PHYSICAL REVIEW VOLUME 81, NUMBER 3 FEBRUARY 1, 1951

Scattering and Absorption of Scalar and Pseudoscalar Mesons by Nucleons

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(Received September 11, 1950)

Cross sections have been calculated for scattering of pi-mesons by nucleons and for absorption of the charge of the meson in the scattering process. Computations are carried out to second order in g^2, using scalar and pseudoscalar fields with scalar and pseudoscalar coupling, respectively. It is found that there are strong qualitative differences in the cross sections depending upon field type and upon the type of mixture where neutral mesons are concerned.

I. INTRODUCTION

THE present status of the weak-coupling meson theories of nuclear forces is far from satisfactory. Many theories agree qualitatively with the experimental information, but no theory has yet been devised which presents a consistent quantitative interpretation of all the well established observations. It is most reasonable to attribute the failures of the theories to large coupling constants. However, it has not yet been possible to formulate a strong coupling theory in a relativistic manner, and in the absence of any other model of action-at-a-distance forces, it seems useful to investigate further the consequences of the weak-coupling theories in the hopes of finding some clearcut agreement or disagreement.

It is entirely possible that the difficulties encountered in the past are a result of considering the relatively complicated problem of forces between nucleons and then deducing the properties of the mesons in an indirect way. This hypothesis could be tested by performing experiments dealing directly with mesons. Such experiments, while characteristically more difficult to carry out in the laboratory, show at least some prospect of unambiguous theoretical interpretation. The purpose of this paper is to study the theoretically simplest problem of the latter kind; the second order scattering of mesons by nucleons.

We shall assume that pi-mesons of mass $276m_e$ are the only field particles interacting with nucleons, regarded as Dirac particles of mass $1836m_e$. We shall be concerned primarily with charged mesons, and shall assume that neutral mesons differ from them only in their lack of charge. Finally, we shall deal with spinless mesons (i.e., scalar and pseudoscalar fields) whose coupling constant will be assumed to be small in spite of the indirect experimental evidence to the contrary.

II. THE MATRIX ELEMENTS

We shall use the following notation. Three-vectors are represented in ordinary boldface, 4-vectors in italics. The time component is real, and we use the convention $a\cdot b=a_4b_4-\mathbf{a}\cdot\mathbf{b}\cdot a$ is the length of the three-vector $\mathbf{a}$. We treat the matrices $\gamma_\mu=\beta\alpha_\mu$ ($\mu=1, 2, 3, 4$) as a four-vector. $\gamma_\mu\gamma_\nu+\gamma_\nu\gamma_\mu=2g_{\mu\nu}$ where $g_{44}=1$, $g_{11}=g_{22}=g_{33}=-1$, and all other $g_{\mu\nu}=0$. Thus by our convention $\gamma_\mu\gamma_\mu=+4$. $\gamma_5=\gamma_1\gamma_2\gamma_3\gamma_4$ and its adjoint $\bar{\gamma}_5=\gamma_4\gamma_3\gamma_2\gamma_1=+\gamma_5$. If a is an ordinary four-vector, $\mathfrak{a}=a_\mu\gamma_\mu=a_4\gamma_4-a_1\gamma_1-a_2\gamma_2-a_3\gamma_3$. Boldface capitals like $\mathbf{M}$ will denote transition matrix elements. In the scattering problem, $k=(\mathbf{k}, \omega)$ is the momentum-energy vector of the incident meson, $k'=(\mathbf{k}', \omega')$ that of the scattered meson, and similarly $p=(\mathbf{p}, E)$ and $p'=(\mathbf{p}', E')$ for the nucleon. We use natural units throughout, i.e. $\hbar=c=1$, g in esu.

The Feynman diagrams for the scattering process are shown in Fig. 1. Picture 1 refers to the scattering of a positive meson by a neutron or of a negative meson by a proton. Picture 2 is for scattering of a negative meson by a neutron or a positive meson by a proton. For scattering of a neutral meson or for conversion of a charged to a neutral meson in the scattering process, the amplitudes for the two diagrams must be added with the appropriate relative phase.

The matrix elements for these diagrams are given by Feynman[1] for pseudoscalar and scalar mesons respectively as[2]

$$\mathbf{M}_1{}^P=\bar{u}'\bar{\gamma}_5\frac{1}{\mathfrak{p}+\mathfrak{k}-M}\gamma_5 u, \tag{1}$$

$$\mathbf{M}_2{}^P=\bar{u}'\bar{\gamma}_5\frac{1}{\mathfrak{p}-\mathfrak{k}'-M}\gamma_5 u, \tag{2}$$

$$\mathbf{M}_1{}^S=\bar{u}'\frac{1}{\mathfrak{p}+\mathfrak{k}-M}u, \tag{3}$$

$$\mathbf{M}_2{}^S=\bar{u}'\frac{1}{\mathfrak{p}-\mathfrak{k}'-M}u, \tag{4}$$

where M is the nucleon mass, u, u' the Dirac spinors (normalized so $\bar{u}u=1$) of the initial and final state and $\bar{u}$, $\bar{u}'$ their adjoints ($\bar{u}=u^*\beta$). μ is used for the pi-meson mass.

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[1] R. P. Feynman, Phys. Rev. **76**, 769 (1949).

[2] The absolute sign of the matrix element does not enter the problem to second order, and will not be elaborated upon here. Actually, if the expansion coefficient $-ig^2$ is used, following the formalism of quantum electrodynamics, it becomes necessary to use γ_5 for emission and $\bar{\gamma}_5$ for absorption in the pseudoscalar theory, and 1 and -1 in the scalar theory.

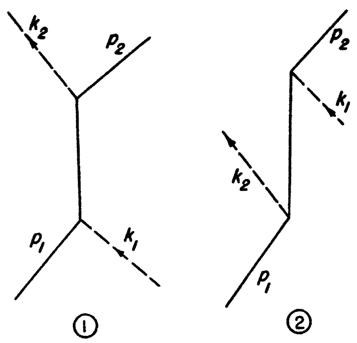

FIG. 1. The Feynman diagrams for second-order scattering. Picture 1 refers to scattering of π^+ by N or π^- by P, while Picture 2 refers to scattering of π^+ by P or π^- by N. For any process involving neutral mesons, the amplitudes for the two processes must be added.

By using the momentum conditions ($p^2=p'^2=M^2$, $k^2=k'^2=\mu^2$, $p+k=p'+k'$) and the Dirac equation ($(\mathfrak{p}-M)u=(\mathfrak{p}'-M)u'=0$), the matrix elements can be simplified to read

$$\mathbf{M}_1{}^P=+(\bar{u}'\mathfrak{k}u)(\mu^2+2k\cdot p)^{-1} \tag{5}$$

$$\mathbf{M}_2{}^P=-(\bar{u}'\mathfrak{k}u)(\mu^2-2k'\cdot p)^{-1} \tag{6}$$

$$\mathbf{M}_1{}^S=(\bar{u}[2M+\mathfrak{k}]u)(\mu^2+2k\cdot p)^{-1} \tag{7}$$

$$\mathbf{M}_2{}^S=(\bar{u}[2M-\mathfrak{k}]u)(\mu^2-2k'\cdot p)^{-1}. \tag{8}$$

The cross sections are proportional to $|\mathbf{M}|^2$, and since we are not interested in the spin states of the nucleons (which can never be turned over by a spinless meson), it will be most convenient to calculate one-half the trace of the operator $\bar{\mathbf{M}}\Lambda'\mathbf{M}\Lambda$, where the projection operator $\Lambda=(\mathfrak{p}+M)/2M$ and $\Lambda'=(\mathfrak{p}'+M)/2M$.

The traces are readily evaluated to give the invariant forms

$$T_1{}^P=\frac{\mu^2(\mu^2-k\cdot k')+2(p\cdot k)(p\cdot k')}{2M^2(\mu^2+2p\cdot k)^2}, \tag{9}$$

$$T_2{}^P=\frac{\mu^2(\mu^2-k\cdot k')+2(p\cdot k)(p\cdot k')}{2M^2(\mu^2-2p\cdot k')^2}, \tag{10}$$

$$T_1{}^S=\frac{2(2M^2+p\cdot k)(2M^2+p\cdot k')-(4M^2-\mu^2)(\mu^2-k\cdot k')}{2M^2(\mu^2+2p\cdot k)^2}, \tag{11}$$

$$T_2{}^S=\frac{2(2M^2-p\cdot k)(2M^2-p\cdot k')-(4M^2-\mu^2)(\mu^2-k\cdot k')}{2M^2(\mu^2-2p\cdot k')^2}, \tag{12}$$

where

$$T=\tfrac{1}{2}\,\mathrm{Trace}\{\bar{\mathbf{M}}\Lambda'\mathbf{M}\Lambda\}.$$

III. THE DIFFERENTIAL CROSS SECTIONS

The relation between differential cross sections and the matrix elements in the Feynman notation is most usefully written in the covariant form

$$\frac{d\sigma}{d\Omega}=\frac{1}{v}\cdot\frac{g^4}{\omega\omega'}\cdot\frac{M}{E}\cdot\frac{M}{E'}k'^2\left[\frac{k'}{\omega'}-\frac{\mathbf{p}'\cdot\mathbf{k}'}{E'k'}\right]^{-1}T. \tag{13}$$

v is the velocity difference and $d\Omega$ is the element of solid angle corresponding to the scattering angle θ.

Relation (13) can be derived from the Lagrangian method[3] or from the relation to the S-matrix.[4] It is easily understood in terms of the ordinary perturbation theory, since T is proportional to the usual $|H|^2$. The factors ω^{-1} and M/E arise from normalization of the initial state, and all the rest, except $1/v$ for the initial state, is proportional to the density of normalized final states per energy interval. The numerical coefficient can be obtained by requiring conservation of probability.

[3] R. P. Feynman, Phys. Rev. **80**, 440 (1950).
[4] F. J. Dyson, Phys. Rev. **75**, 1736 (1949).

In the laboratory system, Eq. (13) reads

$$\frac{d\sigma}{d\Omega}=g^4\frac{k'}{k}\frac{M}{\omega'}T\left[\frac{M}{\omega'}+\frac{\omega}{\omega'}-\frac{k}{k'}\cos\theta\right]^{-1}, \tag{14}$$

and

$$kk'\cos\theta=\omega\omega'-M(\omega-\omega')-\mu^2.$$

Equations (9) through (12) can be put into (14) to give the differential cross sections in the laboratory system.

$$\frac{d\sigma_1{}^P}{d\Omega}=\frac{g_P{}^4}{2}\frac{k'^3}{k}\times\frac{2M\omega\omega'-\mu^2(\omega-\omega')}{(\mu^2+2M\omega)^2[M\omega\omega'-\mu^2(M+\omega-\omega')]}, \tag{15}$$

$$\frac{d\sigma_2{}^P}{d\Omega}=\left(\frac{2M\omega+\mu^2}{2M\omega'-\mu^2}\right)^2\frac{d\sigma_1{}^P}{d\Omega}, \tag{16}$$

$$\frac{d\sigma_1{}^S}{d\Omega}=\left(\frac{g_S}{g_P}\right)^4\left(1+\frac{8M^2(M+\omega)}{2M\omega\omega'-\mu^2(\omega-\omega')}\right)\frac{d\sigma_1{}^P}{d\Omega}, \tag{17}$$

$$\frac{d\sigma_2{}^S}{d\Omega}=\left(\frac{g_S}{g_P}\right)^4\left(1+\frac{8M^2(M-\omega')}{2M\omega\omega'-\mu^2(\omega-\omega')}\right)\frac{d\sigma_2{}^P}{d\Omega}. \tag{18}$$

It is instructive to examine the limiting forms of the differential cross sections for very high and very low energies. In the nonrelativistic limit, the cross sections are most conveniently written in terms of the fractional momentum transfer $\mathbf{q}=(\mathbf{k}-\mathbf{k}')/k$, which is related to the scattering angle by

$$\cos\theta=[1-\tfrac{1}{2}(1+\mu/M)q^2][1-(\mu/M)q^2]^{-\frac{1}{2}}.$$

Then Eq. (14) takes the form

$$d\sigma/d\Omega=L(q)T, \qquad \text{(19)N.R.}$$

where

$$L(q)=g^4(1-0.150q^2)^{\frac{3}{2}}(1-0.0862q^2)^{-1} \qquad \text{(20)N.R.}$$

and

$$T_1{}^P=0.216M^{-2}[1+0.070(k/\mu)^2 -0.081(k/\mu)^2q^2] \qquad \text{(21)N.R.}$$

$$T_2{}^P=0.292M^{-2}[1-0.081(k/\mu)^2 +0.0815(k/\mu)^2q^2] \qquad \text{(22)N.R.}$$

$$T_1{}^S=\mu^{-2}[1-0.860(k/\mu)^2-0.0004(k/\mu)^2q^2] \qquad \text{(23)N.R.}$$

$$T_2{}^S=\mu^{-2}[1-1.16(k/\mu)^2+0.18(k/\mu)^2q^2]. \qquad \text{(24)N.R.}$$

In the extreme relativistic limit, virtually all the mesons come out in the forward direction because of the motion of the center of gravity. In this case it is more useful to look at the energy distribution of the scattered meson, which can be obtained from Eq. (14) and reads

$$\frac{d\sigma}{d\omega'}=2\pi\frac{d\cos\theta}{d\omega'}\frac{d\sigma}{d\Omega}=2\pi r_0{}^2\left(\frac{M}{\omega}\right)^2T, \qquad \text{(25)E.R.}$$

where $r_0=g^2/M$, the effective nucleon radius for mesons. Then

$$d\sigma_1{}^P=d\sigma_1{}^S=\tfrac{1}{2}\pi r_0{}^2\omega^{-3}\omega'd\omega' \qquad \text{(26)E.R.}$$

$$d\sigma_2{}^P=d\sigma_2{}^S=\tfrac{1}{2}\pi r_0{}^2\omega^{-1}\omega'^{-1}d\omega'. \qquad \text{(27)E.R.}$$

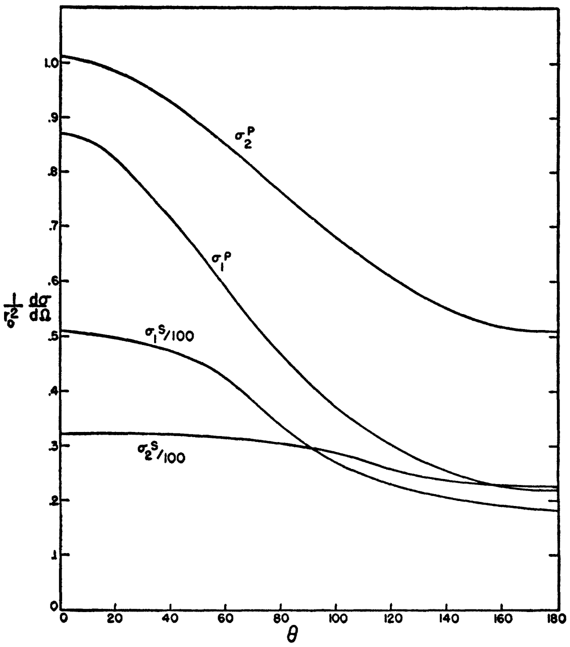

FIG. 2. Angular distributions in the laboratory system for $\omega=2\mu$.

It should be noted that r_0 depends upon g and will be different for the scalar and pseudoscalar cases, and that these equations are valid only for $\omega'\gg M$, so that (27) must not be integrated down to low energies.

The angular distributions for $\omega=2\mu$ are displayed in Fig. 2.

IV. THE TOTAL CROSS SECTIONS

The total cross sections are obtained most easily by carrying out the integration in the center-of-mass system, where Eq. (13) reads

$$d\sigma_c/d\Omega_c=g^4M^2(\omega_c+E_c)^{-2}T.$$

The subscript, c, refers to the center-of-mass system. T, and the total cross section, σ, are invariants. We have then the following relations.

$$\sigma_1{}^P=\frac{2\pi g_P{}^4}{(\omega_c+E_c)^2}\cdot\frac{2\omega_c{}^2E_c{}^2+(2\omega_cE_c-\mu^2)(\omega_c{}^2-\mu^2)}{[(\omega_c+E_c)^2-M^2]^2}, \tag{28}$$

$$\sigma_2{}^P=\frac{\pi g_P{}^4}{(\omega_c+E_c)^2}\cdot\left[\frac{\mu^4}{(2\omega_cE_c-\mu^2)^2-4(\omega_c{}^2-\mu^2)^2}+\frac{2\omega_cE_c+2\omega_c{}^2-\mu^2}{4(\omega_c{}^2-\mu^2)}\ln\frac{2\omega_cE_c+2\omega_c{}^2-3\mu^2}{2\omega_cE_c-2\omega_c{}^2+\mu^2}\right], \tag{29}$$

$$\sigma_1{}^S=\frac{2\pi g_S{}^4}{(\omega_c+E_c)^2}\cdot\frac{2\omega_c{}^2E_c{}^2+(2\omega_cE_c-\mu^2)(\omega_c{}^2-\mu^2)+8M^2E_c(\omega_c+E_c)}{[(\omega_c+E_c)^2-M^2]^2}, \tag{30}$$

$$\sigma_2{}^S=\frac{\pi g_S{}^4}{(\omega_c+E_c)^2}\cdot\left[\frac{(4M^2-\mu^2)^2}{(2\omega_cE_c-\mu^2)^2-4(\omega_c{}^2-\mu^2)^2}+\frac{(\omega_c+E_c)^2-9M^2}{4(\omega_c{}^2-\mu^2)}\ln\frac{2\omega_cE_c+2\omega_c{}^2-3\mu^2}{2\omega_cE_c-2\omega_c{}^2+\mu^2}\right]. \tag{31}$$

These results can be taken over into the laboratory system, making use of the following formulas

$$\omega_c+E_c=(M^2+\mu^2+2M\omega)^{\frac{1}{2}},$$
$$\omega_c=(\mu^2+M\omega)/(\omega_c+E_c).$$

In the extreme cases they become, for $\omega\gg M$,

$$\sigma_1{}^P=\sigma_1{}^S=\tfrac{1}{4}\pi r_0{}^2\omega^{-1}, \qquad \text{(32)E.R.}$$

$$\sigma_2{}^P=\sigma_2{}^S=\tfrac{1}{4}\pi r_0{}^2\omega^{-1}\ln[2M\omega/(M^2-\mu^2)], \qquad \text{(33)E.R.}$$

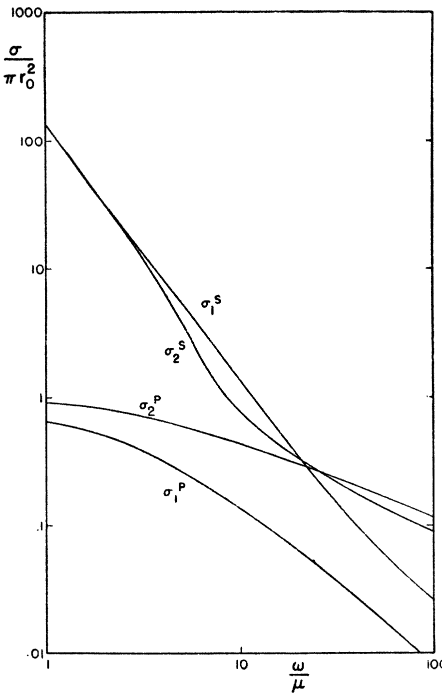

FIG. 3. Total cross section (in units of the nucleon "area") for nuclear scattering *vs.* energy of the incident meson (in units of its mass) for processes 1 and 2, using scalar and pseudoscalar theories. The effect of Coulomb forces has been neglected.

and for $\omega-\mu\ll\mu$,

$$\sigma_1^P=\pi r_0^2[0.654-0.216(\omega-\mu)/\mu], \quad (34)\text{N.R.}$$

$$\sigma_2^P=\pi r_0^2[0.889-0.127(\omega-\mu)/\mu], \quad (35)\text{N.R.}$$

$$\sigma_1^S=\pi r_0^2[136-265(\omega-\mu)/\mu], \quad (36)\text{N.R.}$$

$$\sigma_2^S=\pi r_0^2[136-272(\omega-\mu)/\mu]. \quad (37)\text{N.R.}$$

The behavior of the total cross sections as functions of ω are shown in Fig. 3.

V. CONVERSION PROCESSES

There is also the possibility that a charged meson will lose its charge to the nucleon in the scattering process and be re-emitted as a neutral meson. To calculate the cross section for this conversion it is necessary to include both Feynman diagrams of Fig. 1. The matrix element will then be $M_a=M_1+aM_2$, where a depends on the coupling assumed between mesons and nucleons of the two kinds. In Kemmer's charge-symmetric theory, where the coupling constant of the neutron is equal and opposite to that of the proton, $a=-1$. On the other hand, if we assume "ordinary" neutral mesons (i.e., nucleons having the same coupling constants for neutral and for charged mesons), then $a=+1$. Values other than ±1 are very unlikely, since they would make the neutron-neutron force different from the proton-proton force, contrary to the experience with mirror nuclei.

The differential cross section for the mixed field is given by $\sigma_a=[\sigma_1^{\frac{1}{2}}\pm a\sigma_2^{\frac{1}{2}}]^2$. The relative sign follows from Eqs. (5) through (8), and is negative in the scalar but positive in the pseudoscalar theory. The cross term is integrated in the manner discussed in Sec. IV to yield the result

$$\sigma_a^P=\sigma_1^P+a^2\sigma_2^P+a\frac{2\pi g_P^4}{(\omega_c+E_c)^2}\times\left[1+\frac{\mu^4}{4[(\omega_c+E_c)^2-M^2](\omega_c^2-\mu^2)}\times\ln\frac{2\omega_cE_c+2\omega_c^2-3\mu^2}{2\omega_cE_c-2\omega_c^2+\mu^2}\right], \quad (38)$$

$$\sigma_a^S=\sigma_1^S+a^2\sigma_2^S-a\frac{2\pi g_S^4}{(\omega_c+E_c)^2[(\omega_c+E_c)^2-\mu^2]}\times\left[(\omega_c+E_c)^2-9M^2+\frac{(4M^2-\mu^2)^2}{4(\omega_c^2-\mu^2)}\times\ln\frac{2\omega_cE_c+2\omega_c^2-3\mu^2}{2\omega_cE_c-2\omega_c^2+\mu^2}\right]. \quad (39)$$

These results are plotted as functions of ω in Fig. 4. In every case $\sigma_a=a^2\sigma_2$ in the limit of very high energies, but at low energies there are great differences.

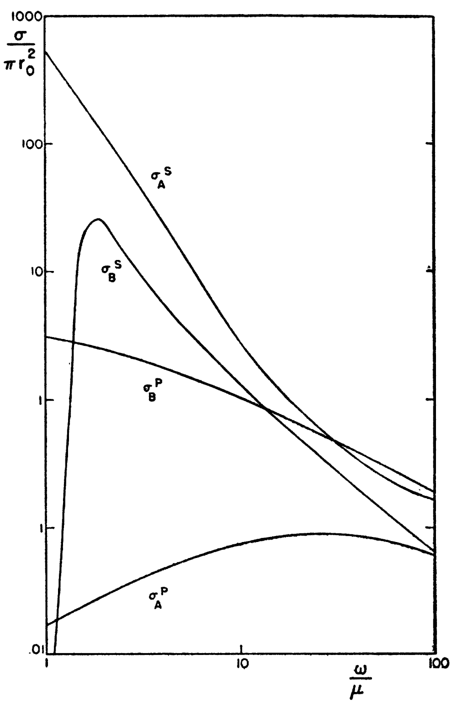

FIG. 4. Total cross section (in units of the nucleon "area") for charge absorption in the scattering process *vs.* energy of the incident meson (in units of its mass), using scalar and pseudoscalar theories. Subscript A refers to the charge-symmetric mixture, B to the "ordinary" mixture.

The "ordinary" mixture leads to cross sections given by the formulas

$$\sigma^P = \pi r_0^2[3.06 - 0.76(\omega-\mu)/\mu], \quad (40)\text{N.R.}$$

$$\sigma^S = \pi r_0^2[1.4(\omega-\mu)^2/\mu^2], \quad (41)\text{N.R.}$$

while the charge-symmetric theory gives

$$\sigma^P = \pi r_0^2[0.0174 + 0.0176(\omega-\mu)/\mu], \quad (42)\text{N.R.}$$

$$\sigma^S = \pi r_0^2[544 - 1077(\omega-\mu)/\mu]. \quad (43)\text{N.R.}$$

VI. RESULTS

There are certain strong qualitative differences among the cases calculated. By comparison of these results with scattering and conversion data, it should be possible to determine whether charged mesons are scalar or pseudoscalar and to find how the neutral mesons are coupled to nucleons of both kinds, provided that neutral and charged mesons are of the same type.

The difference between scalar and pseudoscalar mesons are most striking at low energies. The absolute cross sections will be nearly the same in the two theories despite the apparent disparity because the coupling constants must be adjusted to fit the binding energy of the deuteron. However, it will be about proportional to ω^{-2} for scalar mesons and to $\omega^{-0.2}$ for pseudoscalar. An additional possible distinction is that the total scattering cross sections for scattering of positive and negative mesons by the same type of nucleon are equal for scalar, but differ by about 14 percent for pseudoscalar mesons.

The influence of the mixture on the conversion cross sections is very great at very low energies. The most important feature is that for pseudoscalar theory, the "ordinary" mixture leads to a conversion cross section of four times the scattering cross section, while the charge-symmetric mixture predicts virtually no production of neutral mesons. In the scalar theory, the situation is reversed. This is a particularly useful result since it provides a sensitive measurement of a in case the mixture is of neither of these limiting types.

At intermediate energies, like $\omega=2\mu$, it becomes possible to get information from the angular distribution as well as the total cross section. Then the total cross section for scattering is still an appreciable fraction of the zero-energy cross section for pseudoscalar, but is very much smaller for scalar mesons. While the differential cross section at zero energy is in every case isotropic in the center of gravity system, at somewhat higher energies the pseudoscalar theory gives a stronger backward cross section for process 1 and an equally stronger forward cross section for process 2. The scalar theory shows instead a very weak backward preference for process 1 and an extremely strong forward preference for process 2. In the laboratory system, this means that for σ_1^P, σ_2^P, and σ_1^S, the cross section is peaked forward, while for σ_2^S it is very flat.

The conversion cross section at intermediate energies is again very sensitive to the mixture. For pseudoscalar mesons it rises with the energy in the charge-symmetric mixture, and falls in the "ordinary" theory. For scalar mesons the situation is again reversed.

At extremely high energies, $\omega \gg M$, it is no longer interesting to look at differential cross sections, since the angular distribution is essentially caused by the motion of the center of gravity, and even the energy spectrum is independent of the theory used. However, the total cross section is now most useful, the pseudoscalar being over 100 times larger than the scalar. In each case the conversion cross section is just a^2 times the cross section for scattering by process 2. The absolute sign of a can be determined easily from the direction of the correction due to the linear term in a, as given by Eqs. (38) and (39).

VII. LIMITATIONS

The two principal modifications of these results will arise from radiative corrections and Coulomb effects, the former being both more important and much more difficult to obtain than the latter.

The scattering of charged mesons by protons must, of course, be modified by the Coulomb scattering. Except for very low energies, this will only add some forward scattering. The sign of the interference term between Coulomb and nuclear scattering, if observable, might be of interest because the nuclear scattering of negative mesons by protons (process 1) has the opposite sign for scalar and pseudoscalar theories.

The main limitation of our theory is the neglect of radiative corrections. Some fourth-order corrections have been calculated in the nonrelativistic limit.[5] These indicate that the corrections will be considerable at low energies if g^2 is of the order of unity. Unfortunately there is not at present any estimate of what influence the fourth order has at intermediate or high energies. Higher orders than the fourth, even in the Thomson limit, would be extremely difficult to obtain.

The author wishes to thank Professor Hans A. Bethe of Cornell University, who suggested the problem, for his continued assistance and encouragement.

[5] Ashkin, Simon, and Marshak, Prog. Theo. Phys. (to be published). The author is indebted to them for a comparison of some results.