The errors shown in Fig. 1, as well as those quoted in Table I, are the standard deviations due only to the statistical counting errors. The other errors, which must be superimposed on those shown, are for the most part systematic errors which affect all of the cross sections equally. These errors do not alter the angular distribution very much. We believe that the systematic errors amount to five percent (probable error) in the experiments at 345 Mev, and to 10 percent in the experiments at lower energy.

* This work was performed under the auspices of the AEC.

[1] O. Chamberlain and C. Wiegand, Phys. Rev. 79, 81 (1950).

A Note on Rainwater's Spheroidal Nuclear Model*

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A SPHEROIDAL core offers possibilities for large nuclear quadrupole moments. Coupling between the total angular momentum of the whole nucleus and the symmetry axis of the core is required to maintain a nonspherical charge distribution with respect to external fields. Rainwater[1] has shown that the model of an odd particle in a spheroidal potential well generated by a spheroidal core has the required coupling and a minimum energy associated with a definite departure from the spherical shape. The energy of the spheroidal system contains a quadratic term in the eccentricity from the change in surface and Coulomb energies of the core and a linear term from the shift in the eigenvalue of the extra-core particle. The linear term can be derived simply as follows.

A potential well of constant depth in a spheroidal region of constant volume is defined by the equations

$$\begin{aligned} V(s) &= -D, \quad s<R \\ &= 0, \quad s \geqq R \end{aligned} \tag{1}$$

$$s = [\lambda(x^2+y^2)+z^2/\lambda^2]^{\frac{1}{2}}. \tag{2}$$

The eccentricity, e, is defined by the equation

$$\begin{aligned} e &= -\lambda + 1/\lambda^{\frac{1}{2}} \\ &\cong -\tfrac{3}{2}(\lambda-1) \quad \text{when} \quad |\lambda-1| \ll 1. \end{aligned} \tag{3}$$

For small values of e the eigenvalue of a particle in a spherical well can be computed from first-order perturbation theory employing the normalized solution $\psi_0(x, y, z)$ of the unperturbed problem with the result

$$\begin{aligned} E(e)-E(0) &= \int\int\int |\psi_0(x,y,z)|^2[V(s)-V(r)]dv \\ &= \int\int\int V(r)[|\psi_0(x/\lambda^{\frac{1}{2}}, y/\lambda^{\frac{1}{2}}, \lambda z)|^2 - |\psi_0(x,y,z)|^2]dv \\ &= -\tfrac{1}{3}eD \int\int\int_R \left[\frac{\partial}{\partial x}x + \frac{\partial}{\partial y}y - 2\frac{\partial}{\partial z}z\right] \times |\psi_0(x,y,z)|^2 dv \end{aligned} \tag{4}$$

the integration in the last line of Eq. (4) extending over a spherical region of radius R. Using Gauss' theorem and introducing spherical coordinates, Eq. (4) reduces to

$$\begin{aligned} \Delta E &= -\tfrac{1}{3}eDR^3 \int\int (1-3\cos^2\theta)|\psi_0(R,\theta,\varphi)|^2 \sin\theta d\theta d\varphi \\ &= \tfrac{1}{2}eDR^3 \mathcal{R}_{nl}^2(R) \frac{\frac{1}{3}l(l+1)-m^2}{(l+\frac{3}{2})(l-\frac{1}{2})} \end{aligned} \tag{5}$$

in which $\mathcal{R}_{nl}(r)$ is the normalized radial function.

An asymptotic formula valid for $x_0=(2MD)^{\frac{1}{2}}R/\hbar \gg 1$ is easily derived for levels near the bottom of the well. The result is

$$\tfrac{1}{2}DR^3\mathcal{R}_{nl}^2(R) \cong T_{nl}\left(\frac{x_0}{1+x_0}\right)^2 \tag{6a}$$

$$T_{nl} = \hbar^2\omega_{nl}^2/2MR^2. \tag{6b}$$

Here T_{nl} is the kinetic energy of a particle in a well of infinite depth and ω_{nl} is the nth zero of a Bessel function of order $l+\frac{1}{2}$.

Using $D=28.3$ Mev and $R=A^{\frac{1}{3}}e^2/2mc^2$, the right-hand member of Eq. (6a) is too large by 25 to 40 percent for levels at the top of the occupied spectrum. For example at $A=100$, $x_0=7.64$, $T_{14}=32$ Mev and

$$\begin{aligned} \tfrac{1}{2}DR^3\mathcal{R}_{14}^2(R) &= 20 \text{ Mev} \\ T_{14}[x_0/(1+x_0)]^2 &= 25 \text{ Mev}. \end{aligned} \tag{7}$$

Rainwater finds the asymptotic formula

$$\Delta E_{n,l,\pm l} \cong -\tfrac{2}{3}T_{nl}e \tag{8}$$

(valid for $l\rightarrow\infty$ and $D=\infty$) by a somewhat obscure and complicated procedure. The present calculation checks Rainwater's result under his limiting conditions, but yields a greatly reduced coefficient of e when Eq. (5) is evaluated accurately. Using the numbers quoted above for $A=100$,

$$\begin{aligned} \Delta E_{1,4,\pm 4} &\cong -21e \text{ Mev (Eq. (8))} \\ &\cong -10e \text{ Mev (exact)}. \end{aligned} \tag{9}$$

The possibility of excessive quadrupole moments, mentioned by Rainwater, is probably eliminated by the accurate calculation.

* Assisted by the joint program of the ONR and AEC.

[1] J. Rainwater, Phys. Rev. 79, 432 (1950).

Addendum: The Disintegration of Ti^{45}

[Phys. Rev. 80, 360 (1950)]

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WHEN the proof for the above-mentioned article was returned to the American Institute of Physics, a note to be added to the article was attached. This note is mentioned in footnote 10 but was not printed at the end of the article. Its content was as follows:

Since this paper was submitted, an article describing the work at Illinois has appeared.[1]

Because the elimination of Sc^{46} as an impurity was exceedingly difficult in the current research, the authors feel that the Compton and photo-electrons of Fig. 6 of Kubitschek's paper possibly may be attributed to Sc^{46}. The line at 780 kev is in perfect position to be the K-shell photo-electron line of the 890-kev gamma-ray from Sc^{46}. By drawing the line through the lower energy experimental points in a slightly different manner, it is possible to attribute all to Compton electrons from the 890-kev gamma-ray of Sc^{46}. Also it is difficult to understand the mechanism which would give a more intense L-shell photo-electron line than K-shell photo-electron line as appears to be the case in Fig. 6 of Kubitschek's article.

In a private communication, Dr. Kubitschek has indicated to us that a check of his photo-electron spectrum reveals the possible indication of a bump at the correct momentum to account for a 450-kev gamma-ray of very low intensity.

* Assisted by the joint program of the ONR and AEC.

[1] Kubitschek, Longacre, and Goldhaber, Phys. Rev. 77, 742 (1950); H. E. Kubitschek, Phys. Rev. 79, 23 (1950).

A Method of Isotopic Separation by (n, γ) Recoil*

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STUDIES of gamma-recoil atoms have been confined to systems in which the "hot" atoms remain within the phase boundaries of the target material. The factors of "hot" atom reactivity, ordinary thermal reaction exchange, and physical and chemical separation difficulties operate against efficient isotopic separation.

A new method is proposed for utilizing the recoil energy from gamma-emission for isotopic separation. If targets in the form of very thin films could be bombarded with thermal neutrons in vacuum, it seemed probable that an appreciable percentage of the (n, γ) product atoms would have sufficient recoil energy from the