The Angular Distribution of High Energy Neutrons from Targets Bombarded by 330-Mev Protons*

RICHARD D. MILLER, DUANE C. SEWELL, AND ROBERT W. WRIGHT
Radiation Laboratory, Department of Physics, University of California, Berkeley, California
(Received October 9, 1950)

The angular distribution of neutrons with energies above 20 Mev produced by bombardment with the 330-Mev proton beam in the 184-in. cyclotron of Be, Al, Cu, and U targets has been measured with carbon detectors. The full widths at half-maximum obtained are 54° for Be, 59° for Al and Cu, and 58° for U. Experimental details and the resulting curves are given.

I. INTRODUCTION

WHEN the 184-in. cyclotron was converted to accelerate protons, a beam of high energy neutrons was obtained, as had been the case previously with deuteron acceleration. Preliminary measurements by Hadley of the energy spectrum of the neutrons in the forward direction from a beryllium target bombarded by 345-Mev protons show a distribution extending from about 150 Mev up to 345 Mev with a broad maximum at about 270 Mev.

These neutrons were presumed to be obtained primarily by a single charge exchange collision inside the target nucleus. The n–p scattering data[1] at 90 Mev give evidence of such exchange collisions in approximately 50 percent of the n–p collisions. At the presently available proton energies of greater than 300 Mev, the free particle model of the nucleus as proposed by Serber[2] should be considerably better than it was for the highest previously available nucleon energies of about 100 Mev. There was consequently some hope that the neutron angular distribution could be accounted for by a suitable modification of the free n–p exchange cross section to take into account the effects of binding.

The experiment performed was similar to the measurement of the angular distribution of neutrons from deuteron bombardment by Helmholz, McMillan, and Sewell,[3] which gave results in accord with Serber's stripping theory.[4]

II. EXPERIMENTAL

Detecting System

Relative neutron intensities were measured by the 20.5-min C^{11} beta-activity produced in polystyrene disks $1\frac{11}{16}$ in. in diameter and $\frac{1}{8}$-in. thick. These detectors were mounted on the back of a wall of lead bricks which was sufficiently thick to stop the protons that were also present. It was shown by experiments to be described later that the effects of secondary reactions in the lead absorber were not appreciable.

After a 20- to 40-min bombardment, the eight to ten detecting disks were each counted at two different times on a standard end-window beta-counter. Actual counting rates varied from 500 to 2000 counts/min. After the usual corrections for counter dead time and background, the decay corrections were made using the accepted 20.5-min half-life. Several times the decay of a detector was followed for three half-lives; no evidence of any other activity was found, and the half-life always agreed with the accepted value.

Two further corrections to the weighted average of the two corrected counting rates for each detector were necessary to obtain the relative neutron intensities. The first was the inverse square target distance correction, and the second was the correction for the variation in attenuation of the neutrons by the absorbing lead wall. This latter variation was caused by the slight variation in neutron path length through the absorber with the angle from the target, and the maximum correction for this variation was only three percent.

General Arrangements

Two different arrangements were used. The one with which most of the data were obtained is illustrated schematically in Fig. 1. Protons accelerated in the 184-in. cyclotron impinged on the edge of the target, which was mounted on a probe running into the vacuum tank. The target was set at the 80-in. radius, corresponding to a nominal proton energy of 330 Mev; however, radial oscillations of the protons reduce the mean incident energy by perhaps 10 Mev. A "beam clipper" consisting of two horizontal copper bars, $1\frac{1}{4}$-in. thick and 2 in. apart, and oriented so that it intercepted protons whose vertical oscillations were greater than 2 in., was mounted on a probe on the opposite side of the cyclotron. The purpose of this clipper was to reduce the possibility of an appreciable contribution of neutrons being produced by protons hitting the dee. The center of the neutron beam produced in the target passed through the $1\frac{3}{4}$-in. thick steel wall of the vacuum tank at 10 degrees from the normal.

The 4-in. thick wall of lead and the detectors were placed in a horizontal row against the outside of the tank wall as indicated in Fig. 1. The angles from the target, relative to the incident proton beam direction, at which

* Ths work was performed under the auspices of the AEC.
[1] Hadley, Kelly, Leith, Segrè, Wiegand, and York, Phys. Rev. **75**, 351 (1949).
[2] R. Serber, Phys. Rev. **72**, 1114 (1947).
[3] Helmholz, McMillan, and Sewell, Phys. Rev. **72**, 1003 (1947).
[4] R. Serber, Phys. Rev. **72**, 1007 (1947).

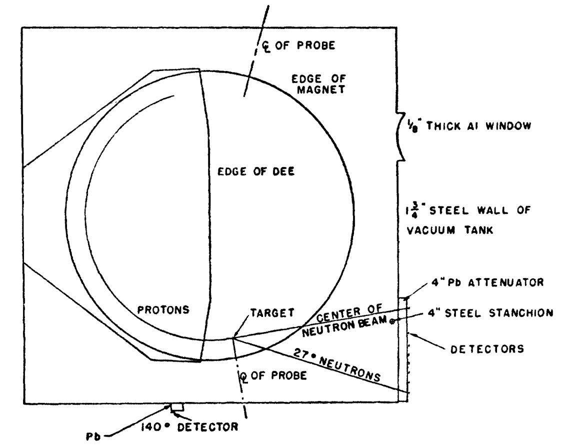

FIG. 1. Schematic drawing of the experimental arrangement.

detecting disks were placed were 0°, 8°, and 12° and at 3° intervals to 27°. No data could be obtained with this arrangement at greater angles because of scattering from the corner of the vacuum tank, and at angles between 0° and 8° because of the absorption and scattering from a stiffening stanchion just inside the tank wall at about the 4 degree position.

The second arrangement was used to obtain data between 0° and 8°, and also, in conjunction with other experiments, to set an upper limit on the contribution of the proton flux to the detector activities because of secondary reactions in the absorber. In this arrangement the positions of the target and "beam clipper" were interchanged, and the direction of the proton beam was reversed. The center of the neutron beam from the target then came out through a $\frac{1}{8}$-in. thick, spun aluminum window in the vacuum tank wall. The only other change was that the lead absorber was made 6 in. thick instead of 4 in.

Data were taken with four different targets: beryllium, aluminum, copper, and uranium. In order to obtain reasonable counting rates, the targets were made as thick as possible consistent with the requirements that the energy loss of the incident protons be small compared with the incident energy, and that they be "thin" for the neutrons produced. The thicknesses used were: beryllium, $2\frac{5}{8}$ in.; aluminum, $1\frac{15}{16}$ in.; copper, $\frac{11}{16}$ in.; and uranium, $\frac{3}{8}$ in. These were all 35 Mev thick for 330-Mev protons. The effective thicknesses, however, were probably smaller, especially for beryllium and aluminum, because of possible deviations of the target edge from tangency to the proton orbits. The nominal thicknesses are all less than 20 percent of the mean free paths, corresponding to the total cross sections for neutrons in the forward direction produced by 345-Mev protons on beryllium, as measured by De Juren.[5] However, for 90-Mev neutrons the total cross sections are larger;[6] for the worst case, beryllium, the nominal thickness was 35 percent of the mean free path. The rms Coulomb multiple scattering of the proton beam in all the targets and the spread caused by radial oscillations of the beam were less than one degree.

Auxiliary Experiments

Absorption cross section of lead. The value of the absorption cross section of lead used in making the path length corrections was measured directly for the neutrons at four different angles from 8 to 27 degrees. An additional wall of 4 in. of lead and 4 additional detectors were mounted directly behind the usual detectors. The ratio of the activity of the detector behind the second absorber to that of the corresponding detector in front of the second absorber was the same at all four positions to within the counting probable errors of two percent. This ratio corresponds to a mean free path of 8.9 in. or a cross section of 1.2 barns. The corresponding cross section of iron was taken by interpolation from the values of De Juren,[5] since the maximum correction caused by the tank wall was only one percent.

Neutron Production in the Lead Absorber

To determine whether or not the protons incident on the lead absorber produced an appreciable number of detectable neutrons in the absorber, three results were necessary: first, the activity produced behind the absorber relative to that in front of it by the mixture of neutrons and protons which were present in the arrangements used; second, the relative activities behind and in front of the same absorber produced by a beam of neutrons only; and third, the relative activities by a beam of protons only.

The second result is that obtained in the neutron attenuation experiment described in the previous paragraph.

The first result was obtained using the second general arrangement described, in which the center of the neutron beam passed through the $\frac{1}{8}$-in. thick aluminum window instead of the steel tank wall. This arrangement was necessary in order not to ignore the neutrons produced in the tank wall. The activity of the detector behind the 6-in. lead absorber was 24 percent of that of the detector in front, at the zero degree position.

To obtain a beam of protons only, it was necessary to use the external, deflected beam of the cyclotron. Since this beam consists of 345-Mev protons, the number of neutrons produced in the absorber in this experiment should be greater than the number produced by the protons in the original arrangements. There, the protons were not over 330 Mev, and were probably about 295 Mev, because of the target thickness. The deflected beam is not large enough in area to perform a "bad geometry" experiment as in the previous cases, so it was necessary to use a number of detectors behind the lead and integrate the distribution of activity to

[5] J. De Juren, Phys. Rev. **81**, 458 (1951).
[6] Cook, McMillan, Peterson, and Sewell, Phys. Rev. **75**, 7 (1949).

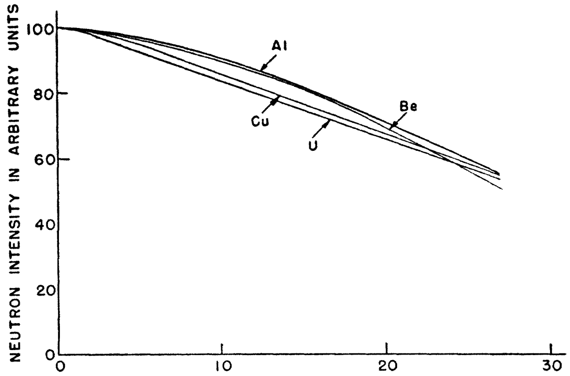

FIG. 2. Angular distributions of neutrons from Be, Al, Cu, and U.

find the total activity which an infinite detector behind the absorber would have received. The incident proton beam was one inch in diameter and was all well within the area of the detector in front of the lead. The integrated activity behind the lead was 3.5±1 percent of the activity of the detector in front.

These three data give the result that at the zero degree position 8.3±2.4 percent of the normal detector activity was due to the protons incident on the absorber. This figure is only an upper limit for three reasons. First, the neutron production in the absorber was measured for 345-Mev protons instead of 295 Mev. Second, the protons that were present were, in all probability, those that were scattered by the target through sufficiently wide angles to escape the magnetic field of the cyclotron. They were then incident on the absorber at rather oblique angles, and the effective absorber thickness for these protons was thus definitely greater than for the neutrons. Third, it should be expected that the substitution of the steel tank wall for the initial part of the lead absorber would produce fewer neutrons. This is because the number of neutrons, in iron nuclei, per unit area of the tank wall, is less than the number per unit area of an equivalent stopping thickness of lead. Also, the proportion of detector activity due to the protons should be greatest at the zero degree position, since the protons at the wider angle detector positions would have to have been scattered through wider angles by the target.

Possible Background Sources of Neutrons

There were three possible sources of background neutrons in these experiments: first, neutrons produced in the cyclotron dee by scattered protons; second, neutrons scattered from the concrete radiation shielding and other objects which were behind the detectors; and third, neutrons scattered from the pole pieces of the cyclotron magnet.

In one run with the beryllium target a detector was placed behind 4 in. of lead on an adjacent wall of the vacuum tank at a position corresponding to 140° from the incident proton beam. Its activity, after being corrected for relative target distance, was eight percent of that of the zero degree detector. As may be seen from Fig. 1, this detector was in a particularly favorable position to see neutrons produced in the dee. Assuming that all of its activity was from this source, it was estimated from geometrical considerations that this source of background contributed less than two percent to the activities of the regular detectors.

Two experiments indicated that the background of neutrons backscattered from objects behind the detectors was negligible. First, several detectors were mounted about four feet above the usual detector positions in such positions that they were well within the shadow of the upper magnet pole piece, but were entirely open to any scattered radiation from the rear. Their activities were all less than 0.5 percent of that of the zero degree detector. Second, it was found that placing an additional 4 in. of lead behind some of the usual detectors did not change their activities measurably.

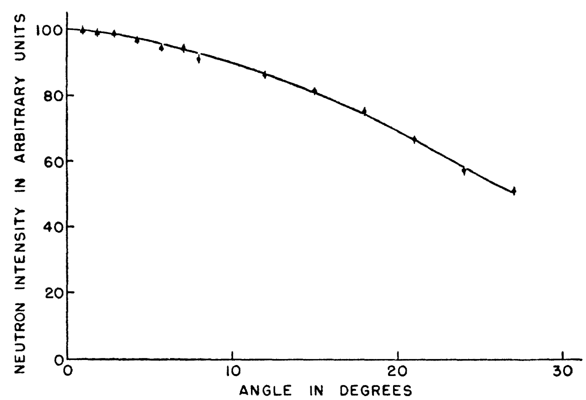

FIG. 3. Angular distribution of neutrons from beryllium with data. Data below 8° were obtained with the reversed beam arrangement.

There is no direct experimental evidence proving that scattering from the pole pieces was negligible. Unfortunately, lack of sufficient intensity makes a measurement of the differential scattering cross sections with these detectors impracticable. However, a computation of the geometrical factors involved in the evaluation of the scattered intensity at the detector positions shows that it is extremely unlikely that this scattered intensity is appreciable. The smallest angle by which any neutron could have been scattered by the magnet into any detector is 22°. For comparison, the first zero in the calculated diffraction scattering pattern of 250-Mev neutrons by iron is at 14°. Detailed calculations showed that in order for the scattered intensity to be three percent of the direct intensity, the scattering cross section per unit solid angle at angles greater than 22° would have to be greater than $1/4\pi$ times the absorption cross section.

III. RESULTS

The resulting angular distributions are shown in Fig. 2. The experimental points and errors due to counting statistics for the beryllium and uranium curves are shown in Figs. 3 and 4. The data for aluminum and copper are similar to those for uranium. Two runs were made with each target except uranium, and the agreement between runs was always within counting errors. The errors shown on the curves are the standard deviations due to counting statistics only.

The full widths at half-maximum, obtained by extrapolation of the curves, were 54° for beryllium, 59° for aluminum and copper, and 58° for uranium. The estimated probable error in these values due to the extrapolation and the scatter of the experimental points is one degree. In the center-of-mass system of the incident proton and the nucleus, the width at half-maximum of the beryllium distribution is increased to 59°.

IV. DISCUSSION

The dependence of the efficiency of the present detecting system on the neutron energy is the principal uncertainty in these measurements. The variation with energy of the neutron absorption cross section of lead is fairly well known,[5,6] the cross section at 270 Mev being about 30 percent lower than at 45 Mev with most of the decrease occurring above 90 Mev. However, there are no experimental data for the excitation function of the $C^{12}(n,2n)C^{11}$ reaction. Theoretical predictions of this excitation function have been made by Heckrotte and Wolff,[7] and by Baumhoff.[8] The theoretical curves

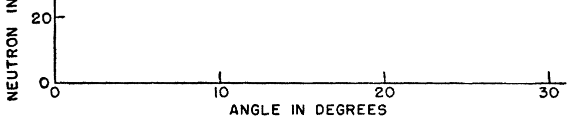

FIG. 4. Angular distributions of neutrons from uranium with data.

show a peak at about 40 Mev, and only a very gradual decrease above 100 Mev. Baumhoff's curve shows a decrease of 25 percent between 100 Mev and 350 Mev. Since this decrease opposes the decrease in the absorption cross section in its effect on the efficiency of the detecting system, it is felt that the detector efficiency is flat to within 15 percent for neutron energies above 100 Mev. It is probable, however, that the detection efficiency for neutrons with energies in the range of 30 to 60 Mev is considerably higher.

[7] W. Heckrotte and P. Wolff, Phys. Rev. **73**, 265 (1948).
[8] L. Baumhoff, private communication.

TABLE I. Nuclear radii divided by the mean free path in nuclear matter.[a]

	Be	Al	Cu	U
90 Mev	0.64	0.91	1.2	1.9
270 Mev	0.4	0.6	0.8	1.3

[a] These values are given by $R=1.37\times10^{-13}A^{\frac{1}{3}}$ cm, $\lambda_{90}=4.5\times10^{-13}$ cm, $\lambda_{270}=6.7\times10^{-13}$ cm.

The most striking feature of the present distributions is their large widths. Using any reasonable momentum distribution for the nucleons in the target nucleus, it appears to be impossible to account for the widths found with the assumption that the detected neutrons were produced in single nucleon-nucleon collisions and escaped the nucleus without further interactions. For instance, the width at half-maximum, due only to the target nucleon momentum distribution, was calculated to be only 36° for a Fermi distribution with the upper limit as high as 30 Mev. The preliminary n-p scattering data[9] at 270 Mev indicate that the exchange collision protons have a width of only about 10°, and thus this source of width contributes only slightly to the total width.

The most obvious further source of broadening is the effect of secondary collisions in the same nucleus in which the initial collision takes place. That this source should be expected to be important is shown in the values of the ratio of nuclear radius to mean free path in nuclear matter given in Table I. The 90-Mev values in this table are those derived by Fernbach, Serber, and Taylor[10] from the 90-Mev neutron cross section data. While they have not been able to fit the 270-Mev data as well with their model, they estimate that the mean free path is about 50 percent greater at this energy.

The low energy partner of the initial collision is particularly likely to suffer further interactions, and thus may escape as a neutron at a wide angle from the incident proton. It is also possible that an appreciable fraction of the incident protons produce more than one detectable neutron, thus emphasizing further the lower energy and wider angle components.

It is of interest to compare the present angular distributions with the angular distributions of protons from targets bombarded by 90-Mev neutrons as measured by Hadley and York.[11] They found the following full widths at half-maximum for protons with energies greater than 20 Mev: 44° for carbon; 46° for copper; and 48° for lead. That the present distributions are wider even though the bombarding momentum is nearly doubled, is strong evidence that secondary collisions are a major source of the width.

It would be of interest to repeat this experiment with higher threshold detectors. It seems likely that the

[9] Kelly, Leith, Segrè, and Wiegand, Phys. Rev. **79**, 96 (1950).
[10] Fernbach, Serber, and Taylor, Phys. Rev. **75**, 1352 (1949).
[11] J. Hadley and H. York, Phys. Rev. **80**, 345 (1950).

angular distributions of neutrons with energies greater than say 90 percent of the incident proton energy should be due principally to the target nucleon momentum distribution, and that consequently such measurements would yield considerable information concerning momentum distributions in nuclei.

The authors wish to express their appreciation to Professor Edwin M. McMillan for his interest and advice, to Dr. Geoffrey Chew and Mr. Lester Baumhoff for valuable discussions on the theoretical aspects of the experiment, and to Mr. James Vale and the cyclotron operating crew for their cooperation and assistance.

PHYSICAL REVIEW VOLUME 81, NUMBER 3 FEBRUARY 1, 1951

Thin Ferromagnetic Films*

MARTIN J. KLEIN AND ROBERT S. SMITH
Department of Physics, Case Institute of Technology, Cleveland, Ohio
(Received June 15, 1950)

The magnetic properties of thin films of ferromagnetic materials have been studied by means of the Bloch spin-wave theory. The spontaneous magnetization depends upon the number of atomic layers, G_z, in the film. For thicknesses below a "critical" thickness, which depends on the temperature and film dimensions, the spontaneous magnetization decreases rapidly as the number of atomic layers is decreased. The temperature dependence of the spontaneous magnetization varies from a $T^{\frac{3}{2}}$ law for very thick films to a linear function of T for a monolayer film. The spontaneous magnetization, at fixed temperature and film thickness, decreases as the film dimension increases.

I. INTRODUCTION

It has been shown by Bloch[1] that one- and two-dimensional lattices, in contrast to three-dimensional lattices, should not exhibit spontaneous magnetization, even when the exchange integral is positive. This conclusion was reached by the use of the spin-wave method, which was introduced by Bloch as an approximate way of treating the Heisenberg[2] model of a ferromagnet. The spin-wave method provides a reasonable approximation to the lowest energy states of the system, as shown by Bethe,[3] but only for these; and conclusions reached by this method are therefore expected to be valid only at low temperatures when the magnetization is near its saturation value. It should be mentioned that Bloch's result on the absence of spontaneous magnetization in two-dimensional lattices has been substantiated by the calculations of Weiss[4] using the Bethe-Peierls method;[5] on the other hand, recent work by Ekstein[6] contradicts this result, leaving this point in doubt.

If one accepts Bloch's result, it follows that the magnetic properties of a slab of ferromagnetic material should vary with the thickness of the slab and should show a transition from ferromagnetic behavior for thick slabs to paramagnetic behavior for sufficiently thin films. We have studied the dependence of the spontaneous magnetization on the thickness of the sample, using the method of spin-waves, in order to determine the nature of the transitional behavior to be expected. An experimental investigation of the magnetic properties of thin films would clearly be desirable, since it could serve as a check on the correctness of conclusions drawn from the spin-wave theory.

II. CALCULATIONS AND RESULTS

On the basis of the usual approximations of the spin-wave method one obtains, in the case of a simple cubic lattice of atoms having spin $\frac{1}{2}$, the spontaneous magnetization:[7]

$$M=\beta N\left[1-\frac{2}{N}\sum_{\lambda_x,\lambda_y,\lambda_z}\left\{\exp\left[\frac{2J}{kT}\sum_{i=x,y,z}\times\left(1-\cos\frac{2\pi\lambda_i}{G_i}\right)\right]-1\right\}^{-1}\right]. \quad (1)$$

Here $2\pi\lambda_x/G_x$ is the x component of the spin-wave vector: G_x is the x dimension of the crystal in units of the lattice parameter and λ_x is an integer ($\lambda_x=0$, 1, 2, $\cdots G_x-1$). β is the Bohr magneton, J is the exchange integral between nearest neighbors, and N is the total number of atoms in the crystal ($N=G_xG_yG_z$).

If G_x, G_y, G_z are large numbers, then the variables $K_\lambda^{(x)}=2\pi\lambda_x/G_x$, etc., change by small steps and one can approximate the sums by integrals. Thus, for a three-

---

* This work was supported in part by the ONR.
[1] F. Bloch, Z. Physik 61, 206 (1930).
[2] W. Heisenberg, Z. Physik 49, 619 (1928). J. H. Van Vleck, Revs. Modern Phys. 17, 27 (1945).
[3] H. Bethe, Z. Physik 71, 205 (1931). A. Sommerfeld and H. Bethe, *Handbuch der Physik*, Band 24, Teil 2, p. 601–613.
[4] P. Weiss, Phys. Rev. 74, 1493 (1948).
[5] H. Bethe, Proc. Roy. Soc. (London) A150, 552 (1935). R. Peierls, Proc. Cambridge Phil. Soc. 32, 477 (1936).
[6] H. Ekstein, Phys. Rev. 80, 122 (1950).
[7] See Sommerfeld and Bethe, reference 3. T. Holstein and H. Primakoff, Phys. Rev. 58, 1098 (1940).