parallel to a field of about 100 gauss, provided by a small permanent magnet. A 2 μfd condenser, initially charged to 8 kv, was discharged through the coil, with 500 ohms in series, in such a sense that the field in the coil reversed to about −100 gauss, with a time constant of about 0.2 μsec and decayed back to the original field with a time constant of 1 msec. The crystal was quickly returned, through the earth's field, to the strong magnet and the Li^7 resonance sampled. The operation could be done in 2 to 3 sec. A reversed deflection was found and it decayed, through zero, to the equilibrium state with the characteristic 5-min time constant. A typical record is shown in Fig. 1.

The state of spin system just after this treatment is thought to be properly described by a negative spin temperature. The system loses internal energy as it gains entropy, and the reversed deflection corresponds to induced radiation. Statistically, the most probable distribution of systems over a *finite* number of equally spaced energy levels, holding the total energy constant, is the Boltzmann distribution with either positive or negative temperature determined by whether the average energy per system is smaller or larger, respectively, than the mid-energy of the available levels. The sudden reversal of the magnetic field produces the latter situation.

One needs yet to be convinced that a single temperature adequately describes the nuclear spin state. Bearing on this is the fact that the crystal passes through the earth's field after the inverted population is produced, on its way back to the main magnet. The retention of the reversed magnetization requires that the spin-only-state, in the earth's field, have an inverted population and be described by a suitably small (~ -1°K) negative temperature. Thus a very short time is required for the attainment of thermal equilibrium within the spin system itself (not the ordinary T_2, however).

A system in a negative temperature state is not cold, but very hot, giving up energy to any system at positive temperature put into contact with it. It decays to a normal state through infinite temperature.

This and related experiments indicate that the spin system is able to follow changes in even a small field adiabatically unless they occur in a time presumed to be less than about 20 μsec.

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Erratum: Experiments on the Effect of Atomic Electrons on the Decay Constant of Be^7. II

[Phys. Rev. **76**, 897 (1949)]

R. F. Leininger, E. Segrè, and C. Wiegand
Radiation Laboratory, Department of Physics, University of California, Berkeley, California

IN Fig. 1 we have erroneously plotted $2\delta e^{\lambda t}$ instead of $\delta e^{\lambda t}$ as indicated on the ordinate scale on the left. The final result is in error by a factor 2 and should read:

$$\lambda(\mathrm{BeO}) - \lambda(\mathrm{BeF_2}) = (0.69 \pm 0.03)10^{-3}\lambda(\mathrm{BeO})$$
$$\lambda(\mathrm{Be}) - \lambda(\mathrm{BeF_2}) = (0.84 \pm 0.10)10^{-3}\lambda(\mathrm{Be}).$$

On the Nuclear Magnetic Moment of Na^{23}

R. A. Logan and P. Kusch
Columbia University, New York, New York*
November 30, 1950

A DISCREPANCY occurs between the nuclear g values of[1] Ga and[2] In as determined from measurements on the hfs spectrum of these atoms in the ground state ($^2P_{1/2}$) and as determined by the nuclear resonance method where the nuclei occur in molecules. Foley[3] has discussed the effect of the partial decoupling by the applied magnetic field of the L and S vectors in the $^2P_{1/2}$ state on the nuclear g-value obtained from observational data under the assumption that decoupling does not occur. He concludes that the diagonal magnetic interaction term $m_I g_I \mu_0 H$ which was assumed in finding g_I from the experimental data is to be modified by a small perturbation term which is, itself, proportional to the applied magnetic field. For the cases of gallium and indium the apparent g-value determined from the hfs of atoms is thus greater than the g-value obtained in the nuclear resonance experiments. Foley indicates a satisfactory agreement between the observed and calculated values of the discrepancy, especially in view of uncertainties in the theoretical calculations and a rather large experimental error.

It is of interest to determine whether or not a similar discrepancy appears for the case of a nucleus which occurs in an atom in the $^2S_{1/2}$ state, where the effect considered by Foley cannot appear. A previous measurement[4] has indicated that the apparent nuclear g-value of Cs is, indeed, the same within a rather large experimental uncertainty when measured in a molecule and when measured in an atom in the $^2S_{1/2}$ state. A precision measurement of the g-value of sodium is reported here.

Essentially use is made of the fact that certain lines $(F, m) \leftrightarrow (F, m-1)$ consist of doublets, one component of which arises in the state $F = I + \frac{1}{2}$ and the other one of which arises in the state $F = I - \frac{1}{2}$. The frequency separation of the doublet is $2g_I\mu_0 H/h$ and the mean frequency of the doublet permits determination of the quantity $x = (g_J - g_I)\mu_0 H/h\Delta\nu$, if $\Delta\nu$ is itself known, and hence of g_I/g_J. The $\Delta\nu$ of Na^{23} was found to be $1771.631 \pm 0.002 \times 10^6$ sec^{-1} by a method previously described[4] which depends on the existence of a maximum in the frequencies of certain lines in the hfs spectrum. The doublet $(F, 1) \leftrightarrow (F, 0)$ was observed at a field of about 6800 gauss where the mean frequency of the doublet is still sufficiently field dependent to permit an accurate determination of x and the doublet separation becomes large enough ($\sim 16 \times 10^6$ sec^{-1}) to permit accurate measurement in the face of a large over-all frequency ($\sim 430 \times 10^6$ sec^{-1}). We find then that

$$g_J(\mathrm{Na}, {}^2S_{1/2})/g_I(\mathrm{Na}, {}^2S_{1/2}) = -2488.39 \pm 0.15.$$

Since[5] $g_I(H)/g_J(\mathrm{Na}, {}^2S_{1/2}) = -15.1927 \times 10^{-4} \pm 0.005$ percent, we find $g_I(\mathrm{Na}, {}^2S_{1/2})/g_I(H) = 0.26451 \pm 0.008$ percent. This is to be compared with Bitter's[6] result $g_I(\mathrm{Na, mole})/g_I(H) = 0.26450 \pm 0.01$ percent. The excellent agreement indicates that the apparent nuclear g-value measured in an atom in the $^2S_{1/2}$ state is, in fact, equal to the true nuclear g-value within the diamagnetic correction. Since no effect is here observed, it appears that the effect discussed by Foley accounts for the entire discrepancy observed for atoms in the $^2P_{1/2}$ state.

Determinations of the spin g-value of the electron reported heretofore depend on a measurement of the ratio of the electronic g_J of atoms in different electronic configurations. A combination of our present result with that of Bitter and with the result of Gardner and Purcell[7] for $2g_L/g_I(H)$ yields $g_s/g_L = 2(1.00107 \pm 0.00012)$ under the assumption that $g_J(\mathrm{Na}, {}^2S_{1/2}) = g_s$ and that no differential diamagnetic correction is to be applied to the nuclear moment of sodium in an atom and in a molecular configuration. It is of interest that this result does not depend on any assumption as to the g_J-values of P-states. The result agrees with other data on the spin moment of the electron. While it could be improved, a very accurate determination is precluded by the nature of the assumptions.

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Numerical Evaluation of the Fermi Beta-Distribution Function

D. G. E. Martin
George Holt Physics Laboratories, University of Liverpool, Liverpool, England
November 30, 1950

ATTENTION has recently been drawn by Feister[1] to methods of calculating the Fermi β-distribution function,

$$f(z, \eta) = \eta^{2+2s} e^{\pi y} |\Gamma(1+s+iy)|^2.$$

An asymptotic expansion of the gamma-function of the complex argument has been used by Hall.[2]

It does not seem to be widely known that the modulus of the gamma-function of the complex argument can be obtained in terms of the polygamma-functions of real arguments.

Let $1+s=x$, a constant, and put $x+iy=z$, then expanding by a Taylor series:

$$\ln\Gamma(x+iy)=\ln\Gamma(x)+iy\left\{\frac{\partial}{\partial y}\ln\Gamma(x+iy)\right\}_0 -\frac{y^2}{2!}\left\{\frac{\partial^2}{\partial y^2}\ln\Gamma(x+iy)\right\}_0+\cdots$$

$$=\ln\Gamma(x)+iy\psi(x)-\frac{y^2}{2!}\psi'(x)-\frac{iy^3}{3!}\psi''(x)+\frac{y^4}{4!}\psi'''(x)+\cdots$$

where $\psi(x)$, $\psi'(x)$, $\psi''(x)$, $\cdots$ are the polygamma-functions which have been tabulated.[3]

Since

$$\ln\Gamma(x+iy)=\ln Re^{i\phi}=\ln R+i\phi$$

we are interested only in the real part of the expansion, so that

$$\ln R=\ln\Gamma(x)-\frac{y^2}{2!}\psi'(x)+\frac{y^4}{4!}\psi'''(x)-\cdots.$$

This series may be made to converge more rapidly by increasing the argument, x.

Thus

$$\ln R=\ln\Gamma(x+1)-\frac{y^2}{2!}\psi'(x+1)+\frac{y^4}{4!}\psi'''(x+1)-\cdots-\tfrac{1}{2}\ln(x^2+y^2)$$

since by the property of the gamma-function

$$\ln\Gamma(z)=\ln\Gamma(z+1)-\ln z$$

and

$$\Re[\ln(x+iy)]=\tfrac{1}{2}\ln(x^2+y^2).$$

This process may be repeated until these terms converge sufficiently rapidly to give the accuracy desired. In this manner for a given value of x (i.e., a given atomic number) a series in y can be obtained for easy computation of the gamma-function.

Thus for $Z=82$, $x=0.80109$

$$\log_{10}R=1.252\,05-0.050\,26y^2+0.000\,44y^4 -\tfrac{1}{2}\log_{10}(x^2+y^2)[(x+1)^2+y^2][(x+2)^2+y^2][(x+3)^2+y^2].$$

I am indebted to Dr. J. C. P. Miller for this method of computation, which has been used in papers by Martin and Richardson[4] to evaluate Fermi functions for $Z=81$, 82, 83, 84.

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On the Decay of the π^- Meson*

L. Lederman, J. Tinlot,† and E. T. Booth
Columbia University, New York, New York

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The spontaneous decay of the π^- meson has been observed directly in a cloud chamber. The decay mechanism has been studied and the mass of the μ^- meson measured.

The decay of the π^+ meson at the end of its range has been studied extensively in cosmic rays[1] and with accelerators.[2] Because of the very large probability for capture of the π^- meson in nuclei,[3] its decay properties can only be studied in flight.

π^- mesons produced by the bombardment of an internal carbon target with 385 Mev protons were collimated by a 2×10 in. hole in the six-foot thick concrete shielding. A 16-in. cloud chamber with a magnetic field of 4040 gauss was exposed to this beam. A preliminary run with a $\frac{1}{4}''$ Pb plate showed that the beam included fast electrons as well as π^- and μ^- mesons. The most probable momentum of the incoming particles was 180 Mev/c.

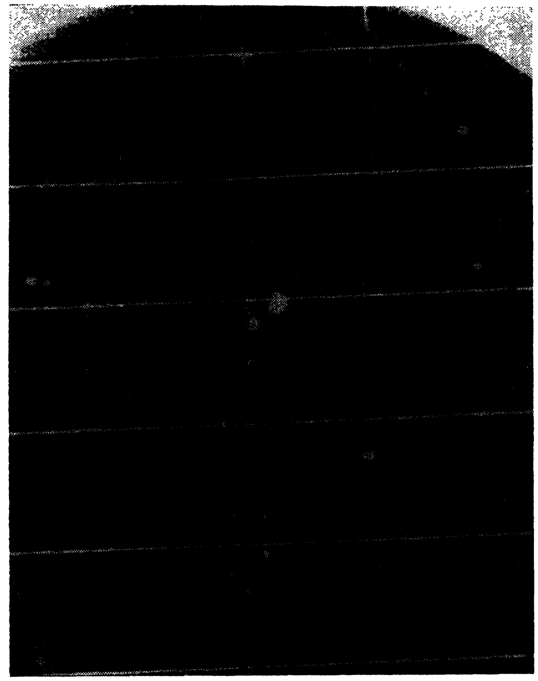

FIG. 1. Cloud-chamber photograph of the meson beam showing a typical decay-in-flight.

Fifty cases of apparent decay-in-flight were observed in the gas of the cloud chamber (Fig. 1). The requirement for identification was an abrupt deflection of a track by more than 5°. The track was required to be well illuminated and at least one inch from the walls of the chamber. The most probable angle of deflection observed was 12°. With the exception of four cases, all of the events are consistent, within the experimental errors, with the kinematics of decay-in-flight for the process:

$$\pi^-\rightarrow\mu^-+\nu^0, \qquad (1)$$

where ν^0 is a neutral particle of small rest mass.

Twenty-eight examples were selected for detailed analysis on the basis of measurability of both arms of the deviated track.

TABLE I. Mass measurements of the μ^- meson.

No.	$M\mu^-$ (Mev)	$\Delta M\mu^-$ (Mev)
1	100.8	10.1
2	126.0	18.3
3	106.2	7.3
4	114.2	42
5	103.5	7.1
6	105.1	14.2
7	106.5	6.1
8	119.5	10.0
9	116.2	11.3
10	112.0	6.5
11	105.7	7.4
12	95.6	23
13	116.8	6.2
14	108.0	6.0
15	114.7	7.0
16	96.6	50
17	104.7	5.0
18	116.7	8.4
19	105.0	7.3
20	98.6	8.9
21	105.9	5.0
22	108.5	5.1
23	104.6	4.0
24	110.5	6.4
25	60.2	10.0
26	Imaginary	[a]
27	Imaginary	[a]
28	Imaginary	[a]

[a] These masses remain imaginary within the limits of the uncertainties in the measurements.