B. Analysis of Experimental Uncertainties

Counter Angle θ

Assuming that Q_2 is the accurately known Q-value, then the error in Q_1 due to the uncertainty in the counter angle θ is

$$(\Delta Q_1)_\theta = (2/A_1)(2E_{p1}E_{d1})^{\frac{1}{2}} \sin\theta_1 \Delta\theta_1 - (2/A_2)(2E_{p2}E_{d2})^{\frac{1}{2}} \sin\theta_2 \Delta\theta_2.$$

Thus for a given error in θ, the minimum error in Q for a heavy nucleus is obtained for small angles for a heavy reference nucleus. However, elements heavier than aluminum either do not have well-resolved ground-state peaks or their Q-values are not accurately known. This is the reason for selecting aluminum for a reference and choosing an angle of 30°. The counter can be aligned to the deuteron beam to an accuracy of about 1°. There are two small effects that change the effective angle of the counter. One effect, owing to the finite solid angle and size of the deuteron beam on the target, increases the angle. At 30° the effective counter angle is increased by about 0.5°. The other small effect is a result of the angular distribution of the protons. Since for light elements the distribution is strongly forward, the effective angle is less than the measured angle. However, since the aluminum ground-state peak has a maximum[44] at 30°, this second effect is not present. For the aluminum ground-state peak at 30° with 13.9-Mev deuterons and 19-Mev protons, an error in angle

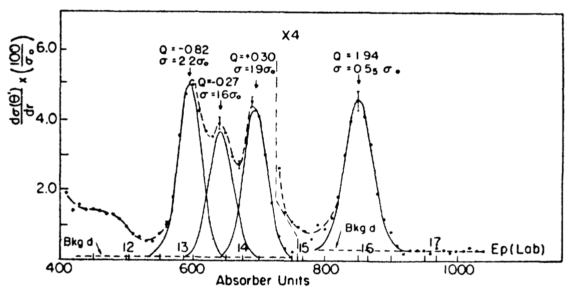

FIG. 1. Proton spectrum at $\theta = 50°$ from a 10-mg/cm² bismuth target bombarded with 14-Mev deuterons from the reaction $Bi^{209}(d, p)Bi^{210}$. The ordinate is the differential cross section in the center-of-mass system per absorber unit in terms of the ground-state proton peak from aluminum $(\sigma_0) \cdot \sigma_0 = 2.5 \pm 1.0 \times 10^{-27}$ cm²/steradian. θ' is the angle measured in the center-of-mass system. The abscissa is in units of 0.416 mg/cm² of Al. The proton energy in the laboratory system is also given. The vertical bar represents the square root of the number of counts.

of 1.5° results in an error in the Q-value for a heavy nucleus of 20 kev.

Range of Protons R

The center of the proton peak is chosen as a measure of the mean range of the proton group, since this corresponds to the protons produced at the center of a thick target. The range is measured in absorber units, which are then converted into mg/cm² of aluminum by the conversion factor that 1 absorber unit = 0.416 mg/cm² of aluminum. To the range measured in the foil changer must be added the thickness of foils and gas in the triple counter (10 mg/cm²). There are two other small corrections to be applied to the range. By changing the gate settings, the peak can be made to shift in range. This correction can be approximately determined by comparing a differential and an integral run. For the gate settings used, this correction amounted to about 4 mg/cm². The other small effect is due to the large solid angle and the size of the deuteron beam on the target. Thus, the protons do not pass through the foils exactly at right angles. The average angle is about 4° from the normal, resulting in a correction of 1.5 mg/cm² for the aluminum ground-state peak. The mean range is accurate to a few mg/cm². The difference between the range of 2 proton peaks is accurate to 1 mg/cm² (20 kev).

Range-Energy Curve

The range-energy curve used is that calculated by Smith.[45] Assuming that the computed values for the energy loss per mg/cm² are accurate to 1 percent, the difference in energy of the two proton groups will be accurate to 1 percent.

Energy Loss in Target

In order to obtain sufficient counting rates, targets were of the order of 20 mg/cm² thick. Since the center of the proton peak is taken as a measure of the proton energy, Q-values must be calculated at the center of the target. Bethe and Livingston[46] have calculated the atomic stopping power of various elements at different proton energies. This can be converted into mg/cm² of the various elements equivalent to 1 mg/cm² of aluminum. For a 20-mg/cm² lead target, the deuteron energy loss is about 200 kev in reaching the center of the target and the proton energy loss another 100 kev. Thus, it is necessary that the measured mass/cm² and the calculated conversion values be accurate to 5 percent. Therefore, it was decided to measure the thickness of all targets for 14-Mev deuterons and for high energy protons. This was done by inserting the target in the deuteron beam or the ground-state aluminum protons and measuring the shift of the aluminum ground-state peak at 30°. Since in both cases the target thickness is measured over the same central region as is bombarded, this eliminates errors due to the non-uniformity of the foils. The target thickness for the deuteron beam and for the protons can be measured to 20 kev, and thus the half-target thickness is accurate to 10 kev. The error due to uncertainty in the target angle is negligible.

C. Factors Contributing to the Spread

Since the proton peaks are broad (full width at half-maximum is 500 kev for the aluminum ground-state

[44] H. E. Gove, Phys. Rev. **81**, 364 (1951).

[45] J. H. Smith, Phys. Rev. **71**, 32 (1947).

[46] M. S. Livingston and H. A. Bethe, Revs. Modern Phys. **9**, 272 (1937).