protons ejected by the sun. Since previous work[1] had shown the hydrogen lines in auroral spectra to be broadened, the writer and D. F. Berkey determined at that time to make a critical test of the hydrogen using two spectrographs. One spectrograph was set up at the airport at Arnprior, Ontario, 330 km north of the second spectrograph at Ithaca, New York.

On the night of September 30, 1950, a moderate aurora appeared with homogeneous arcs and later rayed arcs at elevations of about 10° and 20° above the northern horizon as seen from Ithaca.

The spectrograph (called *B*) at Ithaca was aimed at the arc at 20°, while H. Stevinson of the Flight Research Section at Arnprior operated spectrograph *A* so that it pointed between the zenith and 10° north of the zenith. Telephone communication assured that the spectrographs were aimed at *the identical arc*, that the angle between the lines of sight was between 70° and 90° and that the same part of the arc was observed.

Both spectrographs used Bausch and Lomb replica gratings. The National Geographic Society spectrograph (*A*) had a camera lens of 50-mm focus and aperture *f*:1.6, while the U.S. Signal Corps spectrograph *B* had a camera lens of 90-mm focus and aperture *f*:1.

The results are set forth in Fig. 1. Microphotometer traces

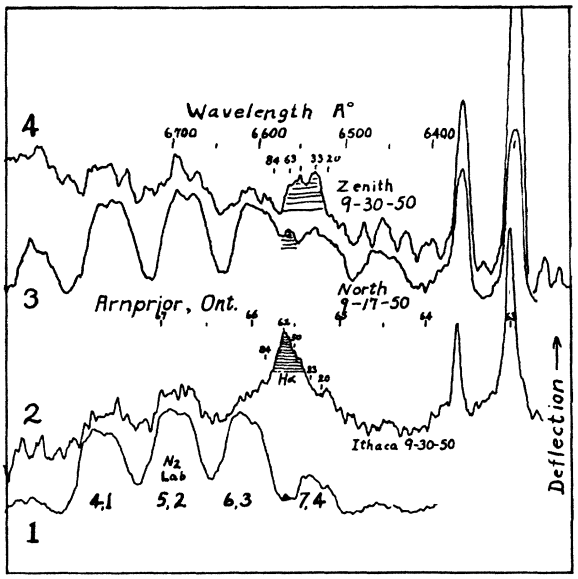

FIG. 1. Microphotometer traces of three aurora spectrograms. Spectra 2 and 4 taken simultaneously. Curves displaced in deflection direction.

were made of the several plates, and these were photographed so that the dispersions were brought to a common scale. Trace 1 is the spectrum of a nitrogen discharge in the laboratory. Trace 3 is from an aurora at elevation 20° above the north horizon at Arnprior. Trace 2 from Ithaca and trace 4 from Arnprior are from the simultaneous spectra of this experiment. In trace 2 the $H\alpha$ line is an outstanding feature and of the same intensity as OI 6363A. The $H\alpha$ is centered on 6562A and is nearly symmetric. The extension on the blue side is presumed to be due to superposition on N_2 band 7,4. The cross lines indicate the wavelength range over which $H\alpha$ was probably spread, that is, from about 6580 to 6535. In trace 4 the $H\alpha$ peak rises sharply from about 6575 to the peak at 6563 and spreads to shorter wavelength to approximately 6520. Thus, there is a clear shift to shorter wavelength on the Arnprior spectrum which is consistent with motion of the hydrogen in spiralling down the lines of magnetic force. The graininess of the records and interference from N_2 bands prevents precision in velocity determination, but preliminary conclusions can be drawn. Trace 2 gives an $H\alpha$ broadening to red and violet of about 10A, indicating a velocity 450 km/sec in the line of sight. This is either the velocity of the scattered hydrogen or the line of sight component of the spiralling velocity. From trace 4 we estimate a mean shift of 15A and a maximum of at least 30A, giving a mean velocity of 675 km/sec and maximum velocity of 1350 km/sec. The experiment will be repeated.

Thus, this experiment establishes that the auroral hydrogen is coming into the earth's atmosphere from outside. Since our previous spectra taken about at right angles to the auroral rays (earth's field) show only a broadening, even when the spectrograph was aimed eastward, while this spectrum taken along the rays shows a broadened shift, we see that there is a strong influence of the earth's field and that the particles are charged; therefore, they must be protons.

Similar results have been obtained by Meinel[2] with two exposures by the same spectrograph, on the great aurora of August 19 (cloudy at Ithaca). His two pictures from the same station show the broadening and a much greater shift. The maximum velocity found was 3200 km/sec. These high velocities occurred during a great aurora (magnetic disturbance $K=9$) while the velocities we find, about 1350 km/sec occurred during a moderate aurora, $K=5$. This suggests that there is a relation between the incoming velocities and the size of aurora, and also that protons may be the immediate cause of the aurora. That the hydrogen behaves in this manner is attested by the broadness of $H\beta$ on all our plates taken since 1940 and by the shift of $H\alpha$ appearing in two auroras at widely separated points at different times.

Thanks are here expressed to D. W. R. McKinley of the National Research Council of Canada and G. S. Levy of the Flight Research Section for use of the facilities at Arnprior and to H. Stevinson for operating the spectrograph.

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Penetration and Diffusion of X-Rays. Calculation of Spatial Distributions by Polynomial Expansion*

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October 9, 1950

THE propagation of x-rays, or of neutrons, through an infinite medium, under the influence of absorption (including pair production) and of multiple scattering, is governed by a well-known transport equation.[1] The successive moments of the space distribution are related by recurrence relationships. These relationships have been recently applied to an analytical formulation of the diffusion of electrons in a homogeneous medium.[2]

The zero-*th* moment, i.e., the simple space integral of the x-ray distribution, has already been calculated numerically in a number of sample cases.[3,4] Other moments can be calculated numerically without excessive labor, by manual operation. Automatic computers might prove useful for the same purpose. Notice that studies of the x-ray propagation must rely on numerical calculations more heavily than do studies of neutron or electron propagation, because of the complicated analytical form of the Klein-Nishina cross section.

It is also widely recognized that a knowledge of the moments of a distribution function enables one, in principle, to reconstruct the function itself. However, little effort seems to have been applied along this line in the study of radiation diffusion problems, presumably because the method was not expected to prove convenient.

This note points out a convenient application of the moment method in obtaining rapidly convergent expansions of the distribution function in suitable polynomial systems. A suitable set of polynomials is one whose "weight function"[5] $w(x)$ serves as a

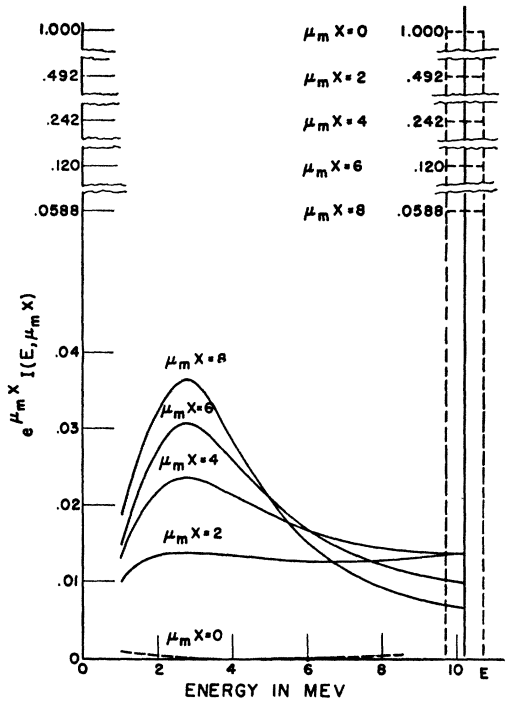

FIG. 1. Differential spectra $I(E, \mu_m x)$ of the x-ray intensity at various distances from a plane monodirectional 10.2-Mev source in Pb. The position of the source spectrum is indicated by the solid vertical line, its intensity by the area of the dotted rectangle. The scale of ordinates is normalized to unit strength of the source at $x=0$; at greater depths it discounts an exponential decay corresponding to the absorption coefficient μ(3.2 Mev) $=\mu_m=0.469$ cm^{-1} of the most penetrating component. The dotted line for $x=0$ departs from the base line only because of the approximations made.

zero-order approximation to the actual distribution function $f(x)$. If the weight function $w(x)$ which one chooses should happen to coincide with $f(x)$ the corresponding polynomial expansion of this function would reduce to a single term $[f(x)=w(x)\cdot 1]$. Even though the weight function departs substantially from the actual distribution, a few terms of the polynomial expansion $\{f(x)\sim w(x)[1+P_1(x)+P_2(x)+\cdots]\}$ appear to compensate the departures effectively over a wide range of values of x.

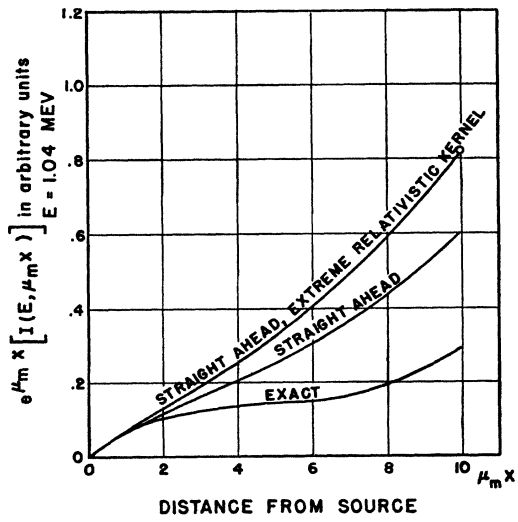

FIG. 2. Differential intensity $I(E, \mu_m x)$ of the 1.04-Mev component of the spectrum of Fig. 1 as a function of the distance from the source, calculated by different methods: (a) as in Fig. 1 ("exact"), (b) disregarding the deflection of scattered photons ("straight ahead"), (c) using the extreme relativistic form of the Klein-Nishina cross section. The point at the end of the top curve is the result of an asymptotic calculation by Karr and Hurwitz (unpublished).

This procedure has been tested by application to standard (Bessel and confluent hypergeometric) mathematical functions. A simple exponential decay proves to be quite adequate as a choice of weight function for the distribution of x-rays up to fairly deep penetration into a material (10 to 15 mean free paths of the primary radiation).

A full report on these calculations will be given in the *Journal of Research* of the National Bureau of Standards.

Two initial numerical applications have been made:

(a) *Plane monodirectional source of 10.2-Mev x-rays in lead.*—This calculation was extended only down to an energy of 1.04 Mev.

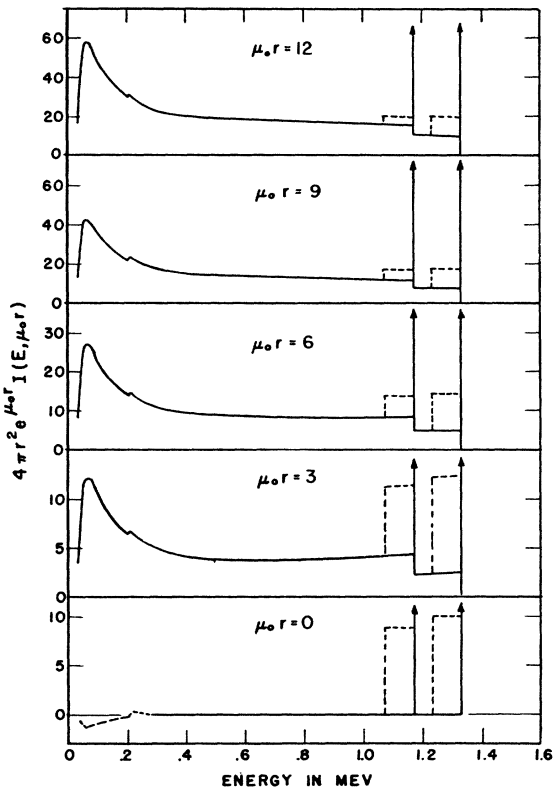

FIG. 3. Differential spectra $I(E, \mu_0 r)$ of the x-ray intensity at various distances r from a Co^{60}-point isotropic source in water. The positions of the two lines of the source spectrum are indicated by the solid vertical lines, their intensities by the area of the dotted rectangles. The scales of ordinates are normalized to unit intensity of the 1.33-Mev component of the source; $\mu_0=0.0612$ cm^{-1} is the narrow-beam absorption coefficient of this component.

Therefore backscattering could be disregarded and one could use an ordinary expansion in Laguerre polynomials. Four terms of the expansion were used. The results tre shown in Figs. 1 and 2.

(b) *Co^{60} isotropic point source of x-rays in water.*—This calculation has been extended down to energies at which the photoelectric absorption suppresses the secondary radiation, so that comparisons could be made with the experimental results.[6] This application required the development of special non-self-adjoint polynomial expansions which utilize only even- or odd-order moments. Four terms of the expansion were used. The results are shown in Figs. 3 and 4.

Analytical formulas have also been developed for plane monodirectional, plane isotropic, and point monodirectional source geometries. The latter geometry is the basic one, in that any source may be resolved into a combination of point monodirectional sources. Sample numerical applications of these formulas are in progress.

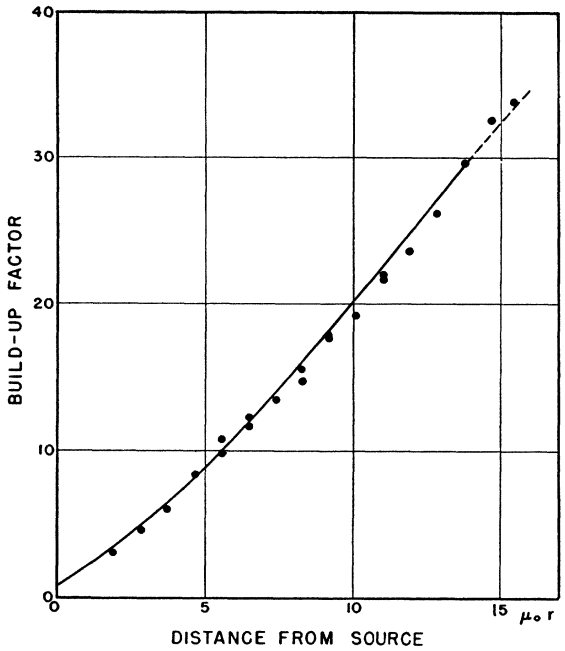

FIG. 4. Comparison of the calculated curve for the "build-up factor" with the experimental results of reference 6. The spectral distributions illustrated in Fig. 3 were weighted with the theoretical response in roentgens of a standard free air ionization chamber and then again with the specific correction factor of the chamber used in reference 6. The integral response over the whole spectrum divided by the calculated response to the primary radiation alone gives the theoretical build-up factor.

These results seem to give some confidence in the following conclusions.

(1) A method is available for calculating the distribution of x-rays in uniform media with a moderate amount of labor up to fairly large depths of penetration.

(2) This technique can presumably be applied with success to the broad class of transport phenomena governed by Boltzmann-type equations. It takes advantage even of rather crude theoretical predictions on the behavior of a distribution function to formulate an initial approximation which can thereafter be rapidly improved by straightforward numerical work.

The present approach is complementary to the earlier papers[7] which dealt primarily with the asymptotic behavior of the photon distribution and which made extensive use of the method of Laplace transforms. The work reported in those papers has now been developed into a more comprehensive treatment which will be the object of a separate report.

We wish to thank Mrs. F. A. Stinson for carrying out most of the numerical work.

* Work supported by the ONR, Applied Mathematics Branch.

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Spontaneous Fission in Cm^{242}

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December 4, 1950

SPONTANEOUS fission has been observed in Cm^{242}. It was detected initially in samples of Am^{241} irradiated with slow neutrons in the Chalk River pile. Chemical separation in ion-exchange resin columns showed that curium was responsible. Irradiations of widely differing intensity yielded curium samples with the same ratio of spontaneous fission rate to Cm^{242} α-activity. This, and the fact that the spontaneous fission rate decayed with the same half-life as Cm^{242}, identified the isotope.

The fission counter was similar to one previously used by Tunnicliffe[1] in this laboratory. The samples, on 1-in. diameter smooth platinum disks, were mounted on the electron collecting electrode, which was located at the center of curvature of the negative H.T. electrode, a hemisphere of radius 2 in. The counter was filled to a pressure of 8 cm Hg with argon containing 5 percent of carbon dioxide, and thus only the first $\frac{1}{2}$ cm (air equivalent) of the fission fragment and α-tracks was effective. This method of improving the discrimination between the two particles was first used in this laboratory by Pontecorvo and West.[2]

The counter was operated with the H.T. electrode at a potential of -400 v, sufficient to ensure minimum collection time.[3] Using a linear amplifier (T.R.E. type 1008A) with rise and fall time constants set to 0.15 μsec the "build-up" of α-pulses was unimportant for source strengths up to about 3×10^8 α-disintegrations/min.

The spontaneous fission rate was measured as 6.2/min per 10^8 Cm^{242} α-disintegrations/min. The over-all limits of error in this figure should not exceed 2 percent. Using a value of 162.5 days for the α half-life[4] the spontaneous fission half-life becomes $(7.2\pm0.2)\times10^6$ years.

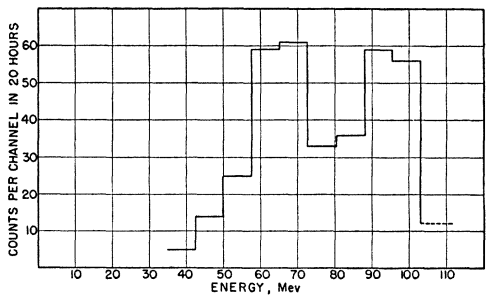

FIG. 1. Energy distribution of the fission pulses.

A rough measurement was made of the energy distribution of the spontaneous fission fragments in an ionization chamber rather similar to that described by Bunemann, Cranshaw, and Harvey.[5] The ion chamber contained two plane parallel electrodes 10 cm square 4.3 cm apart, and a shielding grid placed 1.5 cm in front of the electron-collecting electrode. The source was mounted in in the center of the negative H.T. electrode (cathode) and covered with a simple collimator, a sheet of 0.37-mm brass drilled with 3-mm holes. The chamber was filled to a pressure of one atmosphere with argon containing 5 percent carbon dioxide. A collecting field (between cathode and grid) of 470 volts/cm was used, amply adequate for saturation, and the grid was run at -1.5 kv to prevent its collecting any of the electrons traveling to the collector.

Preliminary experiments on α-particles (using a larger electrode spacing and somewhat lower field strengths) showed that the pulse distribution was well preserved even with rise and fall time constants in the amplifier as low as 0.5 μsec. With this fast response the "build-up" pulses of fission energy size from a source of the strength used (5×10^6 α-disintegrations/min) would be negligible compared with the observed spontaneous fission rate of 18 per hour.

TABLE I. Energies of the fragments in the fission of several nuclei.

Nucleus	Most probable energy (Mev) Light fragment	Heavy fragment	Ratio
Spontaneous fission in Cm^{242}	95 97, 94 91	65 68, 66 65	1.43
$Pu^{239}+n$	94.6	65.2	1.45
$U^{235}+n$	94.5	60.2	1.57
$U^{233}+n$	93.0	56.6	1.64