

From the difference between  $\lambda_A$  and  $\lambda_c$  therefore one can readily calculate the relative difference in the masses  $m^+$  and  $m^-$  of positive and negative electrons. If our observed discrepancy is to be so interpreted in its entirety then

$$(m^- - m^+)/m^- = 2(\lambda_A - \lambda_c)/\lambda_A = 0.82 \times 10^{-4}. \quad (5)$$

The probable error to be assigned to this number we believe lies somewhere between an estimated upper "limit of error" of  $\pm 0.82 \times 10^{-4}$  and a probable error by internal consistency of the individual measurements themselves of  $\pm 0.1 \times 10^{-4}$ . The direction of the discrepancy is consistent with a heavier mass for negative than for positive electrons.

We wish to emphasize that the evidence for this discrepancy ( $\lambda_A - \lambda_c$ ), depends entirely on the possibility of calibrating the 2-meter curved crystal gamma-ray spectrometer with high absolute accuracy by means of x-rays. Other nuclear physics laboratories equipped with  $\beta$ -ray spectrometers may (and we hope will) attempt to verify with all the precision available the ratios of the energies of the various lines we have recently measured such as Au<sup>198</sup>, Cu<sup>64</sup>, Co<sup>60</sup>, Ta<sup>182</sup>, and since our measurements on some of these such as Au<sup>198</sup> are at present somewhat more accurate than our work to date on Cu<sup>64</sup> it may be possible in this way to improve our knowledge of  $\lambda_A$  but the absolute value of  $\lambda_A$  for comparison with  $\lambda_c$  must at present rest on the calibration of our instrument alone.

We plan in the near future to repeat the measurements of  $\lambda_A$  with higher accuracy. Recent very considerable improvements in the sensitivity of our instrument through the use of a crystal scintillation counter and an improved collimator will, we hope, make possible a considerable improvement in resolving power. We plan also to study the effect of changing the atomic number of the substance in which the annihilation takes place. Plans are also under way for a direct precision comparison of the charge-to-mass ratios  $e/m^+$  and  $e/m^-$  by a new method involving the new homogeneous field axial focusing  $\beta$ -ray spectrometer<sup>3</sup> whose construction at this Institute is now nearing completion.

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<sup>1</sup> I am indebted to W. K. H. Panofsky for pointing out to me the importance of this question.

<sup>2</sup> DuMond, Lind, and Watson, Phys. Rev. 75, 1226 (1949).

<sup>3</sup> J. W. M. DuMond and E. R. Cohen, Revs. Modern Phys. 20, 82 (1948).

<sup>4</sup> The shift because of the potential energies of the recombining pair members has only been estimated roughly. For this reason, our program includes study of the annihilation radiation in substances of different atomic numbers.

<sup>5</sup> De Benedetti, Cowan, and Konneker, Phys. Rev. 76, 440 (1949).

<sup>6</sup> J. W. M. DuMond, Phys. Rev. 75, 1226 (1949).

<sup>7</sup> Lind, West, and DuMond, Phys. Rev. 77, 475 (1950).

<sup>8</sup> No description of the design of this new instrument has, as yet, been published. Its design exploits the ideas of the author for obtaining optimum luminosity and energy resolution set forth in a recent paper in Rev. Sci. Instr. 20, 160 (1949).

### The Half-Life of Na<sup>24</sup>

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BEING concerned with the development of precise methods of measuring half-lives, we have used Na<sup>24</sup> as one of the isotopes for checking our approach to the problem. We have obtained a value of  $15.060 \pm 0.039$  hr. This is in accord with the result,  $15.04 \pm 0.04$  hr reported by Solomon.<sup>1</sup> Wilson and Bishop<sup>2</sup> have reported a value of  $14.90 \pm 0.02$  hr, which is at variance with our determination. It appears that an error in their analysis is responsible for the discrepancy. They indicated correctly that the points on the semilogarithmic plot of the activity versus time as obtained in their experiment must be weighted according to the square of the measured activity. For a decaying activity the weighting factors thus decrease with time. Instead of analyzing the decay directly, they compare the activity of the Na<sup>24</sup> source with that of a relatively long-lived source by considering the ratio of the activities. The ratio that they formed inadvertently was that of

the long-lived activity to the Na<sup>24</sup> activity. This function increases with time and thus cannot properly be considered the "activity" as far as the application of the weighting factors in their analytical treatment is concerned. Using their published data we have recalculated the runs, taking as the activity function to be analyzed the ratio  $u = (\text{Na}^{24} \text{ activity})/(\text{reference activity})$ .

With  $t$  the elapsed time in hours, the resulting linear logarithmic equations are:

Series I	$\log u = -1.32143 - 0.04576t$
Series II	$\log u = -0.89174 - 0.04667t$
Series III	$\log u = -1.27935 - 0.04590t$

giving, respectively, a half-life of:

$$15.150 \pm 0.070 \text{ hr}, \quad 14.852 \pm 0.041 \text{ hr} \quad \text{and} \quad 15.102 \pm 0.076 \text{ hr}.$$

The average half-life is thus  $14.96 \pm 0.10$  hr which falls within the range of our measurement. It should be noted that the error in measurement due to statistical variation of the reference activity was considered to be negligible.

The interest and encouragement of Dr. F. N. D. Kurie in this work is gratefully acknowledged, and the verification of our calculations by Dr. G. R. Bishop<sup>3</sup> is appreciated.

<sup>1</sup> A. K. Solomon, Phys. Rev. 79, 403 (1950).

<sup>2</sup> R. Wilson and G. R. Bishop, Proc. Phys. Soc. London A62, 457 (1949)

<sup>3</sup> G. R. Bishop (private communication, November 20, 1950).

### Choice of Gauge in London's Approach to the Theory of Superconductivity

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IT has been pointed out by London<sup>1</sup> that it is possible to derive the phenomenological equation of superconductivity:

$$\text{curl}(\Delta \mathbf{j}) = \mathbf{B}, \quad (1)$$

from quantum theory if it is assumed that the superconducting state is such that the wave function,  $\Psi$ , of the conduction electrons is not altered very much by the magnetic field. The expression for  $\Psi$  depends on the choice of gauge in the vector potential,  $\mathbf{A}$ . London assumes that  $\Psi$  is approximately equal to the wave function for zero field,  $\Psi_0$ , if the gauge is chosen in such a way that

$$\text{div} \mathbf{A}_s = 0; \quad \mathbf{A}_{s\perp} = 0 \text{ on surface.} \quad (2)$$

The subscript  $s$  will indicate this particular choice. For a simply connected region, these conditions determine the gauge uniquely.<sup>2</sup> The current density,  $\mathbf{j}$ , is then proportional to  $\mathbf{A}_s$ ;

$$\mathbf{j} = \mathbf{A}_s / \Lambda, \quad (3)$$

and the curl of this relation gives (1). While this procedure is reasonable, it seems desirable to derive (2) from a gauge invariant formulation of the theory.

Let  $\mathbf{A}$  be the vector potential for arbitrary choice of gauge. Terms in the Hamiltonian which involve the magnetic field are

$$H_m = (1/2m) \sum_{\alpha=1}^n \{ [\mathbf{p}_\alpha + e\mathbf{A}(\mathbf{r}_\alpha)/c]^2 - p_\alpha^2 \}, \quad (4)$$

where  $-e$  is the charge on an electron and the sum is over all electrons. Let us consider the class of wave functions of the form

$$\Psi = \exp[(ie/\hbar c) \sum_{\alpha} \varphi(\mathbf{r}_\alpha)] \Psi_0(\mathbf{r}_1 \cdots \mathbf{r}_n). \quad (5)$$

The exponential factor is of the type which is introduced when a gauge transformation

$$\mathbf{A} \rightarrow \mathbf{A} + \text{grad} \varphi$$

is made, and is required when the gauge is chosen arbitrarily. We shall choose  $\varphi$  in such a way as to make the first-order energy,

$W_m$ , arising from the magnetic field a minimum. This energy is

$$W_m = \int \Psi^* H_m \Psi d\tau = \frac{1}{2m} \Sigma_\alpha \int \Psi_0^* \left[ -\frac{i\hbar}{c} \operatorname{div}(\mathbf{A}_\alpha + \operatorname{grad}\varphi_\alpha) \right] - \frac{2i\hbar e}{c} (\mathbf{A}_\alpha + \operatorname{grad}\varphi_\alpha) \operatorname{grad}_\alpha + \frac{e^2}{c^2} (\mathbf{A}_\alpha + \operatorname{grad}\varphi_\alpha)^2 \Psi_0 d\tau_1 \cdots d\tau_n. \quad (6)$$

After an integration by parts, (6) can be written in the form

$$W_m = \frac{i}{c} \int (\mathbf{A} + \operatorname{grad}\varphi) \cdot \mathbf{j}_0(\mathbf{r}) d\tau - \frac{2}{2mc^2} \int (\mathbf{A} + \operatorname{grad}\varphi)^2 \rho_0(\mathbf{r}) d\tau, \quad (7)$$

where  $\mathbf{j}_0$  and  $\rho_0$  are the current and charge densities in the absence of a field:

$$\mathbf{j}_0(\mathbf{r}) = (i\hbar e/2m) \Sigma_\alpha \int (\Psi_0^* \operatorname{grad}_\alpha \Psi_0 - \Psi_0 \operatorname{grad}_\alpha \Psi_0^*) \times \delta(\mathbf{r} - \mathbf{r}_\alpha) d\tau_1 \cdots d\tau_n, \\ \rho_0 = -e \Sigma_\alpha \int \Psi_0^* \Psi_0 \delta(\mathbf{r} - \mathbf{r}_\alpha) d\tau_1 \cdots d\tau_n. \quad (8)$$

We shall assume that  $\mathbf{j}_0 = 0$  and  $\rho_0 = \text{const.}$  The condition that  $W_m$  be invariant for small changes  $\Delta\varphi$  in  $\varphi$  is then

$$\delta W_m = -(e\rho_0/mc^2) \int [(\mathbf{A} + \operatorname{grad}\varphi) \cdot \operatorname{grad}\Delta\varphi] d\tau = 0,$$

or, after an integration by parts

$$\int \Delta\varphi \operatorname{div}(\mathbf{A} + \operatorname{grad}\varphi) d\tau - \int \Delta\varphi (\mathbf{A} + \operatorname{grad}\varphi)_\perp dS = 0. \quad (9)$$

The second integral is over the surface,  $S$ . If (9) is to be satisfied for all  $\Delta\varphi$  we must have

$$\operatorname{div}(\mathbf{A} + \operatorname{grad}\varphi) = 0; \quad (\mathbf{A} + \operatorname{grad}\varphi)_\perp = 0 \text{ on } S. \quad (10)$$

Choosing  $\varphi$  in this way is equivalent to choosing the gauge so that (2) is satisfied and taking  $\Psi = \Psi_0$ .

We have used<sup>3</sup> this approach in a theory of superconductivity based on interactions between electrons and lattice vibrations.<sup>4</sup>

<sup>1</sup> F. London, Proc. Roy. Soc. A152, 24 (1935); Phys. Rev. 74, 562 (1948).

<sup>2</sup> London (reference 1) has considered the extension of the theory to cover multiply connected regions.

<sup>3</sup> J. Bardeen, Phys. Rev. 80, 567 (1950).

<sup>4</sup> J. Bardeen, Phys. Rev. 79, 167 (1950).

of the magnetic field required to produce ferromagnetic resonance<sup>1,4</sup> at 24,160 Mc. The width of the resonance line,  $\Delta H$ , and the  $g$  factor<sup>5</sup> were also observed. The variations in  $\Delta H$  with crystallographic direction were not large enough to be significant. The results are given in Table I.

TABLE I. Measurements on zinc manganese ferrite.

$K/M$	$M$ (cgs units)	$K$ (ergs/cc)	$\Delta H$ (oe)	$g$
$-21 \pm 5\%$	$181 \pm 10\%$	$-3800 \pm 15\%$	70	1.997

Check measurements were made on a second sample. This sample did not show cubic symmetry magnetically but the magnitude of the variations in the field required for resonance was the same.

Snoek<sup>6</sup> has suggested that the effect field due to magneto-crystalline anisotropy acting on a material in no external field will cause resonance or perhaps relaxation behavior in the initial permeability of the material; this behavior should occur at the frequency of the Larmor precession in the effective field. This effective internal field when the magnetization is along an easy direction ([111] in these crystals) is:<sup>7</sup>

$$H_{\text{eff}} = -1.33K/M.$$

This leads us to expect this behavior at 78 Mc in this material. This frequency may be increased if the effective field is increased by the presence of free poles due to the domain pattern; it is therefore a lower limit in practice.

The authors wish to express their gratitude to Dr. E. A. Wood for x-ray assistance and to H. G. Hopper for technical assistance.

<sup>1</sup> Yager, Galt, Merritt, and Wood, Phys. Rev. 80, 744 (1950).

<sup>2</sup> Galt, Matthias, and Remeika, Phys. Rev. 79, 391 (1950).

<sup>3</sup> C. Guillaud, and M. Roux, Compt. rend. 229, 1133 (1949).

<sup>4</sup> C. Kittel, Phys. Rev. 73, 155 (1948).

<sup>5</sup> C. Kittel, Phys. Rev. 76, 743 (1949).

<sup>6</sup> J. L. Snoek, Nature 160, 90 (1947); Physica 14, 207 (1948).

<sup>7</sup> L. R. Bickford, Jr., Phys. Rev. 78, 449 (1950).

## Crystalline Magnetic Anisotropy in Zinc Manganese Ferrite

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WE have grown crystals of zinc manganese ferrite whose composition is approximately  $\text{Zn}_{0.55}\text{Mn}_{0.45}\text{Fe}_2\text{O}_4$ . From the saturation magnetization,  $M_s$ , and from ferromagnetic resonance data, we have determined the crystalline magnetic anisotropy,  $K$ , of these crystals. The measurements were made at room temperature, by essentially the same procedure as the earlier measurements on nickel ferrite.<sup>1,2</sup> The value of  $M_s$  was measured by comparison with nickel ferrite in a hysteresis loop tracer;  $M_s$  for nickel ferrite has been measured by Guillaud and Roux<sup>3</sup> as well as by us.<sup>2</sup>

The crystals were grown in a manner similar to that by which nickel ferrite crystals were grown earlier.<sup>2</sup> A mix of 15 g borax, 6.4 g  $\text{Fe}_2\text{O}_3$ , 1.6 g ZnO, 1.4 g MnO was heated to 1325°C in a platinum crucible and held for 5½ hours. It was then cooled at approximately 100°C/hr simply by turning off the furnace. The resulting crystals ranged in size up to octahedra 1 mm on an edge. Most of them were not mechanically sound, but one good one was found which showed cubic symmetry magnetically. Chemical analysis showed that this material is approximately  $\text{Zn}_{0.55}\text{Mn}_{0.45}\text{Fe}_2\text{O}_4$ . There were no significant impurities.

The value of  $K/M$  for these materials at room temperature was determined from the variation with crystallographic direction

## The Absence of Cross-Over Transitions in Tellurium Isomers

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A MODIFIED estimate of multipole radiation probabilities has been put forward recently by Weisskopf.<sup>1</sup> According to this theory, the probability  $\lambda$  of electric  $2^e$  pole radiation is given by

$$\lambda = \frac{2(e+1)}{e} [1 \cdot 3 \cdot 5 \cdots (2e+1)]^{-2} \frac{K^{2e+1}}{\hbar} e^2 R^{2l},$$

and the probability of magnetic  $2^e$  pole radiation is lower than that of electric  $2^e$  pole by a factor  $\sim (\hbar/McR)^2$ .  $K$  is the wave number,  $2\pi\nu/c$ , of the radiation,  $M$  is the nucleon mass, and  $R$  is the nuclear radius. It is to be realized that this formula, like previous ones based on older estimates, is claimed to be valid only to within a factor  $\sim 10^{\pm 2}$ . In the following note it is pointed out that Weisskopf's formula, unlike earlier ones, is consistent with the absence of cross-over transitions in the decay of several long-lived tellurium isomers.

Isomers of  $\text{Te}^{121, 123, 125}$  have been observed<sup>2</sup> to decay in two successive transitions, the first being preponderantly magnetic  $2^4$  pole and the second being magnetic dipole. It was puzzling, on the basis of the older radiation formulas, that no cross-over transitions of an electric  $2^5$  character, between the long-lived and the ground states, were observed experimentally.

Relative probabilities of magnetic  $2^4$  and electric  $2^5$  transitions have been calculated and are tabulated in Table I. Columns 2 and 3 give, respectively, the energies of the long-lived magnetic