energy of polarization can be shown to be

$$W_P=-\frac{1}{6}\left[(\alpha_z+2\alpha_x)\left(\frac{e\rho}{r^4}\right)_{Av}\right]+\frac{(\alpha_z-\alpha_x)}{12}\times\left[\frac{e\rho(3\cos^2\theta_m-1)}{r^4}\right]_{Av}P_2(\cos I, J),$$

where α_z = polarizability along the nuclear axis, α_x = polarizability perpendicular to the nuclear axis, ρ = charge density (outside the nucleus considered), r = distance from center of nucleus to charge, θ_m = angle between molecular axis of symmetry and charge, I = nuclear spin, and J = angular momentum of molecule. The quantum-mechanical equivalent of $\alpha_z-\alpha_x$ is

$$\alpha_z-\alpha_x=\frac{2I(I+1)}{2I-1}\sum_n\frac{[|\mu_{on}|_{M=I}]^2-[|\mu_{on}|_{M=I-1}]^2}{E_n-E_0},$$

where E_0 and E_n are the nuclear energies in the ground and excited states, respectively, and $|\mu_{on}|_{M=I}$ is the z-component of the dipole matrix element between these two states for the magnetic quantum number $M=I$.

With the above definition of $\alpha_z-\alpha_x$, the proper expression for the anisotropic part of the polarization energy for a nucleus on the axis of a symmetric molecule is

$$W=\tfrac{2}{3}e(\alpha_z-\alpha_x)\mathrm{p}\left(\frac{3K^2}{J(J+1)}-1\right)\left[\frac{\frac{3}{4}C(C+1)-I(I+1)J(J+1)}{2I(2I-1)(2J-1)(2J+3)}\right],$$

where $C=F(F+1)-I(I+1)-J(J+1)$,

$$\mathrm{p}=\left[\frac{\rho(3\cos^2\theta_m-1)}{r^4}\right]_{Av}=2(E_z^2-E_x^2)_{Av}.$$

E_z is the electric field at the nucleus parallel to the molecular axis due to all charges outside the nucleus.

Hence the polarization coupling constant $\frac{2}{3}e(\alpha_z-\alpha_x)\mathrm{p}$ is equivalent to a quadrupole coupling constant eqQ and distinguishable only because p depends on the inverse fourth power of r, while q depends on the inverse cube. The ratio of apparent quadrupole moments of Cl^{35} and Cl^{37} isotopes will be

$$\frac{Q_{35}+\frac{2}{3}(\alpha_z-\alpha_x)_{35}(\mathrm{p}/q)}{Q_{37}+\frac{2}{3}(\alpha_z-\alpha_x)_{37}(\mathrm{p}/q)},$$

which will vary when p/q varies unless

$$Q_{35}/(\alpha_z-\alpha_x)_{35}=Q_{37}/(\alpha_z-\alpha_x)_{37}.$$

The lowest known energy level of Cl^{35} is 0.6 Mev, while the lowest for Cl^{37} is 2.7 Mev,[3] so that their polarizabilities may be somewhat different. Assuming a single-particle model, the difference of the matrix elements

$$[|\mu_{on}|_{M=I}]^2-[|\mu_{on}|_{M=I-1}]^2$$

will be of the order of $e^2r_n^2$, where r_n is the nuclear radius, and hence $(\alpha_z-\alpha_x)$ for Cl^{35} can be approximated, with $E_1-E_0=0.6$ Mev. Although a number of higher levels may contribute to α_z, they would probably not affect the difference $(\alpha_z-\alpha_x)$ appreciably. From the use of Hartree wave functions[4] for K^+ and K^{++} it is found that for a 3p-electron $\langle 1/r^4\rangle_{Av}\approx 7.5(\langle 1/r^3\rangle_{Av})^{4/3}$, so that p can be estimated from the known value of q. Hence $\frac{2}{3}e(\alpha_z-\alpha_x)\mathrm{p}$ for Cl^{35} in CH_3Cl would be approximately 0.20 Mc, or 1/550 of eqQ. If, therefore, the ratio p/q changes by 90 percent between CH_3Cl and GeH_3Cl, the observed discrepancy of one part in 600 in the $Cl^{35}Cl^{37}$ quadrupole moment ratio would be produced.

In the particular case of K^+ and K^{++}, Hartree wave functions[4] indicate that p/q for the 3p-electrons changes by only three percent. However, this value may differ considerably from the change between the CH_3Cl and the GeH_3Cl molecules.

A rather large possible error should be allowed for the above estimate of the size of the polarization effect because of uncertainties, not only in p/q but also in the dipole matrix elements and the nuclear energy levels. The lowest known Cl^{35} level is 0.6 Mev, but investigation might reveal other lower levels in either Cl^{35} or Cl^{37}.

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1 Geschwind, Gunther-Mohr, and Townes, Phys. Rev. **81**, 288 (1951).
2 H. M. Foley, Phys. Rev. **72**, 504 (1947).
3 D. E. Alburger and E. M. Hafner, Brookhaven Laboratory Report BNL-T-9 (July 1, 1949) (unpublished).
4 W. Thatcher, Proc. Roy. Soc. (London) **172**, 242 (1939).

A Canonical Transformation in the Theory of Particles of Arbitrary Spin

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MUCH of the work in the theory of elementary particles of arbitrary spin[1] is based on the assumption that the relativistic spin operator is of the form

$$t_{\mu\nu}=(\sigma_k,\gamma_k)=\text{const}\cdot i(\beta_\mu\beta_\nu-\beta_\nu\beta_\mu)\qquad k=1,2,3;\quad \mu,\nu=1,2,3,4. \tag{1}$$

It may therefore be of interest to point out a general condition under which this form of the spin operator can be derived as a result. Also, the knowledge of this condition provides a desirable short-cut in the otherwise lengthy derivation of the commutation relations of the β_μ and in work on the solutions of the wave equation, particularly for the higher spins.

The equations expressing the relativistic invariance of the linear first-order wave equation are

$$\begin{aligned}&[\beta_i,\sigma_i]=0,\quad [\beta_i,\sigma_k]=i\beta_l,\quad [\beta_4,\sigma_i]=0,\\&[\beta_i,\gamma_k]=0,\quad [\beta_k,\gamma_k]=i\beta_4,\quad [\beta_4,\gamma_k]=-i\beta_k,\end{aligned} \tag{2}$$

where $[A,B]=AB-BA$, and (i,k,l) are in cyclic order. On the other hand, the commutation relations between the components of the relativistic (6-vector) spin operators are

$$\begin{aligned}&\text{(a)}\ [\gamma_i,\sigma_i]=0;\quad \text{(b)}\ [\gamma_i,\sigma_k]=i\gamma_l,\\&\text{(c)}\ [\gamma_i,\gamma_k]=i\sigma_l;\quad \text{(d)}\ [\sigma_i,\sigma_k]=i\sigma_l.\end{aligned} \tag{3}$$

The similarity existing between Eqs. (2a, b) and (3a, b) suggests that the equations can be solved by the following S-transformation

$$\gamma_k=\lambda S\beta_k S^{-1}, \tag{4}$$

where

$$S\sigma_k S^{-1}=\sigma_k,\quad S\beta_4 S^{-1}=\beta_4, \tag{4'}$$

with λ as a (real) number. It follows from Eqs. (2d, e, f) that

$$[(S^{-1})^2\gamma_k S^2+\gamma_k]\beta_\mu-\beta_\mu[(S^{-1})^2\gamma_k S^2+\gamma_k]=0. \tag{5}$$

The operator $[(S^{-1})^2\gamma_k S^2+\gamma_k]$ thus commutes with the whole complex generated by the β_μ and is, therefore, a multiple of the unit operator. Since from (3b), $\mathrm{Spur}(\gamma_k)=0$, it follows that

$$S^2\gamma_k+\gamma_k S^2=0, \tag{6}$$

whence also

$$S^2\beta_k+\beta_k S^2=0, \tag{7}$$

S^4 thus commuting with all of the β_μ. It can be deduced, therefore, that S^2 is a constant multiple of the operator η_4 the existence of which is necessary for the covariance of the wave equation under reflections of the three space axes.

Applying the canonical transformation (4) and (6) to (2f) and (3c), the important expression (1) is now obtained as a result.

An immediate consequence of the canonical S-transformation is the equality of the characteristic equations of $\lambda\beta_\mu$ and σ_k. For, owing to the complete parity of all four β_μ, the characteristic equations of σ_k and γ_k are the same and, according to (4), so are the characteristic equations of γ_k and $\lambda\beta_k$. Also, the four operators γ_k and $\lambda\beta_4$ satisfy the same commutation relations as do the four $\lambda\beta_\mu$. This knowledge effects considerable simplification not only in the derivation of the commutation relations of the β_μ, but also in the construction of solutions of the wave equation from the elementary solutions in the rest system by Lorentz transformations.

The expressions for the canonical transformation (4) in the cases of the Dirac electron, the Duffin-Kemmer meson, and Bhabha's particle of spin 3/2 are represented by

$$S=\text{const}\cdot\exp(i\tfrac{1}{2}\pi\beta_4). \tag{8}$$

In particular

$$\begin{aligned} S_{\text{electron}}&=(1-i\beta_4)/\sqrt{2}\\ S_{\text{meson}}&=1-i\beta_4-\beta_4{}^2\\ S_{3/2}&=\frac{1}{2\sqrt{2}}\left(\frac{5}{2}-\frac{13}{3}i\beta_4-2\beta_4{}^2+\frac{4}{3}i\beta_4{}^3\right). \end{aligned} \tag{9}$$

The inverse operators are obtained simply by replacing i by $-i$.

[1] H. J. Bhabha, Revs. Modern Phys. 17, 200 (1945).

An Unusual Nuclear Event*

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A NUCLEAR event, unusual in that it produces two electron showers but no charged penetrating particles, has been observed in a cloud chamber operated at sea level. Figure 1 shows one of the stereoscopic views. The nuclear event appears to have been initiated by particle A, which has the proper orientation to have triggered the chamber by passing through the counter telescope directly above. This track is of approximately minimum ionization and therefore it seems improbable that it represents a particle projected upward from the point of interaction. The energies and directions of the two showers, B and C, are consistent with the assumption that each shower was initiated by one of the photons resulting from the decay of a neutral π-meson of extremely short lifetime. No high energy charged particles are produced and therefore most of the kinetic energy of the initiating particle is given to the neutral meson, if one assumes that there are no other energetic neutral particles. It is improbable that there is more than one neutral meson, since then there would be at least two more photons which would have to be emitted in such directions as to escape detection. A rough estimate, based on range and ionization, indicates that the energy of the three heavily ionizing particles, assumed to be protons, is 5 percent$\pm$5 percent of the energy present in the showers. Thus, allowing an equal energy for

FIG. 1. Nuclear interaction producing two energetic photons but no energetic charged particles.

neutrons, the meson receives about 90 percent$\pm$10 percent of the total energy.

The nuclear interaction takes place in a $\frac{1}{4}$-in. aluminum plate. Shower B starts in a $\frac{1}{4}$-in. brass plate, and shower C starts in the first of four $\frac{1}{4}$-in. lead plates. The separation of positive and negative ions in the clearing field indicates that the showers occurred at the same time as the nuclear event. The axes of the showers intersect at approximately the point of the nuclear interaction and make angles of 13° and 4° with the direction of particle A. From the numbers of electrons produced, it was estimated that the energies of showers B and C are 300 Mev and 1000 Mev respectively. Assuming that a meson of 140-Mev rest energy decays into two photons with energies of 300 Mev and 1000 Mev, one finds that the total included angle should be 15°, which compares favorably with the measured angle of 17°.

If the meson receives most of the energy of the incident particle, it should continue very closely in the direction of that particle. Thus a further check can be made of the consistency of the above assumptions. It can be shown that photons emitted at angles of 13° and 4° to the direction of motion of the meson will have energies of 270 Mev and 850 Mev, which values are in good agreement with those given previously.

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