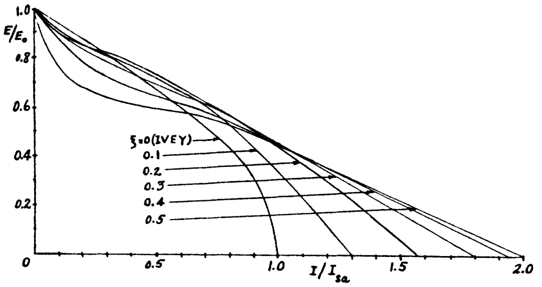

FIG. 2. Dependence of E/E_0 on I/I_{sa} for various values of ξ.

The solution of (2) when inserted into (3) and (4) gives the following general equations[2]

$$(I/I_{sa})^{\frac{1}{2}}=(1-2\psi)(1+\psi)^{\frac{1}{2}}-(\xi-2\psi)(\xi+\psi)^{\frac{1}{2}} \quad (5)$$

$$E/E_0=(4/3)[(1-2\psi)(1+\psi)^{\frac{1}{2}}(\xi+\psi)^{\frac{1}{2}}-(\xi-2\psi)(\xi+\psi)]. \quad (6)$$

Here I_{sa} is the saturation current density with zero initial velocities according to Child's law

$$I_{sa}=(4\epsilon_0/9d^2)(\vartheta/m)^{\frac{1}{2}}u_a^{\frac{3}{2}}.$$

E_0 is the value of E in the absence of current flow (i.e., is equal to u_a/d), ξ is the relative initial velocity of the electrons

$$\xi=(u_1/u_a)^{\frac{1}{2}}\cos\varphi \quad (7)$$

and ψ is a parameter specifying the degree of space charge. For $\psi=-\xi$ it follows that $E=0$, so that the space charge is "complete," and for $\psi\rightarrow\infty$, $E=E_0(I=0)$. With increasing current density ψ decreases from ∞ to $-\xi$ and the cathode field decreases from u_a/d to zero, as is indicated in Fig. 1.

In the general case a relation between I and E can be obtained by elimination of ψ between Eqs. (5) and (6), the nature of the result being shown in Fig. 2. For zero initial velocities ($\xi=0$) the formula of Ivey is regained

$$\frac{I}{I_{sa}}=\frac{1}{2}\left\{1\mp\left[1-\frac{27}{4}\left(\frac{E}{E_0}\right)^2\left(1-\frac{E}{E_0}\right)\right]^{\frac{1}{2}}\right\}. \quad (8)$$

The solution of (2) also permits one to calculate the transit times of electrons with initial velocities, and the electron paths for oblique emission under partial space charge.[2]

[1] H. F. Ivey, Phys. Rev. 76, 554 (1949).
[2] A. O. Barut, Z. Angew. Math. u. Phys. (to be published, 1951).

Diverging Integrals in the Self-Charge Problem

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November 27, 1950

THE polarization of the vacuum arising from the virtual production of electron-positron pairs by an electromagnetic field was examined by Schwinger,[1] who found that the current induced in the vacuum contains a part which leads to a logarithmically divergent but unobservable charge renormalization in the current producing the electromagnetic field. In an investigation of the polarization of the vacuum by the virtual formation of particles with spin zero, Jost and Rayski[2] again found that the induced current includes a logarithmically divergent self-charge term, which moreover has the same sign as that occurring in Schwinger's result. In the calculation of the current induced by the virtual formation of particles with spin 1 having vector coupling with the electromagnetic field, Feldman[3] obtained a quadratic divergence in the self-charge term. He therefore concluded that the cancellation of the diverging self-charge cannot be effected by any superposition of real fields with spin 0, $\frac{1}{2}$, and 1, if no formal regularization procedure is applied.

We have performed the calculation for spin 0 and spin 1 independently, and wish to draw attention to some points in the manipulation of the divergent integrals. For convenience we shall quote from Feldman's paper. In the case of spin 1 he gives the self-charge term of the induced current

$$\alpha/4\pi\cdot(\mathcal{J}_1-\tfrac{1}{2}\mathcal{J}_2)J_\mu(x), \quad (1)$$

where $J_\mu(x)$ is the current producing the external field,

$$\mathcal{J}_1=1/im^2\cdot\int_{-\infty}^{\infty}\exp(im^2z)/z^2\cdot\epsilon(z)dz,$$

$$\mathcal{J}_2=\int_{-\infty}^{\infty}\exp(im^2z)/z\cdot\epsilon(z)dz,$$

$$\epsilon(z)=z/|z|,\quad \alpha=e^2/4\pi,\quad \hbar=c=1.$$

Now these integrals have singularities at $z=0$. Integrating from $-\infty$ to $-z_0'$ and from z_0 to ∞ we deduce that

$$\mathcal{J}_1=2+\mathcal{J}_2+1/im^2\cdot\lim_{z_0\rightarrow 0,\, z_0'\rightarrow 0}(1/z_0-1/z_0'),$$

$$\mathcal{J}_2=-\lim_{z_0\rightarrow 0,\, z_0'\rightarrow 0}(\log\gamma m^2z_0+\log\gamma m^2z_0'),$$

where

$$\gamma=1.78.$$

Expression (1) is equivalent to

$$\{2+\tfrac{1}{2}\mathcal{J}_2+1/im^2\cdot\lim_{z_0\rightarrow 0,\, z_0'\rightarrow 0}(1/z_0-1/z_0')\}(\alpha/4\pi)J_\mu(x). \quad (2)$$

The most strongly divergent terms are imaginary and so must be excluded for physical reasons. In order to make them vanish we are obliged to take the principal values of the diverging integrals, that is, we must put $z_0'=z_0$. Then (2) becomes

$$\{2-\lim_{z_0\rightarrow 0}\log\gamma m^2z_0\}(\alpha/4\pi)J_\mu(x), \quad (3)$$

which diverges logarithmically.

The part of the current induced in the vacuum through the formation of electron-positron pairs, which corresponds to (3), is

$$-(\alpha/6\pi)J_\mu(x)\int_{-\infty}^{\infty}\exp(im_e^2z)/z\cdot\epsilon(z)dz$$
$$=\lim_{z_0\rightarrow 0}(\log\gamma m_e^2z_0)\cdot\alpha/3\pi\cdot J_\mu(x), \quad (4)$$

where m_e is the electron mass. The sign of the logarithmic divergence is different in (3) and (4). Thus on admitting principal values—which, however, is an additional prescription—it will be possible to cancel the self-charge effects by a superposition of fields of particles with spins $\frac{1}{2}$ and 1, or indeed by a superposition of fields with spins 0, $\frac{1}{2}$, and 1.

However, the advantage gained by the compensation of the self-charge term is offset somewhat by the fact that in the case of vector mesons under consideration there appears also a diverging term

$$-(\alpha/24\pi m^2)\mathcal{J}_2\Box J_\mu(x),$$

to which no term corresponds in the spin 0 or spin $\frac{1}{2}$ cases.

I am indebted to Professor W. Heitler, University of Zürich, for helpful comments.

[1] J. Schwinger, Phys. Rev. 75, 651 (1949).
[2] R. Jost and J. Rayski, Helv. Phys. Acta 22, 457 (1949).
[3] D. Feldman, Phys. Rev. 76, 1369 (1949).

Čerenkov Radiation Counter for Fast Electrons*

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November 24, 1950

PROPOSALS have been made by Getting[1] and Dicke[2] to use Čerenkov radiation as a means of counting fast charged particles. Also recently Jelley[3] has reported Čerenkov detection of cosmic-ray particles. This letter is a description of an arrangement with which fast electrons, produced by the 48-Mev bremsstrahlung of a betatron, have been counted by the Čerenkov radiation which they produce in Plexiglas or Lucite. The work was undertaken in the hope of developing a velocity discriminating detector to be used with the 450-Mev proton synchrocyclotron under construction at the University of Chicago.

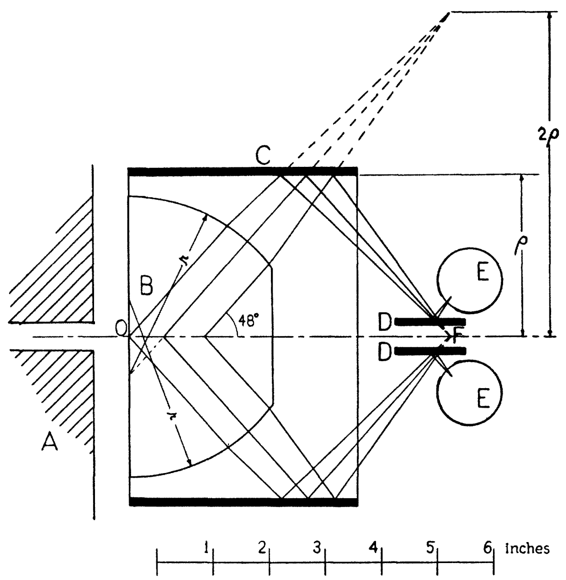

FIG. 1. Counter arrangement (explanation in text).

The arrangement is shown schematically in Fig. 1. The block, A, is a lead collimator through which electrons from a thin ($\frac{1}{8}$-in.) radiator in the betatron beam enter the system. The radiator, B, is made of Lucite or Plexiglas. It is a figure of revolution generated by rotating a circle around an axis in its plane, but not through its center. The radius of curvature, r, is so chosen as to focus the Čerenkov radiation into a sharp ring with a radius of approximately 2ρ, where ρ is the radius of a cylindrical reflector, C, made by silvering a piece of large diameter glass tubing on its inner surface. The cylindrical reflector focuses the light on the axis of the system at F where, in principle, a photo-multiplier could be placed to count the incident particles. Because of noise, it is much preferable to use a fast coincidence system operated by two photo-multipliers. To focus the light on two multipliers, two plane mirrors, D, are used. The photo-multipliers are of type 931-A.

The coincidence and counting circuit can be described briefly as follows. The pulse out of each multiplier was clipped with a 9-in. length of 173-ohm coaxial cable shorted at its far end, and was fed into two Spencer-Kennedy distributed amplifiers in cascade through 173-ohm line. The impedance level was reduced to 100 ohms at the output of the second amplifier and the pulse was clipped again to reduce its width, which at this point was broadened somewhat by the amplifiers. The pulses were then limited in height by a series crystal diode carrying current in a direction opposite to that in the pulse. Finally, the pulses in the two channels were combined at the center of a length of 100-ohm line and fed through a crystal discriminator to a linear amplifier, discriminator, and scaler. This coincidence circuit is similar in principle to one rumored to have been developed by Bell and Jordan.

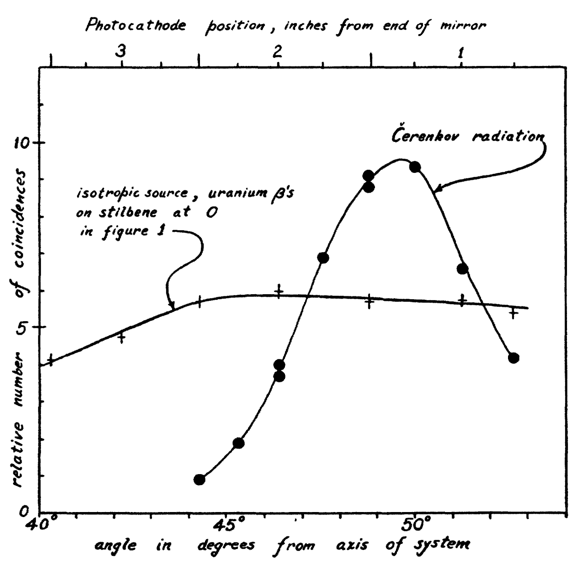

FIG. 2. Angular dependence of coincidence rate.

The Čerenkov counter was set up so that fast electrons, emitted from a lead radiator at about 0.1 radian from the forward direction of a collimated x-ray beam, could enter the system through its collimator after being deflected slightly by a small magnet. The direct x-ray beam was absorbed in 16 in. of lead. Coincidences were observed and it was found that the insertion of enough line in either channel to delay the pulse by 2×10^{-9} sec. reduced the counting rate by a factor 6. The same coincidence circuit excited by uranium betas on stilbene had its counting rate reduced by the same factor by a time delay of about 4×10^{-9} sec. It is apparent that the light pulses are quite short. It is felt that the resolving time obtained is as short as can be expected from the system even from an infinitely fast light pulse.

The counting rate was linear with betatron intensity as indicated by an ionization chamber. An absorption curve in iron taken with good geometry gave a relaxation length of 3.30 ± 0.1 cm, which indicated an effective x-ray energy of approximately 30 Mev.

The position of the multiplier tubes was varied to measure the angle of emission of the light from the forward direction of the electrons. The results are given in Fig. 2. The Čerenkov angle for Lucite ($n=1.5$), with particles of velocity C, should be about 48°. The observed angle indicates a slightly higher index of refraction which may perhaps be explained by assuming that most of the effective light is in the ultraviolet.

A two stilbene crystal coincidence absorber arrangement which should give approximately 100 percent counting efficiency for electrons above 5 or 6 Mev counted at ten times the rate of the Čerenkov counter in the arrangement used. At higher electron energies, the Čerenkov counter should be considerably more efficient than the 10 percent indicated here because of the longer electron ranges and reduced scattering to be expected. A variation of the arrangement described here will be tried with protons from the cyclotron when it comes into operation.

* Work supported in part by the ONR and AEC.
[1] I. A. Getting, Phys. Rev. 71, 123 (1947).
[2] R. H. Dicke, Phys. Rev. 71, 737 (1947).
[3] J. V. Jelley, Harwell Nuclear Physics Conference (September, 1950).

Magnetic Shielding of the Proton Resonance in H_2, H_2O, and Mineral Oil*

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We have measured the differences in magnetic shielding of the proton magnetic resonance in H_2, H_2O, and mineral oil (Nujol). These particular results are reported at this time because the proton resonances in water and mineral oil are used frequently as standards. The theoretical value[1,2] for the magnetic shielding in H_2 can be combined with these data to reduce other observations to the unshielded proton.

Experimental procedures were similar to those described previously.[3] A major improvement was made by reducing the *total variation* in the field of the permanent magnet to 0.15 gauss over a volume 2 in. in diameter and 1 in. thick. Half-maximum line widths of from 0.03 to 0.08 gauss were observed in these experi-