

an important simplification of these calculations by neglecting the spin of the electron; the assumption that this approximation would not lead to a numerical result very different from the correct value seemed to be plausible.

Because of the inherently noncovariant character of the method of computation the renormalization of the infinite self-energy integrals turned out to be ambiguous. Two alternatives offered themselves: the "wave packet" and the "single particle" subtraction prescription. The difference of the  $2s$ - and the  $2p$ -level displacements was found to be

$$D = \frac{1}{3\pi} Z^4 \alpha^3 R y \left( L - \ln 2 + \frac{13}{36} + \frac{4}{15} \ln 2 + \frac{\epsilon}{12} - \frac{1}{40} \right) \quad (1)$$

where  $\epsilon = 2$  and  $-1$ , respectively, for the two alternatives, and  $L$  is the difference of the logarithms of the  $2s_{1/2}$  and  $2p_{1/2}$  levels involving the nonrelativistic average excitation energies  $k_0(n, l)$ . For hydrogen  $L$  was found to be<sup>2</sup>

$$L = \ln \frac{mc^2}{k_0(2, 0)} - \ln \frac{Ry}{k_0(2, 1)} = 7.7215 - 0.0300 \pm 0.0008$$

and therefore Dyson's formula gives

$$D = 1045 \text{ Mc and } 1011 \text{ Mc} \quad (2)$$

for the two subtraction prescriptions.

In order to resolve this ambiguity a covariant calculation is necessary. The Feynman method offers a particularly simple procedure for this purpose. We find for the  $2s$ -level shift

$$D_s = \frac{1}{3\pi} Z^4 \alpha^3 R y \left( \ln \frac{mc^2}{k_0(2, 0)} - \ln 2 - \frac{3}{4} + \frac{5}{6} - \frac{1}{40} \right) \quad (3)$$

and for the  $2p$ -level shift

$$D_p = \frac{1}{3\pi} Z^4 \alpha^3 R y \ln \frac{Ry}{k_0(2, 1)}. \quad (4)$$

The relative shift is therefore

$$D = D_s - D_p = \frac{1}{3\pi} Z^4 \alpha^3 R y \left( L - \ln 2 + \frac{1}{12} - \frac{1}{40} \right). \quad (5)$$

The term  $-3/4$  in Eq. (3) comes from the evaluation of the Feynman diagrams; the term  $5/6$  from the second-order calculation which is nonrelativistic and hence is identical with that for spin  $1/2$ ; the term  $-1/40$  represents the vacuum polarization. For hydrogen one finds from (5)

$$D = 956 \text{ Mc.} \quad (6)$$

As was anticipated, this value is rather close to that found for spin  $1/2$ , which is  $1051.4 - 67.8 = 983.6 \text{ Mc}$ ,  $67.8 \text{ Mc}$  being the contribution of the electron spin.

Comparison of (1) and (5) shows that neither of the two alternative subtraction procedures is correct. The term  $(4/15) \ln 2$  in (1) is easily traced back to a term of the form

$$\int |\nabla^2 \psi|^2 d\tau \quad (7)$$

which originates in a somewhat doubtful expansion in the non-covariant method. It involves the kinetic energy of the free particle and should be omitted. When this is done we notice that Dyson finds a  $p$ -level shift

$$D_p = \frac{1}{3\pi} Z^4 \alpha^3 R y \left( \ln \frac{Ry}{k_0(2, 1)} + \frac{\epsilon}{96} \right)$$

which leads us to take  $\epsilon = 0$  when compared with (4). Formula (1) then has only  $[L - \ln 2 - (1/12) - (1/40)]$  in the bracket, which is still incorrect. We conclude that there seems to be no simple way in which to obtain the correct result from the ambiguous non-covariant calculation.

After this work was completed the authors noticed a paper by Sasaki and Suzuki<sup>3</sup> who obtain instead of (5)

$$D = \frac{1}{3\pi} Z^4 \alpha^3 R y \left( L - \ln 2 + \frac{5}{48} - \frac{1}{40} \right) \quad (8)$$

which yields  $3 \text{ Mc}$  more than (6). They use the Schwinger-

Tomonaga formalism which makes the computations very much more complicated than by the Feynman method, which perhaps increases the chance of making errors.

<sup>1</sup> F. J. Dyson, Phys. Rev. **73**, 617 (1948).

<sup>2</sup> Bethe, Brown, and Stehn, Phys. Rev. **77**, 370 (1950).

<sup>3</sup> M. Sasaki and R. Suzuki, Prog. Theor. Phys. **4**, 485 (1949).

## Two Crystal Spectrometer Results with Uncollimated $\gamma$ -Rays

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FIGURE 1 shows the  $\gamma$ -ray spectra of  $\text{Cs}^{137}$  and  $\text{Co}^{60}$  as obtained on a two crystal Hofstadter<sup>1</sup> type spectrometer. Amplifier gains were different for the two curves. The instrument used differs from that described by Hofstadter in that no collimation was necessary and that considerably larger NaI crystals were used.

The point source of gamma-rays was located several inches from a 1-in. diameter, 1-in. high Compton-electron counting crystal. The degraded-gamma-counting crystal, 1.5 in. in diameter and 1.5-in. high, was located 2.75 in., center to center, from the one-in. crystal and at an angle of about  $150^\circ$  with the direction of the incident gamma-rays. The large crystal was shielded from the direct beam to reduce its count rate. The resolution is essentially equivalent to that obtained here with the same apparatus but employing a well-collimated source. The use of larger crystals and an uncollimated source permit greatly increased sensitivities. The data on  $\text{Co}^{60}$  were taken with a 0.25 mC source and required one hour of counting time. Experimental and calculated efficiencies agree, giving a figure of about 0.003 counts/sec in the peak for unit flux at the Compton scattering crystal.

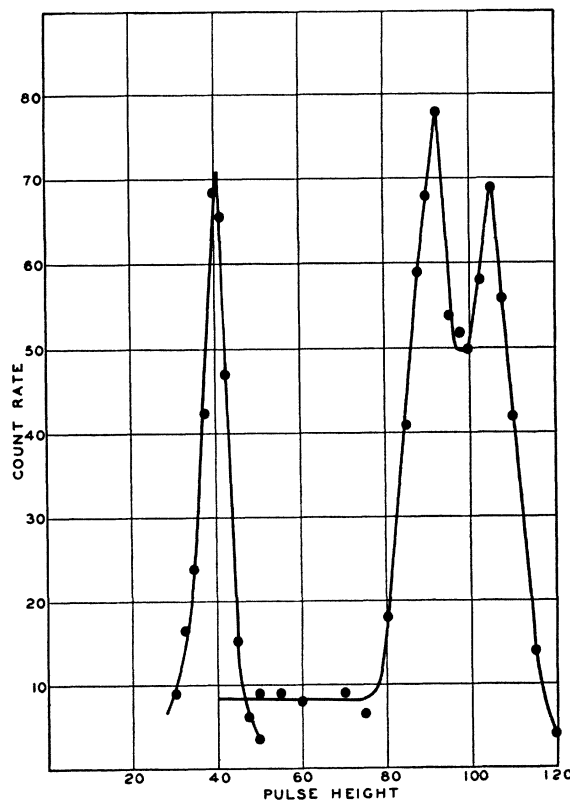


FIG. 1.  $\text{Cs}^{137}$  and  $\text{Co}^{60}$  spectra taken with a two crystal spectrometer using large crystals and an uncollimated source.

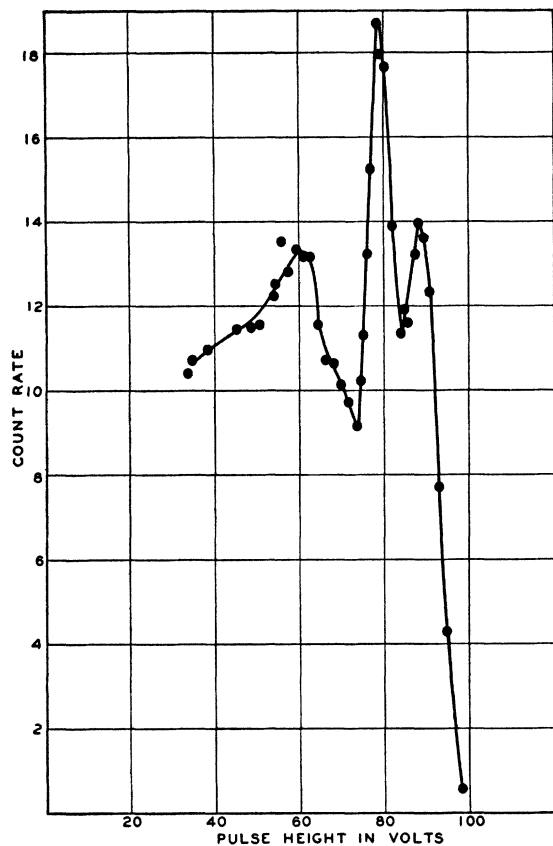


FIG. 2.  $\text{Co}^{60}$  single crystal spectra taken with  $1\frac{1}{2}$ -in. diameter crystal  $1\frac{1}{2}$  in. high and an uncollimated source.

The successful use of large NaI crystals and uncollimated sources is in disagreement with the results obtained by McIntyre and Hofstadter<sup>2</sup> on single crystals but agrees with those obtained by P. R. Bell.<sup>3</sup> Figure 2 shows an uncollimated, single-crystal spectrum of  $\text{Co}^{60}$  obtained with a crystal 1.5 in. in diameter and 1.5-in. high. Presumably the uniformity of the raw crystal material, surface conditions, and mounting have more effect on the resolution than does the crystal size.

<sup>1</sup> R. Hofstadter and J. A. McIntyre, *Phys. Rev.* **78**, 619 (1950).

<sup>2</sup> J. A. McIntyre and R. Hofstadter, *Phys. Rev.* **78**, 617 (1950).

<sup>3</sup> P. R. Bell, *Science* **112**, 7 (1950).

### Erratum: Experiments on the Effect of Atomic Electrons on the Decay Constant of $\text{Be}^7$

[*Phys. Rev.* **75**, 39 (1949)]

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WE have made an error in plotting, as the experimental points of Fig. 2,  $2\delta e^{\lambda t}$  instead of  $\delta e^{\lambda t}$  as indicated. Similarly the final value of  $\Delta\lambda/\lambda$  should be divided by 2 giving

$$\Delta\lambda/\lambda = (-1.5 \pm 0.9)10^{-4}.$$

### Experiments on Elastic $p-p$ Scattering in the Energy Range 120 to 345 Mev\*

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IN a previous paper<sup>1</sup> we described some experiments on  $p-p$  scattering at 340 Mev made with gas proportional counters.

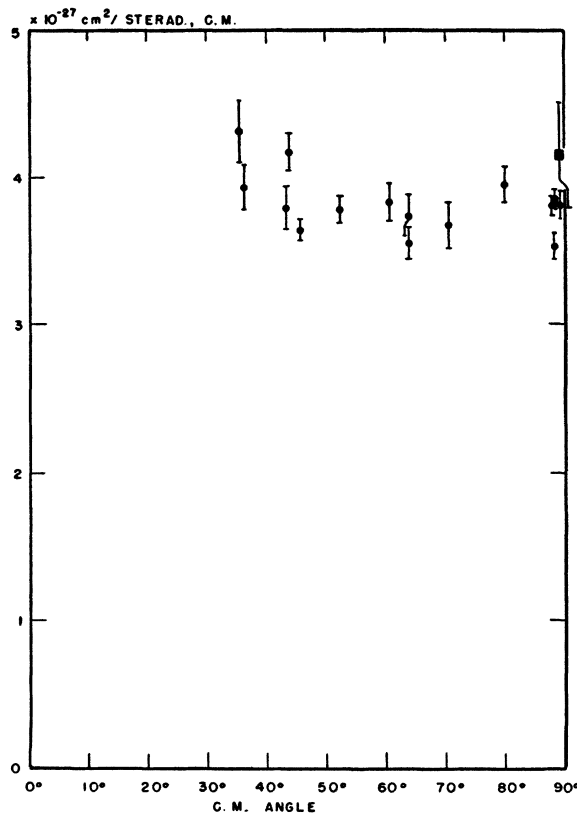


FIG. 1. Differential scattering cross section in the center-of-mass system as a function of the angle in this system.

We have now improved and extended the measurements by using stilbene scintillation counters in coincidence and by varying the energy of the beam by the use of lithium absorbers.

In view of the interest in these results as shown in recent theoretical papers we have decided to publish the results to date. Details on the experiments and extension to smaller angles will follow later.

The results at 345 Mev are summarized in Fig. 1 which gives the differential scattering cross section in the center-of-mass system as a function of the angle in this system. The cross section is normalized in the usual way such that the total scattering cross section  $\sigma_s$  is given by

$$\sigma_s = \frac{1}{2} \int_{4\pi} \sigma(\phi) d\omega_\phi = \frac{1}{2} \int_0^\pi \sigma(\phi) [2\pi \sin\phi d\phi].$$

Table I gives the differential cross sections of  $\sigma(\phi)$  (center of mass system) for incident proton energy,  $E$ , in the laboratory coordinate system, at angle  $\phi$  (center-of-mass system) from the beam direction. The symmetry of the problem in the center-of-mass system guarantees that  $\sigma(\phi) = \sigma(\pi - \phi)$ .

TABLE I. Differential scattering cross sections at reduced energies. Quoted errors are standard deviations from counting statistics only.

$E$ (Mev)	$\phi$ (degrees)	$\sigma(\phi)$ ( $10^{-27} \text{ cm}^2 \text{ sterad}^{-1}$ )
119	63	4.0 $\pm$ 0.4
119	78	4.2 $\pm$ 0.4
119	89	3.95 $\pm$ 0.12
164	61	4.1 $\pm$ 0.4
164	89	3.8 $\pm$ 0.3
249	48	3.5 $\pm$ 0.3
249	63	3.7 $\pm$ 0.2
249	78	3.69 $\pm$ 0.15
249	87	3.64 $\pm$ 0.11