method that the transfer of momentum to the nucleus is represented by the removal of a virtual quantum from the field, and is hence parallel to **k**. Consequently the planes of all pairs produced contain **k** exactly. The difference between this statement and the apparently similar language of the Berlin-Madansky condition may be understood as follows. Presumably in the more accurate theory the probable values of the difference $\phi_+-\phi_--\pi$ are rather small; if this is true the procedure we follow is equivalent to averaging the Bethe-Heitler cross section over the previously difference, rather than considering the case when the difference is equal to zero. The virtual quanta are, on the whole, unpolarized. Hence we need the pair production cross section for two quanta moving in opposite directions, one of them linearly polarized, one unpolarized. This is known[4] to be, in the center of mass of the two quanta:

$$d\sigma=(\beta r_0^2/2x^2)\{(1-\beta^2\cos^2\theta)^{-1}-\tfrac{1}{2}+2\beta^2(1-\beta^2)\times(1-\beta^2\cos^2\theta)^{-2}\sin^2\theta\cos^2\phi\}\sin\theta d\theta d\phi,$$

where βc is the velocity of either electron, $r_0=e^2/mc^2$, $x=h\nu/mc^2=(1-\beta^2)^{-\frac{1}{2}}$, θ is the angle between the electrons and the photons, while ϕ is the azimuth of the plane containing electrons and photons measured from the plane of polarization of the polarized photon. We integrate over θ since we are only interested in the dependence on ϕ. Finally we must transform the frequencies to the system in which one of the quanta has an energy $h\nu_1=mc^2$, and integrate over a spectrum $Cd\nu_2/\nu_2$ for the other quantum. The result is:

$$d\sigma=\tfrac{2}{3}Cr_0^2(1+\tfrac{1}{3}\cos^2\phi)d\phi,$$

which exhibits an azimuthal dependence of comparable magnitude to that found by Berlin-Madansky in their special case. The sign, however, is the opposite: the plane of the pair prefers to be parallel to the electric vector.

The applicability of the Weizsäcker-Williams method to the present problem may be doubted; in particular one may fear that the transverse momentum transfer to the nucleus, which is neglected in this method, might affect the directions of the particles in such a way as to alter the correlation between polarization and directions of motion entirely. It may be pointed out, however, that the transverse momenta of the pair are of the order of mc, while the momentum transfer to the nucleus is much smaller than mc in a majority of the collisions[5] (if $h\nu\ll mc^2$). We believe, therefore, that the Berlin-Madansky conclusions apply only if the condition they postulate ($\phi_+=\phi_-+\pi$) is strictly satisfied. An investigation of the Bethe-Heitler formula under more general conditions is under way.

* This work was performed under the auspices of the AEC.

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The Masses of the Negative and Positive Electrons*

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BECAUSE of its far-reaching theoretical consequences, great importance attaches to the question whether positive electrons differ in mass from negative electrons and the writer has recently been urged[1] to present such experimental evidence as now exists on this point. With E. R. Cohen, he has been engaged for the last several months in the preparation of a completely new least-squares evaluation of all of the atomic constants in the light of a number of new and very precise measurements in the microwave and other fields. These results will soon be released in the form of a preprint. It appears from this study that the present "best" value of the Compton wavelength $h/(mc)$ is

$$\lambda_c=h/(mc)=(2.426067\pm0.000032)\times10^{-10}\text{ cm.}\qquad(1)$$

In this equation, m is clearly the mass of the negative electron because the measurements used involved negative electrons only.

On the other hand, a recent direct measurement[2] by DuMond, Lind, and Watson with the 2-meter curved crystal gamma-ray spectrometer at this Institute of the wavelength λ_A of the annihilation radiation from Cu^{64} yielded the result

$$\lambda_A=(2.4271\pm0.0010)\times10^{-10}\text{ cm}\qquad(2)$$

larger than λ_c by about four parts in 10^4 with an *assigned* uncertainty of about the same order as the difference, $\lambda_A-\lambda_c$.

The uncertainty of the measurement of the annihilation radiation wavelength was very conservatively, perhaps *too* conservatively, estimated with a large allowance for unknown systematic errors. On the basis of only *the internal consistency* of the seven measurements, a calculation of the probable error yields

$$\lambda_A=(2.4271\pm0.00012)\times10^{-10}\text{ cm,}\qquad(3)$$

which is only about one-eighth the error claimed in the paper. In fixing our assigned uncertainty at the higher value, $\pm0.001\times10^{-10}$ cm, two considerations determined our estimate. (1) We were aware of a discrepancy of this order between our observed λ_A and the value of λ_c from the DuMond and Cohen[3] 1947 least-squares evaluation of the constants. (The new reevaluation of the constants has not materially changed this discrepancy.) (2) The uncertainty we assigned (and the discrepancy) correspond to a motion of our source carriage and precision wavelength screw of only 0.01 mm and this did not seem to be an unreasonable safe *upper limit of systematic* error although *we have no direct evidence that errors this large exist.* The difference between λ_c and λ_A corresponds to a quantum energy difference of a little over 200 ev.

Three possible theoretical sources of difference between λ_A and λ_c have been considered and rejected as probably[4] insufficient to explain the discrepancy. Two of these were discussed briefly (pages 1226 and 1237) in the DuMond, Lind, and Watson paper: (1) A possible shift because of the potential energies and (2) a possible shift because of the kinetic energies of the members of the recombining pairs. The low kinetic energies, which we derived from our own observation of the Doppler broadening of the annihilation line itself, have since been further verified in a beautiful independent method by De Benedetti, Cowan, and Konneker[5] with results in accord with ours. The third possible source of shift from Compton modified scattering in the source material has also been analyzed by the author in a recent letter to the editor[6] and shown to be too small to explain the discrepancy.

A possible source of systematic shift has recently been suggested to us by L. Alvarez. Since the line exhibits an observable spectral breadth at half-maximum of the order $\Delta\lambda_A/\lambda_A=0.004$ there will be an appreciable difference in the intensity response of the instrument on the two sides of the line, (1) because, as Lind, West, and DuMond have shown,[7] the reflecting power of the crystal varies about as λ^2, (2) because the multi-cellular counter efficiency depends on some power of the wavelength. This last is difficult to estimate since it depends on the absorption of the radiation in the counter partition walls on the one hand and on the ratio of the range of the ejected electrons to the wall thickness on the other hand. If we assume an over-all instrumental response increasing as λ^3 then the line center at half-maximum height would be shifted toward longer wavelengths by about $0.009a$, where a is the half-breadth at half-maximum, or by a proportionate wavelength shift, $\delta\lambda/\lambda=3.8\times10^{-5}$ which is a ten times smaller quantity than the discrepancy in question. For an instrumental response obeying a higher power of λ then the cube, $\delta\lambda/\lambda$ would only be increased in proportion to the exponent, and it seems very unlikely that this will account for the discrepancy.

The annihilation radiation line, since it results from the recombination of positive with negative electrons, should have a wavelength

$$\lambda_A=h[\tfrac{1}{2}(m^++m^-)c]^{-1}.\qquad(4)$$

From the difference between λ_A and λ_c therefore one can readily calculate the relative difference in the masses m^+ and m^- of positive and negative electrons. If our observed discrepancy is to be so interpreted in its entirety then

$$(m^- - m^+)/m^- = 2(\lambda_A - \lambda_c)/\lambda_A = 0.82 \times 10^{-4}. \qquad (5)$$

The probable error to be assigned to this number we believe lies somewhere between an estimated upper "limit of error" of $\pm 0.82 \times 10^{-4}$ and a probable error by internal consistency of the individual measurements themselves of $\pm 0.1 \times 10^{-4}$. The direction of the discrepancy is consistent with a heavier mass for negative than for positive electrons.

We wish to emphasize that the evidence for this discrepancy $(\lambda_A - \lambda_c)$, *depends entirely on the possibility of calibrating the 2-meter curved crystal gamma-ray spectrometer with high absolute accuracy by means of x-rays.* Other nuclear physics laboratories equipped with β-ray spectrometers may (and we hope will) attempt to verify with all the precision available the *ratios* of the energies of the various lines we have recently measured such as Au^{198}, Cu^{64}, Co^{60}, Ta^{182}, and since our measurements on some of these such as Au^{198} are at present somewhat more accurate than our work to date on Cu^{64} it may be possible in this way to improve our knowledge of λ_A *but the absolute value of* λ_A for comparison with λ_c must at present *rest on the calibration of our instrument alone.*

We plan in the near future to repeat the measurements of λ_A with higher accuracy. Recent very considerable improvements in the sensitivity of our instrument through the use of a crystal scintillation counter and an improved collimator will, we hope, make possible a considerable improvement in resolving power. We plan also to study the effect of changing the atomic number of the substance in which the annihilation takes place. Plans are also under way for a direct precision comparison of the charge-to-mass ratios e/m^+ and e/m^- by a new method involving the new homogeneous field axial focusing β-ray spectrometer[8] whose construction at this Institute is now nearing completion.

* Assisted by the joint program of the ONR and AEC.

[1] I am indebted to W. K. H. Panofsky for pointing out to me the importance of this question.

[2] DuMond, Lind, and Watson, Phys. Rev. **75**, 1226 (1949).

[3] J. W. M. DuMond and E. R. Cohen, Revs. Modern Phys. **20**, 82 (1948).

[4] The shift because of the potential energies of the recombining pair members has only been estimated roughly. For this reason, our program includes study of the annihilation radiation in substances of different atomic numbers.

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[6] J. W. M. DuMond, Phys. Rev. **75**, 1226 (1949).

[7] Lind, West, and DuMond, Phys. Rev. **77**, 475 (1950).

[8] No description of the design of this new instrument has, as yet, been published. Its design exploits the ideas of the author for obtaining optimum luminosity and energy resolution set forth in a recent paper in Rev. Sci. Instr. **20**, 160 (1949).

The Half-Life of Na^{24}

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BEING concerned with the development of precise methods of measuring half-lives, we have used Na^{24} as one of the isotopes for checking our approach to the problem. We have obtained a value of 15.060±0.039 hr. This is in accord with the result, 15.04 ±0.04 hr reported by Solomon.[1] Wilson and Bishop[2] have reported a value of 14.90±0.02 hr, which is at variance with our determination. It appears that an error in their analysis is responsible for the discrepancy. They indicated correctly that the points on the semilogarithmic plot of the activity *versus* time as obtained in their experiment must be weighted according to the square of the measured activity. For a decaying activity the weighting factors thus decrease with time. Instead of analyzing the decay directly, they compare the activity of the Na^{24} source with that of a relatively long-lived source by considering the ratio of the activities. The ratio that they formed inadvertently was that of the long-lived activity to the Na^{24} activity. This function increases with time and thus cannot properly be considered the "activity" as far as the application of the weighting factors in their analytical treatment is concerned. Using their published data we have recalculated the runs, taking as the activity function to be analyzed the ratio $u = (Na^{24}\text{ activity})/(\text{reference activity})$.

With t the elapsed time in hours, the resulting linear logarithmic equations are:

Series I	$\log u = -1.32143 - 0.04576t$
Series II	$\log u = -0.89174 - 0.04667t$
Series III	$\log u = -1.27935 - 0.04590t$

giving, respectively, a half-life of:

15.150±0.070 hr, 14.852±0.041 hr and 15.102±0.076 hr.

The average half-life is thus 14.96±0.10 hr which falls within the range of our measurement. It should be noted that the error in measurement due to statistical variation of the reference activity was considered to be negligible.

The interest and encouragement of Dr. F. N. D. Kurie in this work is gratefully acknowledged, and the verification of our calculations by Dr. G. R. Bishop[3] is appreciated.

[1] A. K. Solomon, Phys. Rev. **79**, 403 (1950).

[2] R. Wilson and G. R. Bishop, Proc. Phys. Soc. London **A62**, 457 (1949)

[3] G. R. Bishop (private communication, November 20, 1950).

Choice of Gauge in London's Approach to the Theory of Superconductivity

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IT has been pointed out by London[1] that it is possible to derive the phenomenological equation of superconductivity:

$$\mathrm{curl}(\Lambda \mathbf{j}) = \mathbf{B}, \qquad (1)$$

from quantum theory if it is assumed that the superconducting state is such that the wave function, Ψ, of the conduction electrons is not altered very much by the magnetic field. The expression for Ψ depends on the choice of gauge in the vector potential, **A**. London assumes that Ψ is approximately equal to the wave function for zero field, Ψ_0, if the gauge is chosen in such a way that

$$\mathrm{div}\mathbf{A}_s = 0; \quad \mathbf{A}_{s\perp} = 0 \text{ on surface.} \qquad (2)$$

The subscript s will indicate this particular choice. For a simply connected region, these conditions determine the gauge uniquely.[2] The current density, **j**, is then proportional to $\mathbf{A}_s$;

$$\mathbf{j} = \mathbf{A}_s/\Lambda, \qquad (3)$$

and the curl of this relation gives (1). While this procedure is reasonable, it seems desirable to derive (2) from a gauge invariant formulation of the theory.

Let **A** be the vector potential for arbitrary choice of gauge. Terms in the Hamiltonian which involve the magnetic field are

$$H_m = (1/2m) \sum_{\alpha=1}^{n} \{[\mathbf{p}_\alpha + e\mathbf{A}(\mathbf{r}_\alpha)/c]^2 - p_\alpha^2\}, \qquad (4)$$

where $-e$ is the charge on an electron and the sum is over all electrons. Let us consider the class of wave functions of the form

$$\Psi = \exp[(ie/\hbar c)\Sigma_\alpha \varphi(\mathbf{r}_\alpha)]\Psi_0(\mathbf{r}_1 \cdots \mathbf{r}_n). \qquad (5)$$

The exponential factor is of the type which is introduced when a gauge transformation

$$\mathbf{A} \rightarrow \mathbf{A} + \mathrm{grad}\,\varphi$$

is made, and is required when the gauge is chosen arbitrarily. We shall choose φ in such a way as to make the first-order energy,