero reflected intensity is expected because the polarized neutrons would then have an index of refraction greater than unity in the analyzer mirror. Neutrons polarized by transmission through iron[18] have already been analyzed

[17] R. M. Bozorth, private communication.

[18] Burgy, Hughes, Wallace, Heller, and Woolf, Phys. Rev. **80**, 953 (1950).

by transmission through a second piece of iron (the "double transmission effect"). The mirror method would be completely analogous to the transmission method, and hence can be considered the "double reflection effect."

The use of the double reflection effect for measurement of polarization would be quite simple were it not for depolarization and reorientation of the neutron spins in the space between the mirrors. Experiments[18] on the double transmission effect had already demonstrated the great importance of stray fields on the spin orientation, and it was no surprise when the first measurements with the apparatus of Fig. 8 showed no intensity change with reversal of field in the analyzer mirror. The failure to observe an effect under these conditions is caused partly by the reorientation of the polarized beam and partly by its depolarization, both phenomena taking place in the stray fields between the mirrors. The field changes direction and magnitude slowly as the neutrons move between the mirrors of Fig. 8, hence they remain aligned with the field (magnetic moment parallel to **H**), rotating adiabatically with it. The neutrons will thus approach the second mirror aligned with the field (instead of antiparallel to it) and will exhibit total reflection. The field may actually reverse several times in the space between the mirrors but the neutrons will always reflect from the analyzer and no double reflection effect will be observed, although the neutrons may be completely polarized.

The reorientation of the spins, which destroys the double reflection effect, will always occur if the field changes slowly for the antiparallel magnetization. In order to maintain the neutron alignment in space while the field changes direction, it is necessary to produce a nonadiabatic transition of the neutron relative to the field; that is, the field must change direction in a time short compared with the Larmor precession time. If the field is made quite small at some point between the magnets and in addition is made to reverse in a few millimeters distance, then the neutrons will pass through the reversal region in a time shorter than the Larmor period and nonadiabatic transitions will result. The neutrons will then strike the second mirror antiparallel to the field and will not reflect, thus exhibiting the desired double reflection effect. If the stray fields are of a random nature some adiabatic changes (reorientation of the beam) will occur as well as nonadiabatic (neutron alignment remaining fixed in space). At some of the field irregularities the field reversal time might be of the same order of magnitude as the precession time and both types of transitions would result, causing partial depolarization of the beam. Thus depolarization (which actually is a partial reorientation of the spins) will cause a decrease in the double reflection effect for the same reason that complete reorientation removes it.

In the double transmission work[18] the nonadiabatic transition region was obtained by use of a magnetic shield to reduce the field and permanent magnets to attain a sharp reversal point. The same method of field adjustment was adapted to the reflection experiments after the first attempts had given a zero double reflection effect. The magnetic shield used was a set of concentric soft iron cylinders surrounding the neutron beam. Small Alnico magnets were placed near the ends of the cylinders and adjusted so that the field inside the central cylinder remained perpendicular to the neutron beam except for a region less than a millimeter in extent where the field reversed. The field was observed during the adjustment by means of a small compass needle which was moved along the neutron path. The field at the reversal point is made perpendicular to the beam because the sharpest reversal point may be obtained with this configuration. The neutrons are actually polarized parallel to the direction of motion at reflection but the stray field near the edge of the mirror magnet turns them through approximately 90° as they leave the mirror vicinity.

With the field adjusted in the manner just described, the double reflection effect was observed but was not as large as that expected from a completely polarized beam. The polarization, P, of the beam incident on the analyzer mirror is calculated from the ratio, R, of the intensity reflected from the analyzer in the parallel to that in the antiparallel case:

$$P=(R-1)/(R+1). \tag{8}$$

This equation is based on the assumption which was later shown to be correct, that only one spin state reflects from the analyzer mirror, and the assumption, which is probably incorrect, that the transitions at the cross-over point are completely nonadiabatic. If the transitions are partly adiabatic, Eq. (8) would require minor modification. The highest values of R obtained experimentally were about 3 (50 percent polarization) and very careful adjustment of the field at the transition point was necessary to attain these values. The two most likely reasons for the low R seemed to be: (a) the neutrons leaving the polarizer might be incompletely polarized, and (b) there might be some adiabatic reorientation of the polarized neutrons in spite of the careful field adjustment. It was impossible to decide from the measured intensities whether improvement was needed at the mirror, such as higher magnetization; or at the transition point, such as better field adjustment.

A method of polarization measurement was then devised which would not depend on sensitive field adjustment at the transition point. In the new method only parallel magnetization is used so that the field between the mirrors can be kept high and fixed in direction. Under such conditions it is easy to insure that no transitions take place in the region between the magnets. The polarization is measured by inserting a thin sheet (about 0.006 in.), of unmagnetized iron in the beam between the mirrors and recording the change in intensity reflected from the analyzer. As the beam is completely depolarized in the iron sheet,[18] the polarized fraction of the beam, which before insertion of the iron sheet reflected completely at the analyzer, has a 50 percent reflectivity with the sheet in place. In other words, the sheet produces (for the polarized fraction of the neutrons) just half the effect that was produced in the old method by reversing the magnetization of the analyzer, but with no uncertainty concerning spin reorientation. The polarization is given directly in terms of the ratio, R_s, of the intensity before, to that after, insertion of the sheet (again with the assumption, later verified, that only one spin state reflects from the analyzer mirror):

$$P = R_s - 1. \tag{9}$$

The sheet scatters some neutrons out of the beam but the reduction in intensity from this cause is easily determined.

Measurement of the polarization with the iron sheet method proved to be simple and it was found that the polarization was 50 percent, in agreement with the results of the double reflection method. It was then certain that the polarization was indeed low and that the first mirror was somehow at fault. The most likely explanations were: (1) the cobalt might be less than 82 percent saturated and as a result both spin states would reflect to some extent and (2) the reflected beam might depolarize in passing near the edge of the polarizer mirror where the field is very inhomogeneous. The variation of the polarization with neutron wavelength was then checked by varying the angles of incidence at the polarizer and analyzer (keeping the angle at the analyzer always slightly less than that at the polarizer). The 50 percent polarization had been obtained for an incident angle of 8 min, or a limiting wavelength (assuming magnetic saturation) of 1.5A. The polarization was then measured for a 4-min angle (0.7A) and it turned out to be extremely small, while a 11.5-min angle (2.1A) gave a polarization of 60 percent.

The finding that higher polarization was obtained for larger incident angles tended to confirm the suspicion that depolarization resulted when the neutrons passed through the strong field inhomogeneities at the edge of the mirror. Adiabatic rotation of the plane of polarization as the neutrons leave the mirror would have no effect on the iron sheet method of polarization measurement. However, nonadiabatic transitions of some of the neutrons would of course reduce the polarization. The transitions should be less likely at larger angles because the neutrons pass the mirror edge at a larger distance, and because they move more slowly through the changing field. The angle was finally increased to 20 min (3.7A limiting wavelength) and the value of R_s increased to 2.01, indicating a polarization of 100 percent within the statistical accuracy of about one percent. No careful attempt was made to demonstrate the complete polarization by means of the double reflection technique because the iron sheet depolarization method was much simpler. Time did not permit rebuilding the cobalt mirror mounting in such a manner as to obtain complete polarization at short wavelengths, but work in this direction is now proceeding. Although the cobalt mirror was certainly not saturated, the attainment of complete polarization demonstrates that the magnetization was certainly above the critical value of 82 percent of saturation.

It is quite certain that beams of completely polarized neutrons, which can now be produced with high intensity (of the order of 10^5 per minute), will be useful in the study of magnetic and nuclear properties. Such studies, which have already been started with incompletely polarized neutrons, will be continued with this new technique of polarization by reflection. We wish to express our thanks to Mr. H. Ross of the Argonne machine shop and Mr. Yoder of the optical shop who prepared the excellent mirrors for these experiments. Dr. M. Hamermesh and Dr. H. Ekstein have aided greatly through many discussions.