those of the even Z isotopes. This may be a result of the closed shells of 50 and 82 protons.

The differential cross sections for these even Z, odd A ground-state peaks are low for probably two reasons. First, since the Q-values for these nuclei are larger than for even Z, even A nuclei, their proton energies will be higher. High energy protons are less probable because of the momentum distribution of the proton in the deuteron. The second reason is that the neutron from the deuteron may not pair up with the odd neutron in the target nucleus for nuclear reasons.

The author is indebted to Professors M. Deutsch and M. S. Livingston under whose direction this research was carried out. He would also like to thank M. Deutsch for many helpful criticisms in the preparation of this paper for publication. The equipment was built in collaboration with K. Boyer and H. E. Gove, with whom the author has had many valuable discussions. The cooperation of all the personnel of the Cyclotron Laboratory, Mr. F. J. Fay, Mr. R. W. Kilson, Mr. E. Pulster, and Mr. E. F. White, is gratefully acknowledged. Many of the targets were rolled foils supplied through the kindness of Dr. J. R. Low of the Knolls Atomic Power Laboratory. The chemistry on the separated lead isotopes was done by Mrs. E. Backofen.

APPENDIX

RELATIVE STOPPING POWER OF VARIOUS ELEMENTS

The relative stopping power (relative to aluminum) was measured for 19-Mev protons and 14-Mev deuterons for nearly all of the targets. The values were plotted *vs* Z and a straight line drawn through the points. A few values are compared to values computed from Livingston and Bethe and also to values obtained by Kelly[64] using 28-Mev alpha-particles.

$E_p = 19.0$ Mev
No. of mg/cm^2 = 1 mg/cm^2 Al

Element	Experimental	L. and B.	Percent difference
Au	1.82±0.02	1.96	7
Ag	1.42±0.03	1.55	9
Cu	1.20±0.03	1.26	5

$E_d = 14$ Mev
No. of mg/cm^2 = 1 mg/cm^2 of Al

Element	Experimental	L. and B.	Percent difference	$E\alpha = 28$ Mev Kelly
Au	2.045±0.01	2.10	2.5	1.99
Ag	1.54 ±0.02	1.61	4.5	1.54
Cu	1.26 ±0.02	1.29	2.5	1.28

[64] E. L. Kelly, Phys. Rev. 75, 1006 (1949).

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Angular Distributions of (d,p) Reactions Using 14-Mev Deuterons*

H. E. Gove
Laboratory for Nuclear Science and Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts
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The angular distribution of (d,p) reactions for a series of target elements, using 14-Mev deuterons has been investigated. The targets chosen were thin foils of carbon, aluminum, nickel, silver, tantalum, gold, and bismuth. These were bombarded with deuterons from the MIT cyclotron in the center of a large scattering chamber shielded from the cyclotron by 4 ft of concrete. The emitted protons were detected in a triple proportional counter which could be set at any angle from 15° to 135° to the beam. In all cases the measured intensity of protons per unit solid angle is found to be greater in the forward direction. The ratio of the area under the distributions from 20° to 90° to that between 90° and 140° varies from 1.6 to 13. For a given element this ratio increases with proton energy. In addition, for carbon and aluminum, in which individual groups can be resolved, the intensity of protons in a given group, in some cases, exhibits maxima at various angles. In the three elements of highest atomic number the intensity rises from back angles to a maximum at some forward angle and then drops off towards zero degrees. The position of this "turn over" angle appears to increase slowly with atomic number. The general features of the distributions appear to be explainable on a stripping picture rather than that of a compound nucleus.

I. INTRODUCTION

CONSIDERABLE work has been reported on (d,p) angular distributions in the range of deuteron energies from zero to 4 Mev and some work has been done with deuteron energies as high as 7.5 Mev. The purpose of the present work was to study the angular distributions of protons from deuteron-induced reactions at a fixed deuteron energy of 14 Mev over a wide range of atomic number. Detailed studies of (d,p) reactions in the energy range near 15 Mev have been largely neglected and to the best of our knowledge there has been no previous work of this nature reported.

Heydenburg and Inglis[1] have investigated the excitation curves and the angular distributions of proton groups from $O^{16}(d,p)O^{17}$ leaving O^{17} in its ground, and first excited state for deuteron energies between 0.6 and 3 Mev. The excitation curves indicate resonances. The

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[1] N. P. H. Heydenburg and D. R. Inglis, Phys. Rev. 73, 230 (1948).

character of the asymmetry about 90° of the angular distributions seems to be a rather random function of the deuteron energy except that the distributions show a tendency to become more nearly symmetric about 90° as the deuteron energy approaches that of the peaks in the excitation curve. In some later work Heydenburg, Inglis, Whitehead, and Hafner[2] studied the reaction $C^{12}(d,p)C^{13}$ for deuteron energies from 1 to 3.5 Mev. In this case the excitation curve shows resonances which agree with energy levels in N^{14} found by measuring γ resonances. Although the angular distributions at various energies were not investigated in such great detail, the results were similar to those from $O^{16}(d,p)O^{17}$ as regards the variation in the character of the asymmetry about 90° with deuteron energy. The angular distribution of protons from $N^{14}(d,p)N^{15}$ leaving N^{15} in its ground and first excited states has been investigated by Wyly[3] for deuteron energies between 1.5 and 3 Mev. The long range group is slightly forward over the whole energy range while the short range group is spherically symmetric. No excitation functions were obtained and there were no marked variations in the distributions with energy. The angular distribution of two proton groups from the reaction $O^{16}(d,p)O^{17}$ and five groups from $Al^{27}(d,p)Al^{28}$ have been reported by Nemilov and Funshtein[4] for 3.9-Mev deuterons. The distributions are markedly forward in all cases except for the two long range groups which are isotropic. The lower energy proton groups are progressively more forward than the high groups; other than this the variation with angle is quite smooth. The angular distribution of the long range proton group from $B^{10}(d,p)B^{11}$ has been investigated by Redman[5] for deuterons between 1 and 3.7 Mev. The distributions when analyzed in an expansion of Legendre polynomials show the presence of a P_1-term giving a forward asymmetry at all but the lowest energy. The highest term required to fit the data was P_2. Whaling and Bonner[6] have studied the disintegration of Li^6 by deuterons. In particular, for the $Li^6(d,p)Li^7$ process the excitation curve for the long and short range proton groups shows no resonance structure for energies up to 1.8 Mev. The angular distributions of these two groups taken at energies between 0.4 and 1.4 Mev show the presence of terms in the Legendre polynomial as high as P_6. Again there is a reasonably smooth change in the shape of the distributions with deuteron energy.

Since the present work was completed the angular distribution of various portions of the proton spectrum from the reaction $Al^{27}(d,p)Al^{28}$ between 25° and 140° has been reported by Holt and Young.[7] Using deuteron energies of 4.6, 5.8, and 7.5 Mev, they find that the distributions are forward in all cases. The angular distribution of the long range proton group for this reaction as measured by the author at 13.8 Mev, and by Holt and Young at 7.5, 5.8, and 4.6, is discussed later in this paper and is illustrated in Fig. 16.

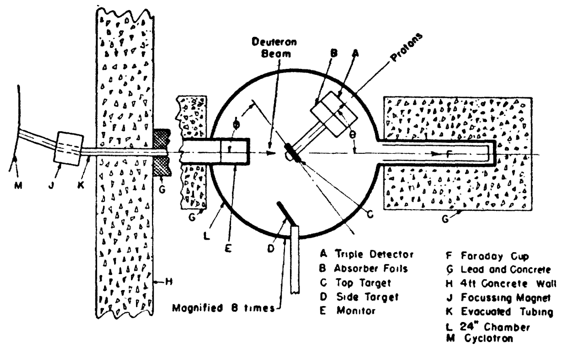

FIG. 1. Schematic diagram of apparatus for measuring angular distributions using a triple proportional counter.

There has been some work done by Helmholz, McMillan, and Sewell[8] on (d,n) angular distributions using very high energy deuterons (about 200 Mev). The distributions are very strongly peaked in the forward direction having angular widths at half maximum around 10°.

Substantial work has been done by Falk[9] on the angular distributions of (d,n) reactions using 15-Mev deuterons. Nine elements ranging from Be to Au were used and the thick target yield of neutrons with angle was measured using the radioactivity produced in silver by the $(n,2n)$ reaction with a threshold of 11 Mev. The measurements were made on both sides of 0° up to 50° and in this range of angle it was found that the intensity rose very sharply toward small angles, in some cases reaching a peak at about 10° and decreasing toward zero and in other cases reaching a maximum at zero. The width at half maximum varies from about 20° to 50°.

This paper reports the yield with angle of protons from reactions in which the neutron was left in a bound state in the residual nucleus. These would be protons of range greater than that of elastic deuterons. In fact, it is quite easy to show that, for deuterons of energy greater than 8 Mev, protons emitted leaving the neutron in a bound state, i.e., with Q-values greater than -2.2 Mev, will have ranges greater than that of the elastically scattered deuteron. The targets chosen were C^{12}, Al^{27}, $Ni^{58,60}$ $Ag^{107,109}$, Ta^{181}, Au^{197}, and Bi^{209} since these covered a wide range of values of Z and were available in thin foil or film form.

II. EXPERIMENTAL ARRANGEMENT

A description of the equipment used in these experiments will be published shortly[10] and only a brief dis-

[2] Heydenburg, Inglis, Whitehead, and Hafner, Phys. Rev. **75**, 1147 (1949).
[3] L. D. Wyly; Phys. Rev. **76**, 104 (1949).
[4] Y. A. Nemilov and B. L. Funshtein, Inst. of Radium, USSR Academy of Science **66**, 609 (1949).
[5] W. C. Redman, Phys. Rev. **79**, 6 (1950).
[6] W. Whaling and T. W. Bonner, Phys. Rev. **79**, 258 (1950).
[7] J. R. Holt and C. T. Young, Proc. Phys. Soc. (London) **A63**, 833 (1950).
[8] Helmholz, McMillan, and Sewell, Phys. Rev. **72**, 1003 (1947).
[9] C. E. Falk, Carnegie Inst. of Tech., Tech. Rep. 5 (July 1, 1950) (to be published).
[10] Boyer, Gove, Harvey, Livingston, and Deutsch (to be published).

TABLE I. Data on target materials, thickness, and deuteron energy.

Target element	$_6C^{12}$	$_{13}Al^{27}$	$_{28}Ni^{58,60}$	$_{47}Ag^{107,109}$	$_{73}Ta^{181}$	$_{79}Au^{197}$	$_{83}Bi^{209}$
Target material	(a) polyethylene (b) aquadag	foil	foil	foil	foil	(a) foil (b) foil	evaporated film
Thickness (mg/cm²)	(a) 3.95 (b) 1	1.9	4	7.3	15.5	(a) 24 (b) 6.6	12
Average deuteron energy at target center	13.8	13.8	13.8	13.8	13.9	13.6	13.8

cussion will be given here. A schematic diagram of the apparatus is shown in Fig. 1. A beam of 14-Mev deuterons from the MIT cyclotron was used to bombard thin targets in the center of a large evacuated chamber located about 12 ft from the cyclotron and separated from it by a 4-ft thick concrete wall. At the entrance to the chamber the deuteron beam passed through a parallel plate ionization chamber which was provided with lead slits, placed before and after, to ensure that all deuterons passing through it struck the target. The output current from this monitor was integrated electronically so that counts were measured for a constant number of deuterons incident on the target. The emitted protons were detected in a triple proportional counter arrangement, the three sections of which were placed in triple coincidence, and which could be set at any angle from 15° to 135° with respect to the deuteron beam. The detector and the associated electronic circuits were designed to count only those protons whose ranges terminated in the detector. A system of graded aluminum foils set in front of the detector was used to scan the spectrum of the emitted protons, and between these and the target an aperture 0.4 in. wide and 0.75 in. high defined a solid angle of 0.013 steradian. The target to be investigated was placed in a target holder inserted through to top lid of the chamber. Fragile targets were mounted in thin aluminum frames large enough so that no deuterons could strike the frame. A separate target holder which could be introduced through the side of the chamber was provided, and an element was chosen for this side target which gave a well-resolved proton group upon deuteron bombardment. This could be used both to line up the equipment initially and to check on drifts. In general it was a 1.9 mg/cm² aluminum foil which gives a fairly well-resolved long range proton group from the reaction $Al^{27}(d,p)$, $Q=5.5$ Mev (Fig. 4). Another convenient side target material was found to be polyethylene of thickness 3.95 mg/cm² carbon content which gives a very well-resolved proton group from the reaction $C^{12}(d,p)$, $Q=2.7$ Mev (Fig. 2).

III. PROCEDURE

The target to be investigated was placed in the top target holder and the discriminator gate settings and amplifier gains for the three sections of the triple chamber were adjusted so that a plot of triple coincidences *vs* range gave a narrow symmetrical peak from the side target. The side target was then removed, the top target inserted, and a plot of triple coincidences *vs* range was recorded for all protons with range greater than that of the elastic deuterons at a series of angles between 15 and 135. The counting time for each point on these plots of the energy spectrum was determined by integrating the deuteron beam current produced in the monitor. Between each run at a given angle on the top target, the side target would be inserted and a plot of the normalization peak would be taken. The top target run would be repeated if the area under this normalization peak had changed more than 10 percent during the run. The target and counter angles are indicated in Fig. 1.

The following relationsip between θ and ϕ was maintained for the reflection and transmission cases

$$\text{for } \theta \leqslant 90° \quad \phi = 90 - \theta/2$$
$$\text{for } \theta \geqslant 90° \quad \phi = 180 - \theta/2.$$

In all of the elements studied, with exception of carbon and the long range proton group from aluminum, the close level spacing and the poor resolution inherent in range measurements made it impossible to resolve individual groups. In these cases angular distributions of segments of the spectrum were measured. This was accomplished by marking on the abscissa which is linear in range (mg/cm² Al), the values of the energy in the center of mass system. These will be nonlinear in the proper way so that the area under the curve measured between any two limits of center of mass energy will be the same as would be the case if the spectrum had been replotted on a scale linear in E with the ordinate multiplied by $\Delta R/\Delta E$. The areas are then multiplied by a factor $\sin\phi$ to compensate for varying thickness presented by the target to the deuteron beam as the angle is varied and a normalization factor N which is the mean area of the normalization peak taken before and after the run to compensate for drift. The third factor applied compensates for the fact that the solid angle in which the protons are accepted by the counter, while constant in the laboratory system of coordinates, varies with angle and energy in the center of mass system. This correction, which has been discussed by several authors,[1] is given by

$$g(\theta) = \frac{\text{center of mass intensity}}{\text{laboratory intensity}} = \frac{\sin\theta d\theta}{\sin\theta' d\theta'},$$

where θ and θ' are the counter angles in the laboratory and center of mass system. This can be written as

$$g(\theta)=[1-2\cos\theta/K(\theta)+1/K^2(\theta)]^{\frac{1}{2}}[1-\cos\theta/K(\theta)],$$
$$K(\theta)=\cos\theta+(\cos^2\theta+G)^{\frac{1}{2}}, \quad (1)$$
$$G=M_c[M_2(1+Q/E_1)-m_1]/m_1m_2.$$

Here m_1, M_c, m_2, M_2 are the masses of the bombarding particle, the "compound" nucleus, the emitted particle, and the residual nucleus, respectively. E_1 is the energy of the bombarding particle in the laboratory system and Q is the reaction energy. For cases of medium atomic number where $G>1$ the expression reduces to

$$g(\theta)\simeq 1-(2\cos\theta/G^{\frac{1}{2}}). \quad (2)$$

For large atomic numbers such as gold, $g(\theta)\sim 1$, and can be ignored.

The areas corrected in this way are plotted against the center of mass angle given by the relation

$$\tan\theta'=K\sin\theta/(K\cos\theta-1). \quad (3)$$

Again for $G>1$ this reduces to $\theta'=\theta$.

Because of the rapid variation of intensity with angle the intensity has been plotted on a logarithmic scale on the ordinates of the figures shown in order that several angular distributions could be displayed on one figure.

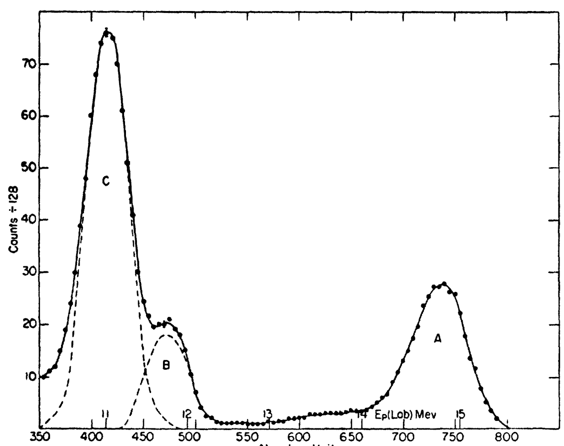

FIG. 2. Proton spectrum from the reaction $C^{12}(d,p)C^{13}$ taken at $\theta=60°$. The ordinate gives the triple coincidence counts on a scale of 128 and the abscissa gives the thickness of absorber inserted between the target and detector in units of 0.416 mg/cm² of aluminum. Also marked on the abscissa is the proton energy in Mev in the laboratory system.

IV. RESULTS

Table I lists the target elements used, along with the target material and thickness and the average deuteron energy at the target center. The measured intensities per unit solid angle for various segments of the emitted proton spectra for the seven elements studied are shown in Figs. 3, 5, 7, 9, 11, 13, and 15. In these figures the intensity has been plotted on a logarithmic scale against the center of mass angle θ' as abscissa or against laboratory angle θ where the difference is negligible.

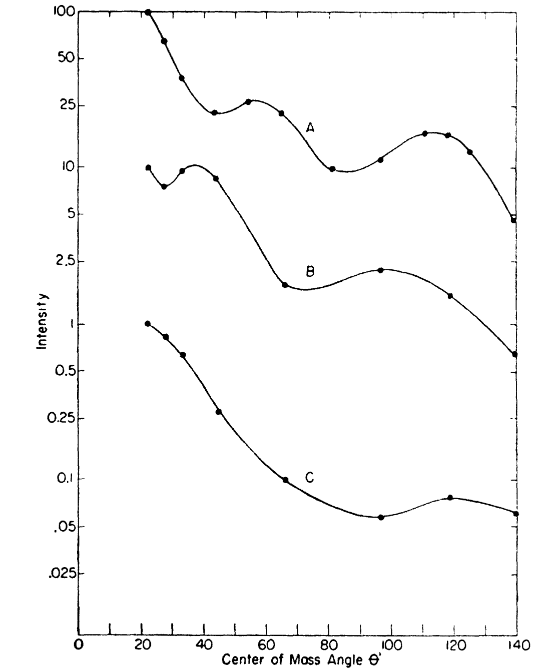

FIG. 3. The angular distributions of protons from the reaction $C^{12}(d,p)C^{13}$ for the three proton groups shown in Fig. 2. The abscissa is proportional to the number of protons emitted at a given angle per unit solid angle. The proton energies for each distribution are the following: (a) 13.5 Mev, (b) 10.7 Mev, (c) 9.9 Mev in the center of mass system.

Typical spectra at one angle for each of the seven targets are shown in Figs. 2, 4, 6, 8, 10, 12, and 14. The choice of segments is shown in each case and lettered alphabetically, corresponding to the lettering on the angular distributions.

Table II lists the mean proton energy in the center of mass system for each of the angular distributions, the average percentage probable error in any of the points plotted on the angular distributions, and finally a factor by which all the points plotted in a given distribution should be multiplied to obtain the differential cross section in units of σ_1, where σ_1 is the differential cross section at 30° for the reaction $Al^{27}(d,p)$; $Q=5.5$ Mev. The absolute value of σ_1 has been measured by the author[11] to be roughly 2.4 millibarns/atom-steradian. The probable error was obtained from the average variation in area of the normalization peak taken before and after a run at a given angle, or where runs have been repeated, from the average variations in the repeated values. Because of the extremely high counting rates, statistical errors can be neglected in most cases. There are, however, two other sources of possible error which cannot be estimated accurately. The first occurs principally at small forward angles and is caused by the presence on the target of carbon and oxygen contamination. These contaminant peaks must be excluded when the area is measured and this introduces some

[11] H. E. Gove and K. Boyer, Phys. Rev. 79, 241A (1950).

TABLE II. Center of mass correction, percentage errors, and cross sections of the angular distributions.

Reaction	Curve	Mean proton energy in center of mass E_p (Mev)	Maximum value of $\|1-g(\theta)\|$ (%)	Average percentage probable error	To obtain cross section in units of σ_1 multiply values on each curve by
$C^{12}(d,p)$	A	13.5	16	±3	0.0266
	B	10.7	18	±5	0.226
	C	9.9	20	±5	19.1
$Al^{27}(d,p)$	A	17.7	7.7	±2	0.010
	B	16.7	8.0	±2	0.020
	C	15.5	8.3	±2	0.155
	D	14.1	8.7	±2	0.849
	E	13.0	9.1	±2	5.04
	F	11.9	9.5	±2	12.5
$Ni^{58,60}(d,p)$	A	19.6	4	±4	0.0297
	B	18.4	4	±4	0.0744
	C	16.9	4	±4	0.561
	D	15.3	4	±4	3.73
	E	13.5	4	±4	10.3
$Ag^{107,109}(d,p)$	A	17.1	2.5	±2.5	0.0348
	B	15.0	2.5	±2.5	0.179
	C	13.5	2.5	±2.5	1.21
	D	12.5	2.5	±2.5	15.3
$Ta^{181}(d,p)$	A	17.1	1.7	±3.5	0.0318
	B	15.8	1.7	±3.5	0.296
	C	14.7	1.7	±3.5	1.63
	D	13.4	1.7	±3.5	6.06
	E	12.1	1.7	±3.5	37.5
$Au^{197}(d,p)$	A	17.2	1.5	±5.0	0.0864
	B	15.8	1.5	±2.5	0.169
	C	14.4	1.5	±2.5	0.826
	D	13.3	1.5	±2.5	5.95
	E	11.9	1.5	±2.5	7.85
$Bi^{209}(d,p)$	A	15.3	1.4	±2	0.0403
	B	13.7	1.4	±2	4.06
	C	12.7	1.4	±2	41.1

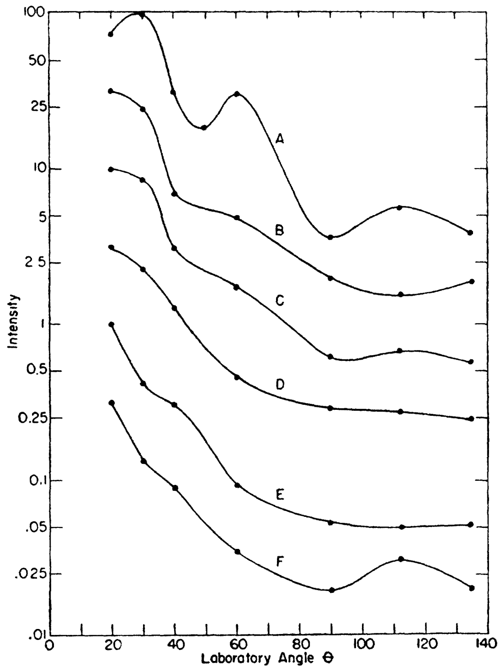

FIG. 5. Angular distributions of protons from the reaction $Al^{27}(d,p)Al^{28}$ for the long range proton group and the segments of spectrum shown in Fig. 4. Mean proton energies in the center of mass system for each distribution are as follows: (a) 17.7 Mev, (b) 16.7 Mev, (c) 15.5 Mev, (d) 14.1 Mev, (e) 13.0 Mev, (f) 11.9 Mev.

uncertainty. The second effect is that of background. As a rule this is measured in two ways, first by raising the target and measuring the counting rate, and secondly by continuing to measure the counting rate well beyond the longest range proton group with the target in the deuteron beam. This latter method is probably a very good measure of the background in the vicinity of the longest range protons. The former method may give erroneous results because the background intensity differs depending on whether the target is in or out of the beam. However, because of the triple coincidence method employed, background rates are less than 1 percent of the actual counting rate and could not give serious errors no matter what technique was used to correct for it.

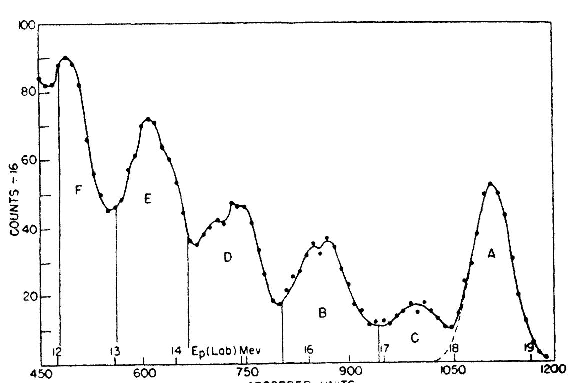

FIG. 4. Proton spectrum from the reaction $Al^{27}(d,p)Al^{28}$ taken at $\theta=60°$. For this and all subsequent figures of proton spectra the abscissa scale is in units of 0.416 mg/cm² and the proton energy scale is given in Mev in the laboratory system.

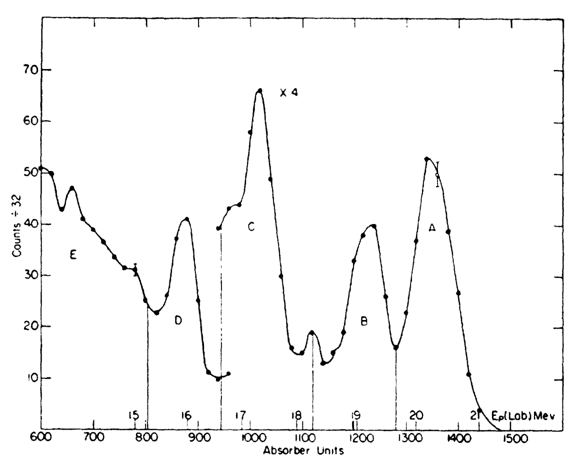

FIG. 6. Proton spectrum from the reaction $Ni^{58,60}(d,p)Ni^{59,61}$ taken at $\theta=30°$.

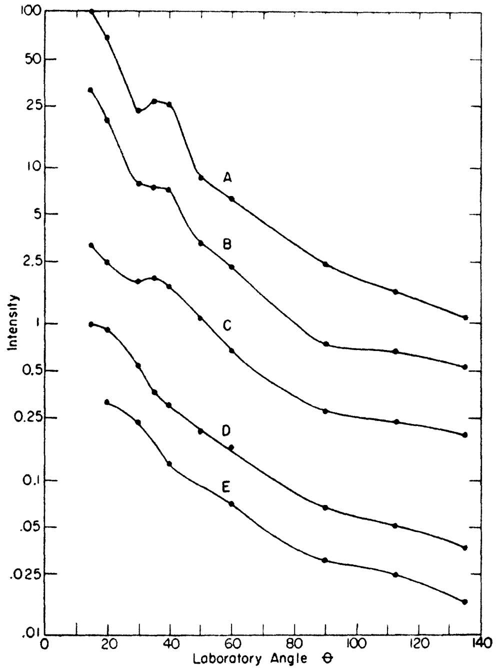

FIG. 7. Angular distributions of the five segments of the proton spectrum shown in Fig. 6 for the reaction $Ni^{58,60}(d,p)Ni^{59,61}$. Mean proton energies in the center of mass system for each distribution are as follows: (a) 19.6 Mev, (b) 18.4 Mev, (c) 16.9 Mev, (d) 15.3 Mev, (e) 13.5 Mev.

V. ANALYSIS AND DISCUSSION OF RESULTS

Possibly the most striking feature of the angular distributions is the rather strong forward intensity, at least for the lighter nuclei. This feature appears very much less pronounced in the figures than it actually is

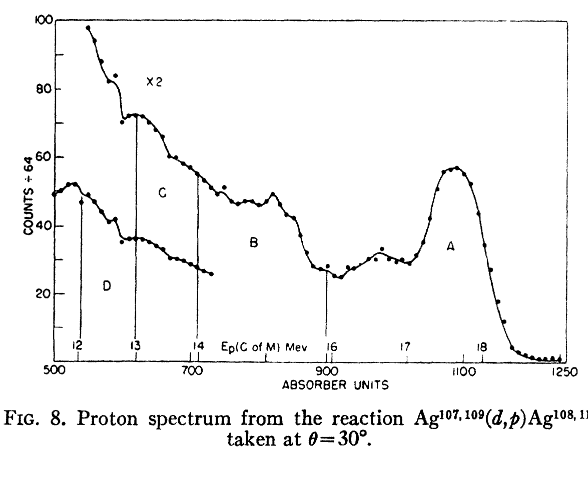

FIG. 8. Proton spectrum from the reaction $Ag^{107,109}(d,p)Ag^{108,110}$ taken at $\theta=30°$.

because of our method of plotting the *logarithm* of the observed intensity per unit solid angle.

A gross measure of the character of this asymmetry about 90° can be obtained by measuring the ratio of the area under the plot of intensity per unit solid angle against angle for $\theta<90°$ to the area for $\theta>90°$. Since the measurements were made in general between 20° and 140° the ratio R of the area under the angular distributions between 20° and 90° to that between 90° and 140° was measured. It is true that this overestimates the contribution from forward directions due to the unequal division of angle but it provides some measure of the asymmetry. Notice that this is not a measure of the number of protons emitted $<90°$ to those emitted $>90°$ since the solid angle factor $\sin\theta$ has not been included.

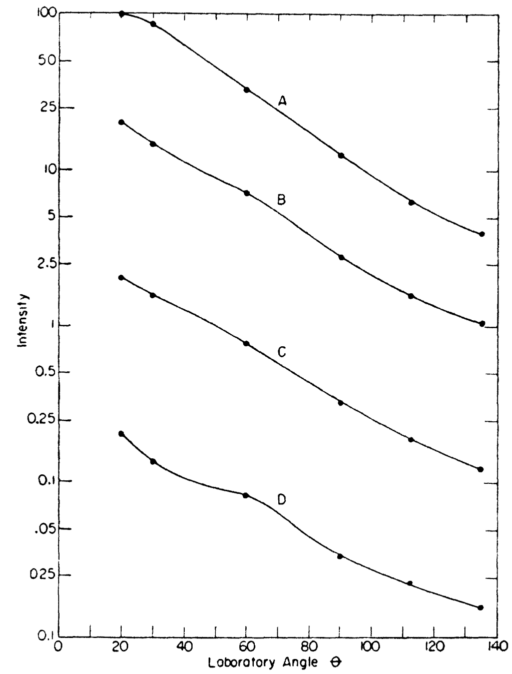

FIG. 9. Angular distributions of the four segments of the proton spectrum shown in Fig. 8 for the reaction $Ag^{107,109}(d,p)Ag^{108,110}$. Mean proton energies in the center of mass system for each distribution are as follows: (a) 17.1 Mev, (b) 15.0 Mev, (c) 13.5 Mev, (d) 12.5 Mev.

It is merely some measure of the asymmetry of the distributions as they are normally plotted; i.e., intensity per unit solid angle as ordinate. This quantity R is listed in Table III with the corresponding mean proton energy in the center of mass system for each segment or

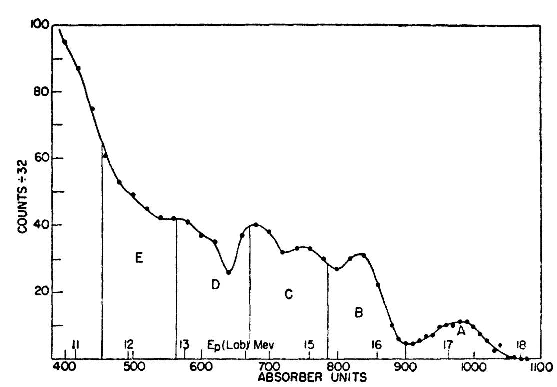

FIG. 10. Proton spectrum from the reaction $Ta^{181}(d,p)Ta^{182}$ taken at $\theta=90°$.

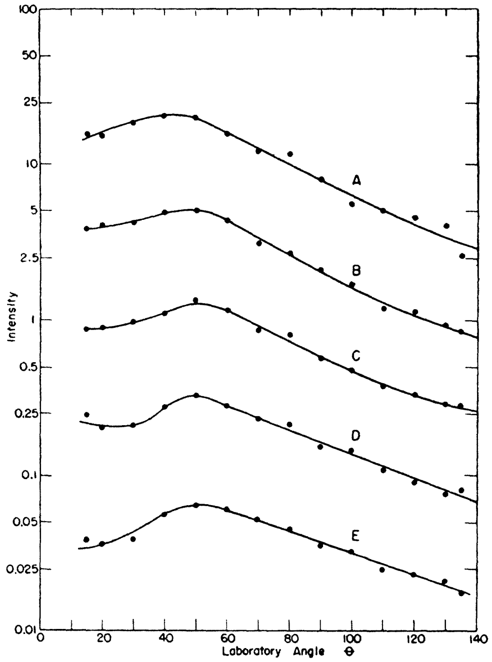

FIG. 11. Angular distributions of the five segments of the proton spectrum shown in Fig. 10 for the reaction $Ta^{181}(d,p)Ta^{182}$. Mean proton energies in the center of mass system for each distribution are as follows: (a) 17.1 Mev, (b) 15.8 Mev, (c) 14.7 Mev, (d) 13.4 Mev, (e) 12.1 Mev.

proton group. Also included is the approximate barrier height for each element studied. The last two columns in Table III list the variation of R with E_p, that is (dR/dE_p) assuming it to be linear, as found by the method of least squares and the product of (dR/dE_p) and barrier height B.

The ratio R measured in this way turns out to be considerably greater than unity in most cases, varying from 1.6 in gold to 13 in nickel. Even if one takes account of the unequal division in the abscissa, the ratio is greater than unity in *all* cases.

For all elements except the lightest one, carbon, the ratio R increases in essentially a linear fashion as the

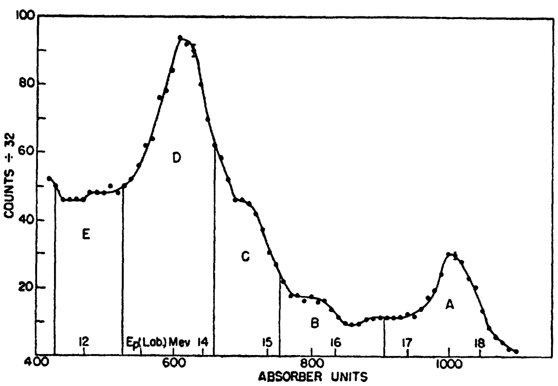

FIG. 12. Proton spectrum from the reaction $Au^{197}(d,p)Au^{198}$ taken at $\theta=30°$.

proton energy increases. Furthermore, (dR/dE_p) seems to decrease with increasing Z. In fact, if we measure E_p in units of the barrier height the quantity $B(dR/dE_p)$ is substantially independent of Z.

A more detailed examination of the angular distributions indicates two further interesting features. The proton groups corresponding to C^{13} being left in its ground state and first excited level (Fig. 3a and 3b), and Al^{28} in its ground state (Fig. 5a) show pronounced angular maxima. These three groups as well as the one leaving C^{13} in its second excited state (Fig. 3c) probably involve only one level in the residual-nucleus although it has recently[12] been shown that the long range proton group from $Al^{27}(d,p)Al^{28}$ is actually two groups of 30-kev separation.

The long range proton group from $Al^{27}(d,p)$ is of particular interest because of the work reported by

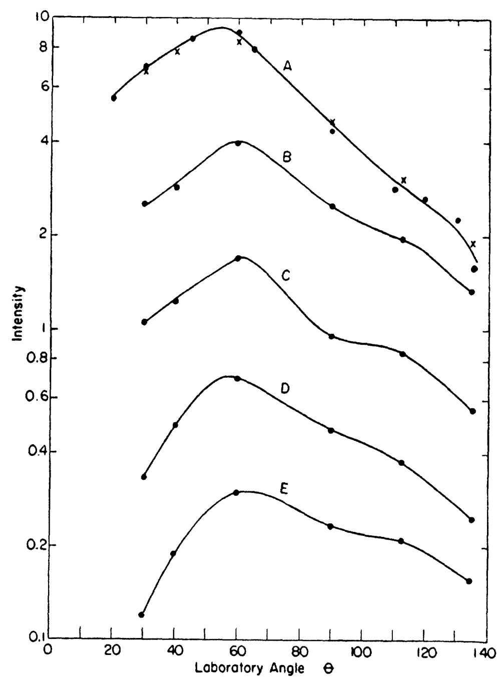

FIG. 13. Angular distributions of the five segments of the proton spectrum shown in Fig. 12 for the reaction $Au^{197}(d,p)Au^{198}$. Mean proton energies in the center of mass system for each distribution are as follows: (a) 17.2 Mev, (b) 15.8 Mev, (c) 14.4 Mev, (d) 13.3 Mev, (e) 11.9 Mev.

Holt and Young[7] at three lower deuteron energies where they also find rather pronounced angular maxima. Figure 16 shows a plot of the distributions for the four energies. Nemilov and Funshtein[4] find that the long range group from this reaction is emitted with spherical symmetry within the limits of accuracy of their results for 3.9-Mev deuterons. At 4.6 Mev a single maximum appears at about 60° with a rise at forward energies. At 5.8 Mev there are three maxima at about 50°, 85°, and 135°. At 7.5 Mev these maxima have shifted forward to

[12] Preliminary measurements, MIT High Voltage Laboratory.

40°, 80°, and 130°. These three measurements are due to Holt and Young. In each case the distribution rises at forward angles. The intensity at 0° for the 7.5-Mev case is reported[7] to be 30 times the value at 25°. At 13.8 Mev the three maxima have shifted forward to 30°, 60°, and 120°. There is some evidence to show that the intensity at 15° is again rising. These maxima give a definite impression of waves of intensity advancing along the θ axis as the deuteron energy is increased. This analogy has been used previously by Heydenburg and Inglis[1] in describing the results of their angular distribution measurements for $O^{16}(d,p)O^{17}$.

TABLE III. Analysis of data in terms of the proton energy, the ratio R and the barrier height.

Target element	Atomic number Z	Barrier height B (Mev)	Mean proton energy in center of mass E_p (Mev)	Ratio $\frac{\text{area } 20°\text{–}90°}{\text{area } 90°\text{–}140°}$ R	Slope $\frac{dR}{dE_p}$	$B\frac{dR}{dE_p}$
Carbon	6	3.0	9.9	9.1	Neg.	
			10.7	4.6		
			13.5	3.4		
Aluminum	13	4.5	11.9	4.0	0.79	3.5
			13.0	6.2		
			14.1	5.3		
			15.5	7.8		
			16.7	7.6		
			17.7	10.4		
Nickel	28	7.3	13.5	6.6	0.61	4.4
			15.3	7.4		
			16.9	6.5		
			18.4	10.0		
			19.6	13.0		
Silver	47	9.8	12.5	5.6	0.87	8.5
			13.5	7.0		
			15.0	7.7		
			17.1	9.6		
Tantalum	73	12.8	12.1	2.7	0.45	5.7
			13.4	3.2		
			14.7	3.7		
			15.8	4.4		
			17.1	4.5		
Gold	79	13.4	11.9	1.6	0.32	4.2
			13.3	2.9		
			14.4	2.3		
			15.8	2.4		
			17.2	3.6		
Bismuth	83	13.9	12.7	2.0	0.39	5.4
			13.7	2.5		
			15.3	3.1		

The second feature is evident in the angular distributions of the three heaviest elements, Ta, Au, and Bi. In these cases the intensity, instead of continuing to rise with decreasing angle, reaches a maximum between 40° and 60° and then falls as the angle decreases. In this respect the results bear a resemblance to the work of Falk[9] who found a similar effect for (d,n) distributions except that in all cases the "turn over" angle is about 10°.

A conventional method of evaluating the complexity of angular distributions is to expand the observed intensity per unit solid angle $I(\theta)$ in terms of Legendre polynomials.

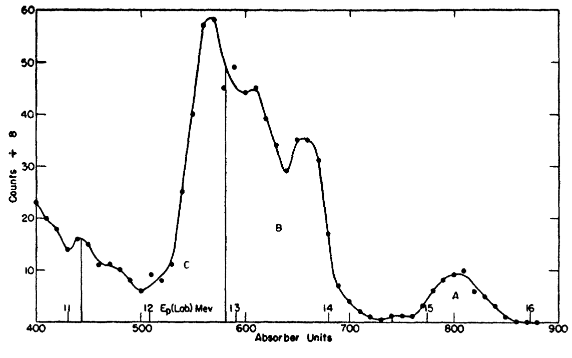

FIG. 14. Proton spectrum from the reaction $Bi^{209}(d,p)Bi^{210}$ taken at $\theta=90°$.

$$I(\theta)=\sum_l A_l P_l(\cos\theta), \quad (4)$$

where

$$A_l=\tfrac{1}{2}(2l+1)\int_{\theta=0}^{\pi} I(\theta)\sin\theta P_l(\cos\theta)d\theta. \quad (5)$$

This has been done for the case of $C^{12}(d,p)$ (Fig. 2a), $Al^{27}(d,p)$ (Fig. 4a), and $Au^{197}(d,p)$ (Fig. 12a). The integration has been carried out by first multiplying the observed intensity by $\sin\theta$ and extrapolating the resultant curve to zero at $\theta=0$ and $\theta=180°$. Intervals of θ of 5° were taken and the quantity $I(\theta)\sin\theta P_l(\cos\theta)$ was plotted against θ for values of l from 0 to 11. The area under each of these curves was measured with a planimeter and multiplied by the statistical factor $(2l+1)$. Table IV lists the coefficients obtained for the

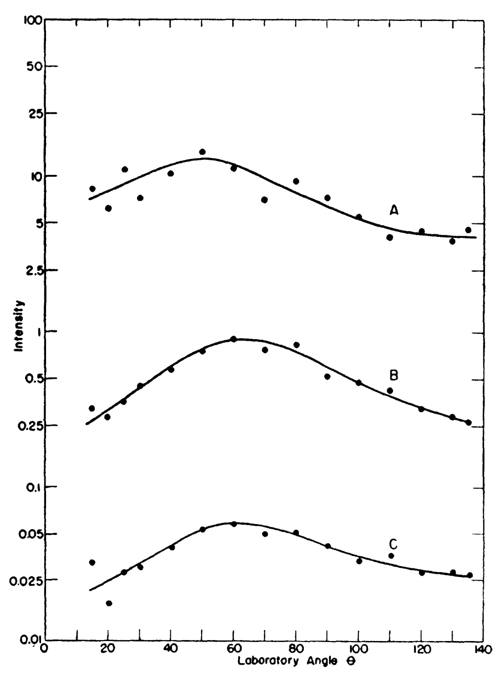

FIG. 15. Angular distributions of the three segments of the proton spectrum shown in Fig. 14 for the reaction $Bi^{209}(d,p)Bi^{210}$. Mean proton energies in the center of mass system for each distribution are as follows: (a) 15.3 Mev, (b) 13.7 Mev, (c) 12.7 Mev.

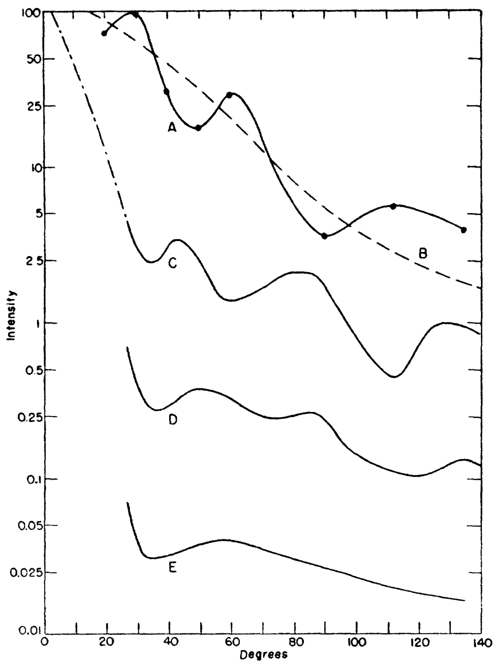

FIG. 16. Comparison of the angular distribution of the long range proton group from the reaction $Al^{27}(d,p)Al^{28}$ taken at 13.8 Mev (curve A) with the theoretical prediction of French and Goldberger (curve B) and with the experimental results of Holt and Young at deuteron energies of 7.5 Mev (curve C), 5.8 Mev (curve D), and 4.6 Mev (curve E). The dotted portion of curve C is based on the measurements made by Holt and Young of the ratio of the intensity at 0° to that at 25° which they found to be 30. At 3.9 Mev Nemilov and Funshtein find the distribution to be spherically symmetric.

distributions that have been analyzed in this way. In all cases A_0 has been normalized to unity. For the $C^{12}(d,p)$ case the coefficients are relatively constant up to A_7 after which they fall off rapidly. A_3 and A_6 are especially favored because of the two maxima in the distribution (Fig. 2a). In the case of $Al^{27}(d,p)$ (Fig. 4a) coefficients near A_8 are favored because of the maxima in the distributions while those near A_5 are depressed. Again A_1 is favored because of the forward asymmetry. The coefficients for $Au^{197}(d,p)$ (Fig. 12a) fall off rather rapidly past A_1, none of them being greater than A_0; values as high as A_8 are significant. It should be pointed out that Legendre polynomials are quite sensitive functions of angle near zero and 180°. Because this is just the region in which extrapolated values are used, this leads to a rather large uncertainty in the results. Hence it would be unwise to attach too much significance to any analysis of this nature. In spite of this, it is probably safe to say that the presence of terms in the expansion as high as P_7 to P_9 indicates that at least g-wave deuterons are effective in the reactions.

VI. THEORY

The emission of protons from deuteron-induced reactions can be described by two different models. The first model is based on the Bohr[13] concept of the compound nucleus and the second on the Oppenheimer-Phillips[14] stripping process.

Let us first consider the compound nucleus case. When the deuteron enters the target nucleus a compound nucleus of rather high excitation energy is formed (about 28 Mev for deuterons of 14-Mev energy). This energy is quickly shared among the constituent particles. For excitation energies of this magnitude the levels in the compound nucleus would be expected to be broad and overlapping so that even for a monoenergetic deuteron beam a rather large number of values of total angular momentum would be involved in the compound nucleus. Recent calculations have been made by Wolfenstein[15] on the angular distribution of deuteron-induced reactions in the region of deuteron energy. In this work he assumed the formation of a compound nucleus. Since, for all l-values of the incoming deuterons only three m-values are permitted, the angular momentum of the compound nucleus will be polarized. He concludes that this will result in an anisotropy in the angular distribution of the outgoing particles. It will, however, be characteristically symmetric about 90°. This latter condition presumably results from the assumption that many levels in the compound nucleus may be effective and that for every case of two levels differing in parity which interfere to give a forward distribution there will be some other combination of interfering levels which give a backward distribution. The net result gives the required anisotropy required by the polarization, but still maintaining the symmetry about 90°. The condition is somewhat different when lower energy deuterons and lighter target nuclei are employed. Here the presence of resonances in the excitation functions indicates that more widely spaced levels in the compound nucleus are involved. The proton angular distributions show a tendency to become more symmetric about 90° at deuteron energies corresponding to resonances in the excitation curve since one value of l for the compound nucleus will predominate over all others at such an energy. In the regions between resonances the distribu-

TABLE IV. Coefficients of the expansion in Legendre polynomials.

Reaction	Figure	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}
$C^{12}(d,p)$	2A	1.00	1.37	1.26	1.64	1.19	1.03	1.52	0.72	0.07	0	−0.39	−0.18
$Al^{27}(d,p)$	4A	1.00	1.78	1.55	0.89	0.22	−0.06	−0.04	−0.81	−1.28	−1.01	−0.49	0.61
$Au^{197}(d,p)$	12A	1.00	0.85	−0.20	−0.42	−0.38	−0.10	0.16	0.41	0.41	−0.14	−0.12	−0.12

[13] N. Bohr, Nature **137**, 344 (1936).
[14] J. R. Oppenheimer and M. Phillips, Phys. Rev. **48**, 500 (1935).
[15] L. Wolfenstein, Phys. Rev. **78**, 322A (1950).

tions show either forward or backward asymmetry in a somewhat random fashion due to interference effects between two levels of different parity. Analyzing the distributions in terms of either an expansion in powers of $\cos\theta$, or of Legendre polynomials, gives some indication of the possible values of angular momentum of the incoming deuteron involved in the reaction and, in a few simple cases, of the total angular momentum of the excited level in the compound nucleus.

The second possible mode of reaction for (d,p) and (d,n) processes was first proposed by Oppenheimer and Phillips.[14] Subsequent considerations of this process have been made by several authors, in particular by Volkoff.[16] In this article references can be found to previous theoretical work on this subject. Application to high energy deuterons was made by Serber[17] to explain the results of Helmholz *et al.*[8] An explanation of deuteron excitation functions for a wide range of atomic number and for deuteron energies up to 15 Mev using the stripping model has been proposed by Peaslee.[18] An extension of the Peaslee stripping theory to the angular distribution of protons from (d,p) reactions at 14 Mev has been made by French and Goldberger.[19] In this model the deuteron is pictured as breaking up, with the neutron at the surface of the target nucleus (where it must be in order that there exist an appreciable probability for it to be captured) and the proton of the order of a deuteron diameter away. At breakup the proton's momentum consists of a contribution from the forward momentum of the deuteron and a contribution from the internal motion of the deuteron. It has been shown[19] that under these conditions the angular distribution would manifest a strong preference for forward directions. Furthermore, it turns out that this preference increases with proton energy. On the other hand, it is rather unlikely that protons will be emitted at zero degrees because both the incoming deuteron and the outgoing proton are deflected by the Coulomb potential of the target nucleus. Hence at some forward angle the rising intensity must reach a maximum and there decline towards 0°. Clearly this angle at which the "turn-over" occurs will increase for increasing atomic number. It is true, of course, that one can designate this assembly of particles comprising the target nucleus and the deuteron at breakup as a compound system, but it differs from the conventional concept of the compound nucleus in two respects, first, there is no thorough sharing of energy among all the particles, and, secondly, the possible values of total angular momentum of the system will be somewhat greater in consequence of the larger size. This latter difference is actually just a question of degree whereas the former is a real difference which would not allow the character of the asymmetry about 90° of the proton distribution to vary in a random fashion as the deuteron energy is changed.

VII. CONCLUSIONS

The experimental results presented in this paper are clearly not to be explained by the conventional compound nucleus model. On the other hand, if we exclude the carbon case, the principal features of the distribution, namely, the ratio R greater than unity in all cases, the variation of R with E_p, the apparent lack of dependence of (dR/dE_p) on the atomic number Z when E_p is measured in units of the barrier height, and the "turn over" in heavy elements at forward angles are, qualitatively at least, explained by French and Goldberger on a stripping model. The tantalum, gold, and bismuth cases indicate that this "turn-over" angle does increase with Z as predicted. It will however be necessary to measure the position of this angle for lighter Z-elements which involves the use of targets in the rare earth region of atomic numbers and some method for measuring proton intensity for angles from 15° to zero.

The results in the case of carbon are not completely compatible with the stripping picture because the higher energy protons are less forward than the lower energy ones. This same result was found at much lower energy by Nemilov and Funshtein which, similarly, does not lend itself to interpretation by a stripping model.

The rather sharp angular maxima which are observed in the proton distributions from lighter target nuclei have not yet been satisfactorily explained due, no doubt, to the fact that deuterons are not in general very amenable to theoretical interpretation. It is conceivable that these maxima may not be incompatible with the stripping concept. On the other hand, they may be attributable to the competing compound nucleus process. The agreement between theory[19] and experiment for 14-Mev deuterons is shown for the long range proton groups from $Al^{27}(d,p)$ in Fig. 16a.

The effect of these maxima shifting toward smaller angles as the energy of the deuterons is increased can be attributed to an effect of the Coulomb potential. The same intensity maximum would be expected to occur at more forward angles as the deuteron energy increases since both the higher energy deuterons and protons will suffer smaller deflections from the Coulomb field. Such an effect points again to a stripping picture rather than that of a compound nucleus.

It is a sincere pleasure to acknowledge the guidance and encouragement provided by Professor M. Deutsch and Professor M. S. Livingston throughout this investigation. The many informative discussions with Dr. J. B. French are greatly appreciated. The building and testing of equipment was performed in collaboration with Dr. J. A. Harvey and Dr. K. Boyer who have provided a considerable amount of advice and assistance. The writer wishes to thank Mr. K. Huang and Mr. M. Zimmerman for their assistance in analyzing some of the data. He would also like to express his gratitude to the personnel of the cyclotron laboratory, in particular to Mr. F. J. Fay, Mr. R. W. Kilson, and Mr. E. F. White who have assisted in the work in innumerable ways.

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[17] R. Serber, Phys. Rev. **72**, 1008 (1947).
[18] D. C. Peaslee, Phys. Rev. **74**, 1001 (1948).
[19] J. B. French and M. Goldberger (to be published).