tering cross section, 6 percent uncertainty in the purity of the tritium sample, and 5 percent statistical error. Errors introduced by Rutherford scattering are estimated to be negligible. The finite angular resolution will cause some broadening of the peak in the angular distribution at $\phi = 180°$.

For the $n-T$ angular distribution measurements the background count observed with the platinum shutter in place averaged 30 percent for triple coincidences and 20 percent for quadruples. Accidental coincidences were calculated to contribute a background of less than one part in 10^4. For the data with hydrogen in the gas cell the corresponding backgrounds were about 20 percent and 10 percent. These backgrounds were lower because the higher counter gas pressure (63 cm instead of 15 cm) did not allow short range background particles to traverse all counters.

Integration of $\sigma(\phi)$ as plotted in Fig. 2 from 90° to 180° gives a value of 0.17 barn. If, as might be expected, the total cross section of tritium is greater than that of hydrogen (0.64 barn), there must either be a large peak in the angular distribution curve for scattering in the region from 0° to 90°, or a disintegration process which contributes a large fraction to the total cross section.

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Velocity Dependent Interactions and Nuclear Shells*

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An investigation is made of the energy splitting of the two possible j values, $l \pm \frac{1}{2}$, of a single nucleon outside a closed shell resulting from the assumption of certain velocity dependent terms in the nucleon-nucleon interaction.

I. INTRODUCTION

THE modification[1] by Mayer of the nuclear shell model demands spin-orbit coupling such that the single nucleon state with $j = l + \frac{1}{2}$ is displaced in energy well below the state with $j = l - \frac{1}{2}$. It was suggested in a previous communication[2] that the splitting might be accounted for by terms in the nucleon-nucleon interaction which are linear in the nucleon momenta.

II. THEORY

We have considered the system of a single nucleon outside a nuclear core consisting of a closed shell of nucleons, and have investigated the splitting between the two possible j values which results from the velocity dependent interaction of the outside nucleon with the closed shell. The following notation is used for the quantum numbers involved:

k = orbital angular momentum of each of the $4(2k+1)$ nucleons in the closed shell (two neutrons and two protons in each of the $2k+1$ states).

l = orbital angular momentum of the outside nucleon.

$j = l \pm \frac{1}{2}$ = total angular momentum of the outside nucleon, and hence, of the entire system.

Let $U(k, l, j)$ be the average velocity dependent interaction energy of the outside nucleon with the closed shell. The difference

$$\Delta U(k, l) = U(k, l, l+\tfrac{1}{2}) - U(k, l, l-\tfrac{1}{2})$$

gives the first-order splitting. Neither central nor tensor static forces can contribute to the splitting in first order.

It follows from the fact that the average velocity dependent interaction energy of an *entire* k shell with the l shell must vanish because of the transformation properties of the interactions, and from the ratio of the degeneracies of the $j = l + \frac{1}{2}$ and $j = l - \frac{1}{2}$ states, that

$$(l+1)U(k, l, l+\tfrac{1}{2}) = -lU(k, l, l-\tfrac{1}{2}).$$

Thus

$$\Delta U(k, l) = [(2l+1)/l]U(k, l, l+\tfrac{1}{2})$$

and only the case $j = l + \frac{1}{2}$ need be evaluated.

The velocity dependent forces may be enumerated as

TABLE I. Energy splitting due to velocity dependent interactions.

n	$S_{\alpha\beta}{}^{(n)}T_{\alpha\beta}{}^{(n)}$	$\Delta U(k, l)$
1	$(\boldsymbol{\sigma}_\alpha + \boldsymbol{\sigma}_\beta)\frac{1}{2}(\tau_{\alpha 3} + \tau_{\beta 3})$	$Q(A-B)$
2	$(\boldsymbol{\sigma}_\alpha + \boldsymbol{\sigma}_\beta)\frac{1}{2}(1 + \tau_{\alpha 3}\tau_{\beta 3})$	$A-B$
3	$(\boldsymbol{\sigma}_\alpha + \boldsymbol{\sigma}_\beta)\frac{1}{4}(3 + \boldsymbol{\tau}_\alpha \cdot \boldsymbol{\tau}_\beta)$	$\frac{3}{2}(A-B)$
4	$(\boldsymbol{\sigma}_\alpha + \boldsymbol{\sigma}_\beta)\frac{1}{2}(1 - \tau_{\alpha 3}\tau_{\beta 3})$	A
5	$(\boldsymbol{\sigma}_\alpha - \boldsymbol{\sigma}_\beta)\frac{1}{2}(\tau_{\alpha 3} - \tau_{\beta 3})$	QA
6	$[\boldsymbol{\sigma}_\alpha \times \boldsymbol{\sigma}_\beta]\frac{1}{2}[\boldsymbol{\tau}_\alpha \times \boldsymbol{\tau}_\beta]_3$	QB

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[1] M. G. Mayer, Phys. Rev. **78**, 16 (1950).
[2] Blanchard, Avery, and Sachs, Phys. Rev. **78**, 292 (1950).

TABLE II. Values of A and B for particular values of k and l.

k	l	A/V_0	B/V_0
0	1	0.25	−0.25
	2	0.18	−0.06
	3	0.11	−0.01
	4	0.06	−0.003
	5	0.03	−0.0008
1	2	0.28	−0.11
	3	0.22	−0.06
	4	0.14	−0.02
2	3	0.20	−0.04

in reference 2:

$$V_{\alpha\beta}{}^{(n)}=J(|\mathbf{r}_\alpha-\mathbf{r}_\beta|)[(\mathbf{r}_\alpha-\mathbf{r}_\beta)\times(\mathbf{p}_\alpha-\mathbf{p}_\beta)]\cdot\mathbf{S}_{\alpha\beta}{}^{(n)}T_{\alpha\beta}{}^{(n)}$$

with n running from 1 to 6. The results of the calculation of the splitting are summarized in Tables I and II.

In Table I, $Q=+1$ if the outside nucleon is a neutron and -1 if it is a proton. A and B are the following integrals over all space:

$$A=\frac{2(2k+1)(2l+1)\hbar}{(4\pi)^2}\int\int[f(r_1)]^2[g(r_2)]^2 \times J(|\mathbf{r}_1-\mathbf{r}_2|)\left\{1-\frac{r_1}{r_2}\mu\right\}d^3r_1d^3r_2,$$

$$B=\frac{2(2k+1)(2l+1)\hbar}{l(l+1)(4\pi)^2}\int\int f(r_1)g(r_2)J(|\mathbf{r}_1-\mathbf{r}_2|) \times\Bigg\{l(l+1)\left(1-\frac{r_1}{r_2}\mu\right)P_k(\mu)P_l(\mu) +(1-\mu^2)\left(1-\frac{r_1}{r_2}\mu\right)P_k'(\mu)P_l'(\mu) -\frac{r_1}{r_2}(1-\mu^2)P_k(\mu)P_l'(\mu)+r_1r_2(1-\mu^2)P_k(\mu)P_l'(\mu) \times\left[\frac{1}{r_2}\frac{\partial}{\partial r_2}-\frac{1}{r_1}\frac{\partial}{\partial r_1}\right]\Bigg\}f(r_2)g(r_1)d^3r_1d^3r_2,$$

where $\mu=(\mathbf{r}_1\cdot\mathbf{r}_2)/r_1r_2$; $f(r)$, $g(r)$ are the normalized radial wave functions for the k orbits and l orbit, respectively; P_m is the Legendre polynomial of degree m, and P_m' its derivative.

The integrals A and B were evaluated for a few low values of k and l. In order to do so, it is of course necessary to assign definite radial dependences to the wave functions and to the interaction. The choice made was:

$$f(r)\sim r^ke^{-ar};\quad g(r)\sim r^le^{-ar};\quad J(r)=V_0e^{-br}/\hbar br,$$

where V_0 has the dimensions of energy; $a/b=1$.

With this choice of parameters, the numerical results obtained are given in Table II.

The results given in Table II depend on a and b only through their ratio, but they are extremely sensitive to the choice made, the splitting decreasing very rapidly if a/b is decreased.

If the strength, V_0, of the velocity dependent interaction is taken to be comparable to that of the usually assumed static forces, then splittings of the order of magnitude of those hypothesized for the spin-orbit coupling shell model can be expected. For the interactions $V^{(2)}$, $V^{(3)}$, $V^{(4)}$, V_0 must be negative if the state $j=l+\frac{1}{2}$ is to lie lower. For $V^{(1)}$, $V^{(5)}$, $V^{(6)}$, however, the presence of the factor Q has the following consequence. With a definite sign for V_0, the total angular momentum state that is lowest depends on whether the outside nucleon is a neutron or a proton. Since no such charge-dependent doublet inversion has been observed experimentally,[3] this argues strongly against $V^{(1)}$, $V^{(5)}$, $V^{(6)}$ as possible interaction functions.

Since $V^{(4)}$ can be discarded by the considerations of reference 2, the only velocity dependent interactions which remain with no serious shortcomings are $V^{(2)}$ and $V^{(3)}$. $V^{(2)}$ is charge dependent, giving the same forces for the neutron-neutron and proton-proton interactions but no forces in the neutron-proton interaction. $V^{(3)}$ is charge independent; $\mathbf{S}_{\alpha\beta}{}^{(3)}T_{\alpha\beta}{}^{(3)}$ can be written as $(\boldsymbol{\sigma}_\alpha+\boldsymbol{\sigma}_\beta)(1-P_{\alpha\beta}{}^X)$, where $P_{\alpha\beta}{}^X$ is the space exchange operator. It is interesting that $V^{(3)}$ is just the interaction inferred from high energy scattering data by Case and Pais.[4]

It is to be noted that rejection of $V^{(5)}$ and $V^{(6)}$ would weaken one of the arguments of reference 2 for the introduction of velocity dependent interactions: the possibility of resolving the magnetic moment anomalies of H^3 and He^3 through interaction moments. It was found that only $V^{(5)}$ and $V^{(6)}$ allowed such a possibility.

The suggestions and advice of Professor R. G. Sachs have been of value throughout the course of this work.

[3] $V^{(1)}$ gives a clearcut contradiction with observation, since it gives interactions only between like nucleons. Thus, for a particular choice of V_0, if the next neutron added after a closed shell of neutrons went into the $j=l+\frac{1}{2}$ state, then the corresponding proton state would be $j=l-\frac{1}{2}$, and neutron and proton magic numbers would not coincide. $V^{(5)}$ and $V^{(6)}$, on the other hand, give interactions only between unlike nucleons. Accordingly, we compare the spins of two nuclei: (N_1, Z_1) and (N_2, Z_2), where $Z_1=N_2$, and N_1 and Z_2 correspond to closed shells in the sense of the present calculation. Only one applicable case has been found. $Sb^{121}(I=5/2)$ and $Zr^{91}(I=5/2)$. This case certainly gives no indication of charge dependent doublet inversion. For purposes of interpretation, it does not seem to be necessary to have closed shells in the strict sense of the calculation, since the splittings found are proportional to the size of the core. Thus we conclude that $V^{(5)}$ and $V^{(6)}$ would actually give effects like those of $V^{(1)}$, and can be excluded.

[4] K. M. Case and A. Pais, Phys. Rev. **80**, 203 (1950).