On the Displacement of Corresponding Energy Levels of C^{13} and N^{13}

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It is investigated to what extent the change in boundary conditions at the nuclear surface due to Coulomb wave function distortion in the external region can explain the relative displacement of the first excited states of C^{13} and N^{13}. It is found that the calculated displacement is in the right direction and of a sufficiently large magnitude, but rather sensitively dependent on the definition of nuclear radius.

I. INTRODUCTION

IT is well known that the energy differences between the ground states of the members of a pair of mirror nuclei can be explained, apart from the neutron-hydrogen mass difference, as due to the Coulomb repulsion existing between a pair of protons. This means that the nuclear radii as calculated from the Coulomb energy differences can be expressed by an equation of the form

$$r=r_0A^{\frac{1}{3}}, \tag{1}$$

where A is the mass number of the nucleus, and r_0 is fairly constant for all mirror nuclei, being equal[1] to about 1.45×10^{-13} cm. Moreover, the spins and parities are presumed to agree for both mirror nuclei, and there seems to be no definite experimental evidence to contradict this assumption.

These circumstances may be explained by assuming that the force between two protons is the same as that between two neutrons except for the Coulomb repulsion. From this assumption it would then seem to be plausible to anticipate that also in the excited states of mirror nuclei there will be a similar correspondence in the positions of energy levels, provided that the radii in the excited states are the same as in the ground states. Now, while in the ground states the energy differences can be fitted with the above-indicated variation of the nuclear radius to within 100 kev (except in the case of $Al^{27}-Si^{27}$, where[2] it is 150 kev), in the excited states there appear some very large discrepancies.

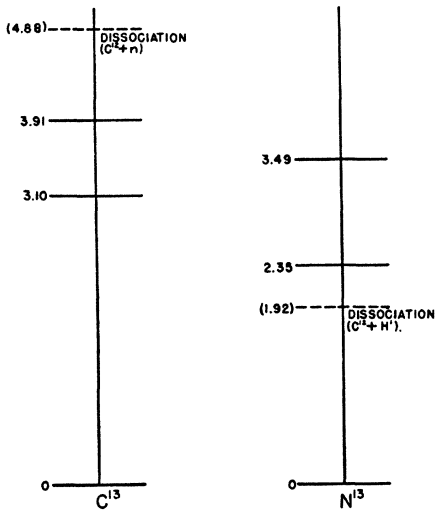

FIG. 1.

In the case of $C^{13}-N^{13}$, the first two excited levels[3] of the two nuclei lie at about 3.10 Mev and 3.91 Mev for C^{13}, and at 2.35 Mev and 3.49 Mev for N^{13} (Fig. 1). A level in C^{13} at 0.8 Mev found by the reaction $B^{10}(\alpha,p)C^{13}$ has not been found by other reactions and will not be considered here. If we now make the tentative assumption that the above mentioned levels are really corresponding levels in the two nuclei, then, at first sight, it would appear that not only must the nuclear radius be considered to be larger in the excited states than in the ground states, but this change of radius would not even be monotonic in going from one level to the next.

However, a more careful analysis indicates that the displacement between the corresponding energy levels should not be due entirely to a Coulomb energy difference varying inversely as the nuclear radius and otherwise independent of energy. It is postulated that the elements of the dispersion-theoretic R-matrix on the nuclear surface as a function of energy may be the same for both mirror nuclei, once the ordinary Coulomb energy difference has been taken into account. If there were no Coulomb repulsion between a proton pair, and exact equality of n-n and p-p forces, then the nuclear levels would occur at the same energies (except for the constant neutron-hydrogen mass difference displacement) and the two nuclei would be described by the same wave functions at any energy, the Hamiltonian being symmetric in the exchange of a neutron with a proton. In this case, the R matrix would certainly be the same function of energy for both mirror nuclei. Now, for light nuclei the Coulomb forces inside the nucleus are very much smaller than are the purely nuclear forces, so that they will hardly influence the nuclear wave function inside the nucleus. Their only

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[1] J. M. Blatt, *On the Interpretation of Energy Levels in Light Nuclei*, unpublished.

[2] Barkas, Creutz, Delsasso, Sutton, and White, Phys. Rev. 58, 382 (1940). G. Kuerti and S. N. Van Voorhis, Phys. Rev. 58, 614 (1940).

[3] W. F. Hornyak and T. Lauritsen, Revs. Modern Phys. 20, 191 (1948). T. Lauritsen, *Energy Levels of Light Nuclei*, Preliminary Report No. 5, Nuclear Science Series, Nat. Res. Council.

appreciable effect will be a constant displacement of the energy levels. This will result in a displacement of the R matrix curves along the energy axis by a constant amount. If the assumption is made that the protons are distributed uniformly throughout a sphere of radius r, then the above displacement,

$$\Delta = 6Ze^2/5r, \tag{2}$$

where Z and $Z+1$ are the numbers of protons in the two mirror nuclei. The radius thus obtained has about the $A^{\frac{1}{3}}$ dependence on mass number, as discussed below. This postulate is therefore a reasonable form in which to express an assumption concerning the equality of n-n and p-p forces, without using any model to discuss wave functions in the interior of the nucleus. (In the case of the shell model this would mean that the shapes of the potentials in the nuclear interior are the same, the potential for the proton being displaced upward by a constant amount with respect to that for the neutron.)

On the basis of the above assumption, a calculation has been carried out to determine to what extent the 750-kev discrepancy between the first excited states of C^{13} and N^{13} can be explained while adhering to the assumption that the excited state radius is equal to the ground state radius. In applying the above mentioned postulate there is some ambiguity as to the exact value which ought to be taken for the radius of the nuclear surface along that channel of the configuration space which is of interest. For a system which, in the dissociated state, consists of two nuclei, of mass numbers A_1 and A_2, it may be most reasonable to take $r=r_0(A_1^{\frac{1}{3}}+A_2^{\frac{1}{3}})$. In the present case, $A_1=12$ and $A_2=1$. However, there may be effects which tend to decrease or increase this value, for example, taking some sort of auxiliary potential in the interior of the nucleus to represent electrostatic repulsion in the case of the proton, and polarization of the C^{12} core, respectively. Moreover, the results obtained here depend rather critically on the value taken for the radius. The calculations were carried out for a very low value, $r=r_0(12)^{\frac{1}{3}}$, and for the higher value $r=r_0[(12)^{\frac{1}{3}}+(1)^{\frac{1}{3}}]$. Here r_0 was taken to be 1.40×10^{-13} cm, which is a rather low value, and also as 1.46×10^{-13} cm in the only important case, to show the critical dependence on the radius. The value $r=r_0(12)^{\frac{1}{3}}$ will turn out to be quite unusable.

The boundary condition postulate predicts a shift in the positions of the energy levels of C^{13} with respect to the corresponding levels of N^{13}, in addition to the ordinary Coulomb energy difference and the neutron-hydrogen mass difference, especially for states near the dissociation energy. It turns out that for the first excited state this shift is in the right direction to explain the experimentally observed results. The ground state, however, shows a shift in the same direction, although a smaller one, thus reducing the energy discrepancy between the first excited states which this consideration is able to explain. This means that the ordinary Coulomb energy difference is smaller than the actual energy difference, the remainder constituting a "boundary condition energy difference" which arises as a result of the Coulomb wave function distortion, as compared with the neutron case, in the external region of configuration space. If the ordinary Coulomb energy difference is still to be given by the old formula, a somewhat larger than the usual value of the nuclear radius must be assumed in the ground state.

II. FORMALISM OF THE R MATRIX FORMALISM

The following derivation employs the results of the article of Wigner and Eisenbud,[4] and that of Feshbach, Peaslee, and Weisskopf.[5] In a system having two channels of disintegration, let φ denote r times the radial part of the wave function. Now in each channel let radial wave functions D and V be defined so that

$$\begin{array}{ll}(rD)\,|_{r=a}=0 & (d/dr)(rD)\,|_{r=a}=(M/\hbar)^{\frac{1}{2}},\\ (rV)\,|_{r=a}=(M/\hbar)^{\frac{1}{2}} & (d/dr)(rV)\,|_{r=a}=0,\end{array} \tag{3}$$

where M is the reduced mass for the channel, and a is the nuclear radius. Then, for each channel, equations of the form[6]

$$\varphi_1=(rD)_1+R_{11}(rV)_1+R_{12}(rV)_2 \tag{4a}$$

$$\varphi_2=(rD)_2+R_{21}(rV)_1+R_{22}(rV)_2 \tag{4b}$$

can be used to define an R matrix. It can easily be shown that, like the R matrix of Wigner and Eisenbud, our R matrix also can be written as

$$R=\sum_\lambda(\gamma_\lambda\times\gamma_\lambda)/(E_\lambda-E), \tag{5}$$

where γ_λ, E_λ are independent of energy. One then defines the quantities

$$\begin{array}{ll}B=k^{\frac{1}{2}}(F'^2+G'^2)^{\frac{1}{2}}, & A=(i)^l k^{-\frac{1}{2}}(G+iF),\\ \omega=(-i)^l(F'+iG')(F'^2+G'^2)^{-\frac{1}{2}}, & C=-GG'-FF',\end{array} \tag{6}$$

where the prime means $d/d(kr)$; l is the orbital angular momentum; and F and G represent the regular and irregular confluent hypergeometric functions defined according to the convention of Yost, Wheeler, and Breit.[7] All functions are to be evaluated for the argument equal to ka. Let u and v be incoming and outgoing waves with the asymptotic behavior $s^{-\frac{1}{2}}\exp(\mp ikr)$ at infinity, s being the velocity at infinity. If φ is to represent that function which has an incoming and an outgoing wave in channel 1, but only an outgoing wave in channel 2, then one can write $\varphi=\varphi_1+\mathfrak{C}\varphi_2$, and obtain $\mathfrak{C}$ by equating to zero the coefficient of u_2 on

[4] E. P. Wigner and L. Eisenbud, Phys. Rev. **72**, 29 (1947).
[5] Feshbach, Peaslee, and Weisskopf, Phys. Rev. **71**, 145 (1947).
[6] In Eq. (4a) and below, suitably normalized functions of the angular and of the internal variables have been omitted for the sake of brevity of notation, since no confusion arises. In detail, Eq. (4a) should read

$$\varphi_1=[(rD)_1+R_{11}(rV)_1]\omega_1+[R_{12}(rV)_2]\omega_2,$$

where ω_1 depends on the internal and angular variables of channel 1, and similarly for ω_2.
[7] Yost, Wheeler, and Breit, Phys. Rev. **49** 174 (1936).

the right-hand side of the following equation:

$$\begin{aligned}\varphi=&[(rD)_1+R_{11}(rV)_1+R_{12}(rV)_2]\\&+\mathfrak{C}[(rD)_2+R_{21}(rV)_1+R_{22}(rV)_2]\\=&\tfrac{1}{2}i[A_1(-i)^{l_1}u_1-A_1^*(+i)^{l_1}v_1]\\&+\tfrac{1}{2}R_{11}[B_1\omega_1^*(-i)^{l_1}u_1+B_1\omega_1(+i)^{l_1}v_1]\\&+\tfrac{1}{2}R_{12}[B_2\omega_2^*(-i)^{l_2}u_2+B_2\omega_2(+i)^{l_2}v_2]\\&+\tfrac{1}{2}i\mathfrak{C}[A_2(-i)^{l_2}u_2-A_2^*(+i)^{l_2}v_2]\\&+\tfrac{1}{2}R_{21}\mathfrak{C}[B_1\omega_1^*(-i)^{l_1}u_1+B_1\omega_1(+i)^{l_1}v_1]\\&+\tfrac{1}{2}R_{22}\mathfrak{C}[B_2\omega_2^*(-i)^{l_2}u_2+B_2\omega_2(+i)^{l_2}v_2]. \quad (7)\end{aligned}$$

Therefore,

$$\mathfrak{C}=-R_{12}[R_{22}+iA_2(B_2\omega_2^*)^{-1}]^{-1}. \quad (8)$$

In the application to be made here, the second channel will refer to the emission of a photon. Since the γ-ray width will be extremely small compared with the width for channel 1 (elastic proton or neutron scattering), it will be sufficient for the present derivation to treat the photon as if it were described by the Schrödinger equation for zero angular momentum and no Coulomb field. (It will turn out that the exact nature of this assumption will not enter the final result. Only the smallness of the γ-ray width and γ-ray wave number (compared with a nuclear wave number) will be employed.) Hence, $C_2=0$, $A_2A_2^*=k_2^{-1}$, k_2 being the γ-ray wave number. If one next defines φ to be that part of the previously defined φ referring to channel 1 only, and $f(E)=a\varphi^{-1}d\varphi/dr$, then it follows by straightforward calculation that

$$f(E)=a/(R_{11}-R_{12}^2(R_{22}+ik_2^{-1})^{-1}), \quad (9)$$

or, since $k_2^{-1}\gg R_{22}$,

$$f(E)=a(R_{11}+ik_2R_{12}^2)^{-1}. \quad (10)$$

If $f(E)=f_0(E)-ih(E)$, where f and h are real, then

$$f_0(E)=aR_{11}(R_{11}^2+k_2^2R_{12}^4)^{-1}\cong aR_{11}^{-1} \quad (11a)$$

$$h(E)=k_2aR_{12}^2(R_{11}^2+k_2^2R_{12}^4)^{-1}\cong k_2a(R_{12}/R_{11})^2. \quad (11b)$$

The absorption cross section (transition from channel 1 to channel 2) becomes, on noting that $k_2(R_{12}/R_{11})^2 \ll k(G^2+F^2)^{-1}$:

$$\sigma_{abs}=\frac{4\pi}{k^2}(2l+1)ka\frac{k_2a(R_{12}/R_{11})^2(G^2+F^2)^{-1}}{[ka(G^2+F^2)^{-1}]^2+[(a/R_{11})-ka(GG'+FF')(G^2+F^2)^{-1}]^2}, \quad (12)$$

l being the orbital angular momentum in channel 1. (The confluent hypergeometric functions are, of course, to be evaluated for the argument ka.) For low kinetic energies and high potential barrier (Coulomb and centrifugal), F and F' are negligible compared with G and G', and the expression for $(1/R_{11})_{res}$ becomes the same as the one which would be obtained for resonance if only elastic scattering were considered,

$$(1/R_{11})_{res}=(kG'/G)_{res}, \quad (13a)$$

the F and F' having already been neglected. However, the variation of k, G, and G' between E_{res} and $E_{res}+\frac{1}{2}\Gamma$ cannot be neglected[8] in the computation of the variation of R. Hence,

$$\left[\left(\frac{1}{R}-\frac{kG'}{G}\right)^2+\frac{k^2}{G^4}\right]_{E_{res}+\frac{1}{2}\Gamma}=\left(\frac{2k^2}{G^4}\right)_{E_{res}}.$$

$f(E)$ is proportional to the logarithmic derivative of the wave function in channel 1, while the R matrix has elements referring to all channels. Equation (9) shows the relation between them. The absorption cross section, of course, could have been obtained directly from the ratio of the coefficients of v_2 and u_1 in Eq. (7).

From these equations and a knowledge of the location of the maximum and of the width of the absorption resonance, it is possible to determine R_{11} for two values of the energy. The matrix element R_{12} is not necessary for this consideration for the reasons already mentioned above, while R_{22} is even more negligible.

For a bound state, R_{11} is easily determined by using the Schrödinger wave functions, whose asymptotic behavior for large r is similar to $\exp(-\beta r)$. In the neutron case, these functions are of the form $\exp(-\beta r)\cdot P(\beta r)$, where $P(\beta r)$ is a polynomial in $(\beta r)^{-1}$. In the proton case, these functions are the functions $W_{k,m}(z)$ given by Whittaker and Watson,[9] where $z=2\beta r$, $m=l+\frac{1}{2}$, and $k=-ZZ'e^2M/\hbar^2\beta^2$, Ze and $Z'e$ being the charges, M the reduced mass of the system, and $-\hbar^2\beta^2/2M$ its energy.

If a diagonal element of the R matrix given by Eq. (5) is approximated by summing only over those terms for which E_λ is smaller than a given value E_0, the result will be algebraically smaller than the correct value,

TABLE I. R values and calculated energy displacements for first excited states.

	$-R_{res}$	$-R_{res+\frac{1}{2}\Gamma}$	$-R_{Cis}$	Displacement for R linear in E (kev)	Displacement for R^{-1} linear in E (kev)
$l=0$ $r=0.48$	0.55814	0.55338	0.355	750	1160
$l=0$ $r=0.46$	0.53720	0.53417	0.355	1050	1570
$l=0$ $r=0.32$	0.42423	0.42494	0.355	...	...
$l=1$ $r=0.46$	0.32027	0.32178	0.266	...	...
$l=1$ $r=0.32$	0.23446	0.23526	0.224	...	...

[8] The author wishes to thank Mr. R. G. Thomas, whose work parallels the present discussion in some respects, for bringing this essential point to his attention (see Phys. Rev. 80, 136 (1950)).

[9] Whittaker and Watson, *Modern Analysis* (Cambridge University Press, 1927), fourth edition, p. 340.

and will increase less rapidly as a function of E than the correct function as long as E is less than E_0. In particular, in an energy range in which only one E_λ lies below E, a one-level approximation to R with that E_λ gives an increase of R with E slower than the correct one. This turns out to be important in interpreting the results of the calculation given in the next section.

III. RESULTS

Table I lists the values of R_{11} in units of 10^{-12} cm corresponding to the N^{13} nucleus at 2.35 Mev (E_{res}) and at (2.35+0.0175) Mev, which is $E_{res}+\frac{1}{2}\Gamma$, and to the C^{13} nucleus at 3.10 Mev. The cases considered are $l=0$ with $r=0.32$, 0.46, and 0.48×10^{-12} cm, and $l=1$ with $r=0.32$ and 0.46×10^{-12} cm. The cases in $l=1$ and the case $l=0$, $r=0.32\times10^{-12}$ cm must be ruled out, since they make dR_{11}/dE negative. If, for N^{13}, R_{11} were known to be a function of E involving only two parameters, these two could then be determined from the two known points of the curve, for $E=2.35$ Mev and $E=(2.35+0.0175)$ Mev. The energy at which R_{11} for N^{13} attains the value which it actually has for C^{13} at 3.10 Mev is then called the position of the energy level predicted for C^{13}, since this value of R_{11} is the boundary condition which is experimentally known to be required for a level. (R_{11} will henceforth be written as R.)

Table I shows that the calculated correction depends quite sensitively on the value taken for the "radius." Since the 2.35-Mev level of N^{13} is of even parity ($l=0$), while the ground state is generally assumed to have odd parity, the 2.35-Mev level may be assumed to be the lowest level of its kind (even parity, $J=\frac{1}{2}$). Hence, a one-level approximation for the form of the R matrix allows one to obtain an upper bound for the amount by which the C^{13} level is calculated to lie above the N^{13} level. Figure 2 indicates schematically the form of R as a function of energy (dotted curve), its intersection with the full curve indicating the position of (s-proton) resonances for the case of positive energy and of bound states for negative energy (of which there are none in this case). The value of γ^2 calculated from such a one-level formula is about equal to the quantity given by the sum rule over processes,[10,11] $\sum_s \gamma_{\lambda s}^2 \cong 3\hbar^2/2Ma$, where s represents modes of disintegration, whether energetically allowed or not at the energy considered, and λ may be considered to refer to the E_λ most closely connected with this resonance. As a matter of fact, for $r=0.46\times10^{-12}$ cm, γ^2 is 1.13 times $3\hbar^2/2Ma$, while for $r=0.48\times10^{-12}$ cm, it is 0.81 times $3\hbar^2/2Ma$. This means that if the compound nucleus picture is to be used at all, the one-level approximation can hardly be a valid one here. Moreover, the γ^2 obtained[12] from the one-level equation $\Gamma=2k\gamma^2A^{-2}$ (where A^{-2} is the penetrability) does not agree with the above γ^2 but is smaller by a factor of about 4, proving again that a one-level approximation is not valid in this case. The possibility of $l=1$ could have been ruled out previously from the result of the experiment of Hall and Fowler[13] on the radiative capture of protons of energy near 100 kev by C^{12}. Their cross section is larger than the one calculated theoretically from the assumption $l=0$ for the 2.35-Mev level of N^{13} by a factor of less than 2 (not unsatisfactory, because of the approximations in their calculations), but is of much larger order of magnitude than would be calculated from $l=1$. Moreover, the sum rule would be even more seriously infringed by the $l=1$ assumption, in addition, of course, to dR/dE being negative at the 2.35-Mev resonance. The one-level correction indicated in Table I (R^{-1} linear in E) is therefore only to be considered as an upper bound to the correction resulting from the use of the true function of energy $R(E)$. The example R linear in E is given as an illustration.

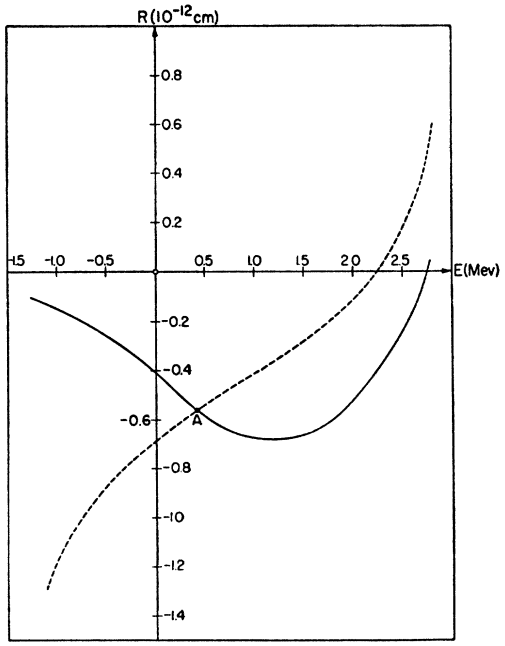

FIG. 2. Schematic graph of R for N^{13} as a function of energy (broken curve). Point A corresponds to the 2.35-Mev level in N^{13}; i.e., to the proton-C^{12} resonance at about 430 kev. If at any energy R has the value given by the solid curve, there is a bound state of N^{13} at that energy if $E<0$ and a virtual state if $E>0$. The whole figure refers to $L=0$, $r=0.48\times10^{-12}$ cm. Both curves are schematic.

One must remember, however, that in order to calculate a displacement which can be compared with the experimentally determined value, it is necessary to make the same boundary condition adjustment for the ground states as was done for the first excited states, since the experimental values are referred to a ground state displacement zero. Table II lists the values of R for the ground states, $l=1$. In this case, since both states involved are bound states, there is no experimentally determined width. The energy displacement implied by the difference in R values can be estimated,

[10] T. Teichmann, Ph.D. Thesis, Princeton University (1949).
[11] E. P. Wigner, Am. J. Phys. 17, 99 (1949).
[12] L. Eisenbud, Ph.D. Thesis, Princeton University (1948).
[13] R. N. Hall and W. A. Fowler, Phys. Rev. 77, 197 (1950).

TABLE II. R values and calculated energy displacements for ground states.

	$R_{C^{13}}$ ground	$R_{N^{13}}$ ground	γ^2 (10^{-13} erg-cm)	ΔE (one-level estimate) (kev)
$r=0.46$	−0.187	−0.212	0.57	195
$r=0.32$	−0.169	−0.178	0.96	155

at least with respect to order of magnitude, by using a one-level formula with γ^2 being estimated from the first excited state of N^{13} by $\Gamma=2k\gamma^2A^{-2}$. (Although this is not exactly applicable here, it can be used for such an order of magnitude estimation.) The results are also listed in the table. The γ^2 thus obtained is still about 0.25 of $3\hbar^2/2Ma$, which may very well be larger than the γ^2 appropriate for the ground state, so that the ΔE listed in Table II may be too large. At any rate, the displacement is in the direction to cut down the displacement of the first excited levels relative to the ground states here explained.

IV. CONCLUSIONS

It has been shown that the boundary condition postulate introduced here predicts a displacement of the first excited levels of C^{13} with respect to N^{13} in the right direction, and of sufficient magnitude to explain the experimentally known displacement. The critical dependence on the nuclear radius rules out any very definite result. It may be, of course, that there exist as yet undiscovered levels in C^{13} and N^{13} which would alter the whole analysis, and that the 3.10-Mev level of C^{13} is not really the one which corresponds to the 2.35-Mev level in N^{13}. Recent experiments on the elastic scattering of protons by C^{12} up to a proton energy of about 4 Mev (with energy resolution of order 1 kev) have failed to reveal additional N^{13} levels in the range in question here.[14]

It is a pleasure to thank Professor Wigner for suggesting this problem, and for continued advice and encouragement. I also wish to thank Professor Sherr for his interesting discussions of the experimental aspects of this problem, and Professor Breit for making available to me his tables of confluent hypergeometric functions.

[14] The author wishes to thank G. Goldhaber and R. Williamson for informing him of their results prior to publication.

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Internal Excitation and Apparent Range of Nuclear Forces in Scattering Experiments

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The effect of a speculatively considered internal excitation of nucleons on the scattering of elementary particles is examined by means of a schematic model. It is found that the relation between the integral of the square of the wave function describing the normal nucleonic state and the apparent range of force differs from that expected on the ordinary potential energy description. The effect is of the order of eight percent if one assumes the excitation to be 275 mc^2. The direction of the effect is such as to decrease the apparent range as though the meson mass were increased, somewhat as indicated by a comparison of the mass of a pi-meson with experiments on scattering. A possible necessity of corrections for the velocity dependence of nuclear forces which is considered above is briefly discussed in connection with the correlation of data on scattering with that on binding energies.

I. INTRODUCTION

IT has become customary to describe interactions between nuclear particles by means of potential energies. That such a description is of a provisory character has been well realized since the first introduction of exchange forces by Heisenberg[1] and Majorana.[2] These forces can, of course, be introduced by means of certain additive terms in the Hamiltonian without any reference to their origin. Nevertheless, Heisenberg's arguments were partly based on an expected analogy to interactions between atoms which take place in molecules. The same general feature is present in meson theories of nuclear forces.[3] According to these theories the origin of the force is not the interaction between nucleons with each other but a more primary type of process consisting in the production of mesons by nucleons. Only in special cases can the static part of the nuclear potential be split off from the relativistic part by a definite transformation. In the general Møller-Rosenfeld mixture, the potential employed in attempts to explain scattering or binding energies is only a part of a more general interaction which has not been evaluated on

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† Assisted by the joint program of the ONR and AEC.
[1] W. Heisenberg, Z. Physik 77, 1 (1932).
[2] E. Majorana, Z. Physik 82, 137 (1933).
[3] H. Yukawa, Proc. Phys. Math. Soc. Japan 17, 48 (1935). C. Møller and L. Rosenfeld, Proc. Cop. 17, No. 8. G. Wentzel, *Einführung in die Quantentheorie der Wellenfelder* (Franz Deuticke, Wien, 1943).