Letters to the Editor

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On the Existence of Rossi Second and Third Maxima

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RECENTLY the controversial existence of a Rossi second maximum has been claimed to be re-established[1–3] along with a third maximum. It appears from the results of Hess[4] and his co-workers that they obtained some evidence of the third maximum at 25 cm of lead, using a triple coincidence arrangement similar to that of Clay.[1] We also have definitely established the existence of second and third maxima. From a careful analysis it seems that the failure of some workers to confirm these maxima may be due to overlapping, firstly by oblique showers when the three counters are arranged very close together near the absorber, and secondly by a greater percentage of side showers of external origin when all the three counters without sufficient vertical separation are placed far below the absorber for narrow-angle showers. Full details will be published soon. We were led to these investigations by the correspondences of these maxima with the anomalies of RaC gamma-ray absorption in lead reported by the author.[5] The absorption coefficient between 16 and 20 cm, particularly at about 24 cm, was found to be far below the theoretical minimum value as shown in Fig. 1, with a rise again after this. This was attributed to some neutral particle either emitted by RaC or created by a photon. This anomaly almost exactly corresponds to the third maximum in the *d*-curve of Bothe[3] which he indicates is due to a long-lived neutretto. Soddy and Russell[6] obtained a hump in the log intensity curve exactly where the Rossi second maximum is obtained and they interpreted these to be due to some peculiar secondary radiation generated in lead and manifesting itself in this locality. They could eliminate these by bringing the electroscope very near the absorber. From these correspondences we[5] suggested that these anomalies and the two maxima have the same origin and probably are due to two groups of unstable neutral particles of low mass produced by the photon. Since RaC gamma-rays are of maximum energy only 2.4 Mev, therefore these neutral particles might be simply positron-electron dipoles behaving like the former before annihilation. According to Heitler[7] the probability of a positron capturing an electron is a maximum when its kinetic energy is 0.5 Mev and consequently a positron-electron dipole with very slight interaction with matter and with this much energy requires a life only about 10^{-9} sec to produce these anomalies and maxima. The two groups may be due to two different lives depending on spin. Wheeler[8] showed that a positron-electron can only form an atom of life about 10^{-6} sec with parallel spin and the probability of its formation is one in a million.

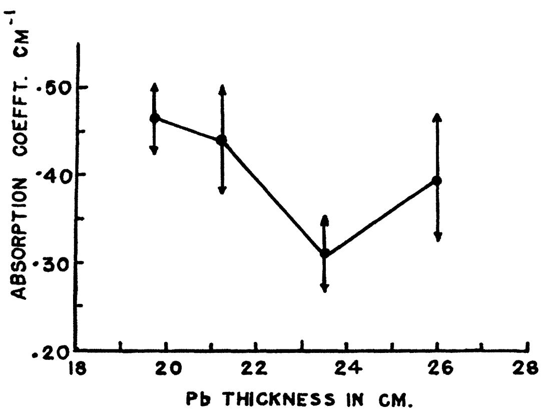

FIG. 1. Absorption coefficient of the RaC γ-ray in lead.

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5 P. K. Sen Chaudhary, Ind. J. Phys. 22, 341 (1948); Science and Culture 15, 38 (1949).
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The Cathode Field in Diodes under Partial Space-Charge Conditions with Initial Velocities

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A FORMULA has been given by Ivey[1] which relates the cathode field, E, in a diode under partial space-charge conditions to the actual diode current density, I, if the initial velocities of the electrons are neglected. We have obtained a more general formula, inclusive of the effects of a homogeneous initial velocity distribution, by a method described below.

If the velocity vector of an electron is denoted by

$$\mathbf{V}=f(x)\mathbf{i}+k\mathbf{j} \tag{1}$$

the equations of motion, and those of the electric field, yield for the function $f(x)$ the differential equation[2]

$$f'^3+4ff'f''+f'''f^2=0. \tag{2}$$

With the initial condition $f(0)=(\vartheta u_1/m)^{\frac{1}{2}}\cos\varphi$, where u_1 is the initial velocity, the field strength, $E(x)$, and the current density can be calculated in terms of $f(x)$ and its derivatives by the relations

$$E(x)=-(m/e)ff', \tag{3}$$

$$I=\rho V=-(m\epsilon_0/2e)(ff'^2+f^2f''). \tag{4}$$

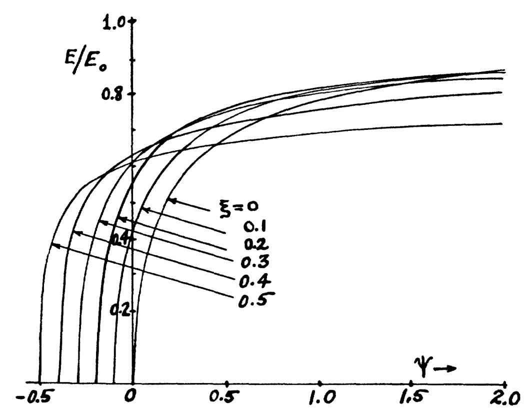

FIG. 1. Dependence of E/E_0 on ψ for various values of ξ.

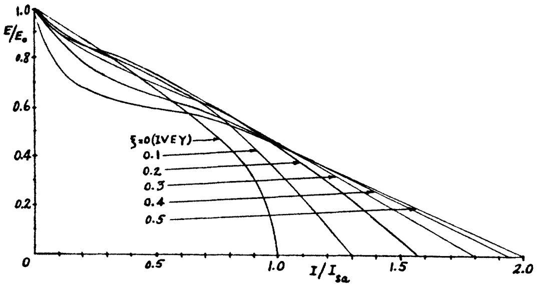

FIG. 2. Dependence of E/E_0 on I/I_{sa} for various values of ξ.

The solution of (2) when inserted into (3) and (4) gives the following general equations[2]

$$(I/I_{sa})^{\frac{1}{2}}=(1-2\psi)(1+\psi)^{\frac{1}{2}}-(\xi-2\psi)(\xi+\psi)^{\frac{1}{2}} \tag{5}$$

$$E/E_0=(4/3)[(1-2\psi)(1+\psi)^{\frac{1}{2}}(\xi+\psi)^{\frac{1}{2}}-(\xi-2\psi)(\xi+\psi)]. \tag{6}$$

Here I_{sa} is the saturation current density with zero initial velocities according to Child's law

$$I_{sa}=(4\epsilon_0/9d^2)(\vartheta/m)^{\frac{1}{2}}u_a^{\frac{3}{2}}.$$

E_0 is the value of E in the absence of current flow (i.e., is equal to u_a/d), ξ is the relative initial velocity of the electrons

$$\xi=(u_1/u_a)^{\frac{1}{2}}\cos\varphi \tag{7}$$

and ψ is a parameter specifying the degree of space charge. For $\psi=-\xi$ it follows that $E=0$, so that the space charge is "complete," and for $\psi\rightarrow\infty$, $E=E_0(I=0)$. With increasing current density ψ decreases from ∞ to $-\xi$ and the cathode field decreases from u_a/d to zero, as is indicated in Fig. 1.

In the general case a relation between I and E can be obtained by elimination of ψ between Eqs. (5) and (6), the nature of the result being shown in Fig. 2. For zero initial velocities ($\xi=0$) the formula of Ivey is regained

$$\frac{I}{I_{sa}}=\frac{1}{2}\left\{1\mp\left[1-\frac{27}{4}\left(\frac{E}{E_0}\right)^2\left(1-\frac{E}{E_0}\right)\right]^{\frac{1}{2}}\right\}. \tag{8}$$

The solution of (2) also permits one to calculate the transit times of electrons with initial velocities, and the electron paths for oblique emission under partial space charge.[2]

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Diverging Integrals in the Self-Charge Problem

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THE polarization of the vacuum arising from the virtual production of electron-positron pairs by an electromagnetic field was examined by Schwinger,[1] who found that the current induced in the vacuum contains a part which leads to a logarithmically divergent but unobservable charge renormalization in the current producing the electromagnetic field. In an investigation of the polarization of the vacuum by the virtual formation of particles with spin zero, Jost and Rayski[2] again found that the induced current includes a logarithmically divergent self-charge term, which moreover has the same sign as that occurring in Schwinger's result. In the calculation of the current induced by the virtual formation of particles with spin 1 having vector coupling with the electromagnetic field, Feldman[3] obtained a quadratic divergence in the self-charge term. He therefore concluded that the cancellation of the diverging self-charge cannot be effected by any superposition of real fields with spin 0, ½, and 1, if no formal regularization procedure is applied.

We have performed the calculation for spin 0 and spin 1 independently, and wish to draw attention to some points in the manipulation of the divergent integrals. For convenience we shall quote from Feldman's paper. In the case of spin 1 he gives the self-charge term of the induced current

$$\alpha/4\pi\cdot(\mathcal{G}_1-\tfrac{1}{2}\mathcal{G}_2)J_\mu(x), \tag{1}$$

where $J_\mu(x)$ is the current producing the external field,

$$\mathcal{G}_1=1/im^2\cdot\int_{-\infty}^{\infty}\exp(im^2z)/z^2\cdot\epsilon(z)dz,$$

$$\mathcal{G}_2=\int_{-\infty}^{\infty}\exp(im^2z)/z\cdot\epsilon(z)dz,$$

$$\epsilon(z)=z/|z|,\quad \alpha=e^2/4\pi,\quad \hbar=c=1.$$

Now these integrals have singularities at $z=0$. Integrating from $-\infty$ to $-z_0'$ and from z_0 to ∞ we deduce that

$$\mathcal{G}_1=2+\mathcal{G}_2+1/im^2\cdot\lim_{z_0\rightarrow0,\,z_0'\rightarrow0}(1/z_0-1/z_0'),$$

$$\mathcal{G}_2=-\lim_{z_0\rightarrow0,\,z_0'\rightarrow0}(\log\gamma m^2z_0+\log\gamma m^2z_0'),$$

where

$$\gamma=1.78.$$

Expression (1) is equivalent to

$$\{2+\tfrac{1}{2}\mathcal{G}_2+1/im^2\cdot\lim_{z_0\rightarrow0,\,z_0'\rightarrow0}(1/z_0-1/z_0')\}(\alpha/4\pi)J_\mu(x). \tag{2}$$

The most strongly divergent terms are imaginary and so must be excluded for physical reasons. In order to make them vanish we are obliged to take the principal values of the diverging integrals, that is, we must put $z_0'=z_0$. Then (2) becomes

$$\{2-\lim_{z_0\rightarrow0}\log\gamma m^2z_0\}(\alpha/4\pi)J_\mu(x), \tag{3}$$

which diverges logarithmically.

The part of the current induced in the vacuum through the formation of electron-positron pairs, which corresponds to (3), is

$$-(\alpha/6\pi)J_\mu(x)\int_{-\infty}^{\infty}\exp(im_e^2z)/z\cdot\epsilon(z)dz=\lim_{z_0\rightarrow0}(\log\gamma m_e^2z_0)\cdot\alpha/3\pi\cdot J_\mu(x), \tag{4}$$

where m_e is the electron mass. The sign of the logarithmic divergence is different in (3) and (4). Thus on admitting principal values—which, however, is an additional prescription—it will be possible to cancel the self-charge effects by a superposition of fields of particles with spins ½ and 1, or indeed by a superposition of fields with spins 0, ½, and 1.

However, the advantage gained by the compensation of the self-charge term is offset somewhat by the fact that in the case of vector mesons under consideration there appears also a diverging term

$$-(\alpha/24\pi m^2)\mathcal{G}_2\Box J_\mu(x),$$

to which no term corresponds in the spin 0 or spin ½ cases.

I am indebted to Professor W. Heitler, University of Zürich, for helpful comments.

[1] J. Schwinger, Phys. Rev. 75, 651 (1949).
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Čerenkov Radiation Counter for Fast Electrons*

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PROPOSALS have been made by Getting[1] and Dicke[2] to use Čerenkov radiation as a means of counting fast charged particles. Also recently Jelley[3] has reported Čerenkov detection of cosmic-ray particles. This letter is a description of an arrangement with which fast electrons, produced by the 48-Mev bremsstrahlung of a betatron, have been counted by the Čerenkov radiation which they produce in Plexiglas or Lucite. The work was undertaken in the hope of developing a velocity discriminating detector to be used with the 450-Mev proton synchrocyclotron under construction at the University of Chicago.