The Angular Distribution of the Neutrons Produced in the Photo-Disintegration of the Deuteron by the 2.51-Mev Gamma-Rays of Ga^{72}

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The angular distribution of the neutrons resulting from the photo-disintegration of the deuteron by the 2.51-Mev gamma-rays of Ga^{72} has been studied. The ratio of the magnetic to the electric cross section has been determined to be 0.61 ± 0.04. A discussion is given of the present status of the problem.

I. PROCEDURE

WE have determined the angular distribution of the neutrons produced in the photo-disintegration of deuterons by the 2.51-Mev γ-rays of Ga^{72}. Figure 1 shows the experimental arrangement. A thin-walled (0.2 mm) copper ring R of square cross section (6 mm$\times$6 mm) contained heavy water. It was fixed perpendicular to the plane of an aluminum ring, Al by Nylon threads. Spheres of metallic gallium alloyed with 10 percent Ni (diameter 16 mm) could be placed in positions A, A', and B. The photo-neutrons liberated in the heavy water ring were detected by measuring the radioactivity of iodine I^{128} (25-min half-life) produced in two glass cylinders J_1 and J_2, each of which contained a solution of 5 mg of iodine in 2.5 liters of ethyl iodide, and was covered by cadmium foil. The free radioactive iodine and carrier in J_1 and J_2 were extracted with sodium sulphite solution by a method described elsewhere.[1] Owing to the bulk of the ethyl iodide, it was found necessary to shake the contents of each container consecutively with the same extracting agent, in order to eliminate errors arising from incomplete mixing of

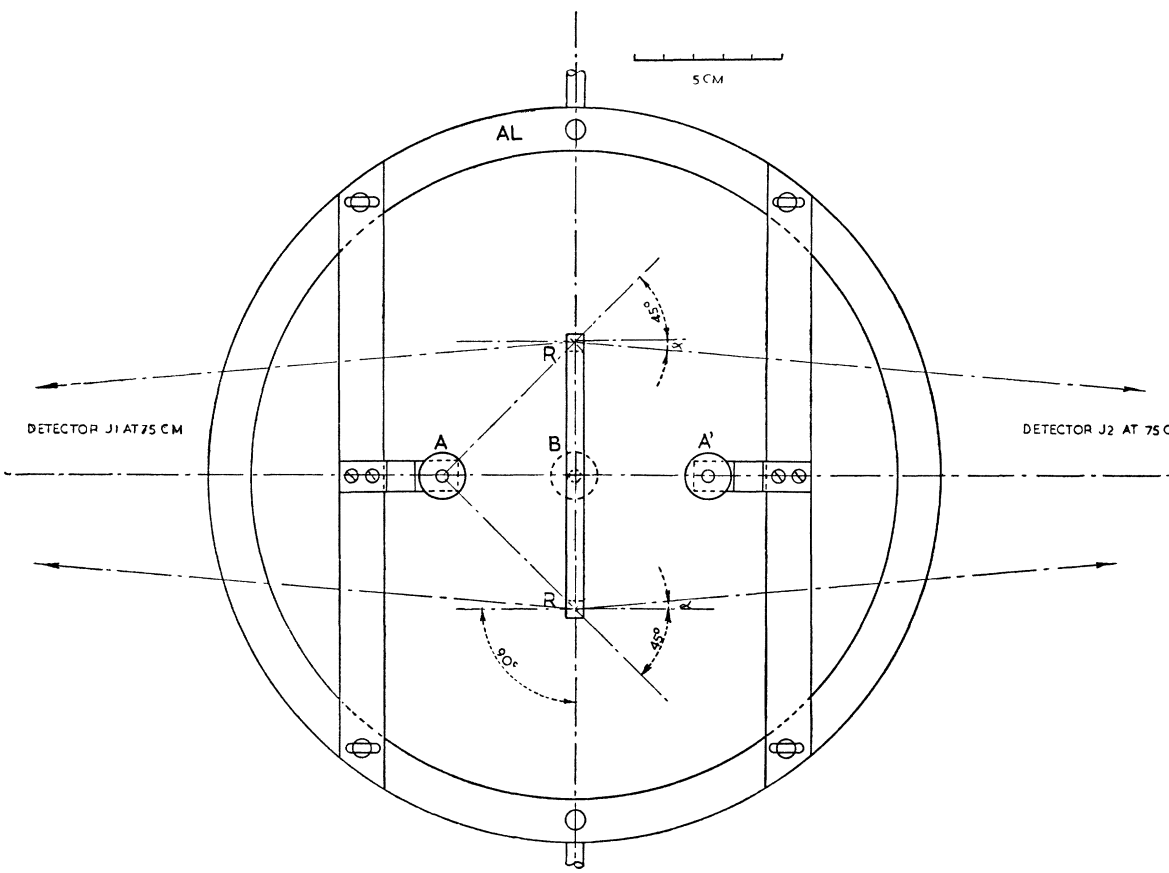

FIG. 1. Experimental arrangement.

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† Revised version received September 29, 1950.
[1] P. F. D. Shaw and C. H. Collie, J. Chem. Soc. 264, 1217 (1949).

the two liquids. The radio-iodine and carrier were precipitated as silver iodide; and this was placed, in a brass slide, between two "end-on" Geiger-Müller counters (G.E.C. Type G.M. 4) which were 6 mm apart in a lead castle. The activity was counted for 30 to 45 min.

An average angle of 45° was obtained between the primary rays and the photo-neutrons detected in J_1 and J_2, by placing the two gallium spheres in positions A and A'. For an average angle of 90°, only one source was placed in the position B (consecutive measurements were made on each of the sources used in the 45° position).

The gallium spheres were irradiated in the pile at A.E.R.E. Harwell to a total activity of about three Curies in a neutron flux of 10^{12} neutrons/cm^2/sec. Two separate runs were carried out. Each consisted of six measurements in the 45° position, and three for each of the two sources in the 90° position. Each irradiation was made for one hour. In order to evaluate a correction for the scattering of photo-neutrons inside the heavy water ring, a similar series of measurements was made with a ring of the same mean diameter (9 cm) but with cross section 12 mm×12 mm. Comparison of the measurements for each ring allows an extrapolation for the ratio of intensities $I_{45°}/I_{90°}$ to zero thickness of heavy water.

Scattering of photo-neutrons by the surrounding material was reduced to a minimum by carrying out the irradiations on an aluminum structure rising 5 meters above the flat roof (10 m×23 m) of a tower of the laboratory (17 m from ground level). Measurements were made to estimate the number of neutrons scattered from the roof into the detectors, by supporting a neutron source and the detectors at heights of 1.4, 3, and 5 m, from the roof. The results showed that the correction for scattering was zero within the experimental error (<1 percent) of the measurements.

II. RESULTS AND CORRECTIONS

The theoretical distribution of the photo-neutrons is given by the relation

$$\eta_\theta = a + b\sin^2\theta, \quad a/b = 2\sigma_m/3\sigma_e, \tag{1}$$

where η_θ is the number of photo-neutrons ejected into unit solid angle at an angle θ from the direction of the incident γ-rays, σ_m is the photo-magnetic cross section, and σ_e is the photoelectric cross section.

Before the observed intensities, $I_{45°}/I_{90°}$ are converted to the ratio a/b, via the relation

$$I_{45°}/I_{90°} = (a+\tfrac{1}{2}b)/(a+b), \tag{2}$$

a number of corrections must be made. These are as follows.

(1) *The change of solid angle presented by the heavy water ring to the sources in the 45° and 90° positions.* A correction to the inverse square law is made for the finite sizes of ring and sources using a formula derived by integration over the volumes.

$$\text{Mean } (1/r^2) = (1/r^2)[1+(a^2/5r^2)-(x^2/3r^2)],$$

where a is the radius of the source and $2x$ is the thickness of the ring,

(2) *The spread in angle θ about the mean angles defined by the relative positions of ring, sources, and detectors, owing to their finite sizes.* This correction differs for the 45° and 90° positions.

45° position: For the symmetrical arrangement of sources and detector, the mean angles for each source are $45°+\alpha$ and $135°-\alpha$, (Fig. 1) and the sines become $\sin(45°\pm\alpha)$. Since the distribution of Eq. (1) is symmetrical about 45°, the average intensity $\bar{I}_{45°}$ is independent of the angle α, and of the angular spread, because the corrections are equal and of opposite sign and cancel out. Then

$$\bar{I}_{45°} = (a+\tfrac{1}{2}b).$$

90° position: The finite sizes of sources, rings, and detectors introduce a spread in angle about the mean angle θ, where θ is given by

$$2\theta = \theta_1 + \theta_2,$$

and the average intensity I_θ is

$$I_\theta = \int_{\theta_1}^{\theta_2} (a+b\sin^2\theta)\sin\theta d\theta \Big/ \int_{\theta_1}^{\theta_2} \sin\theta d\theta$$
$$= a+b-\tfrac{1}{3}b(\cos^2\theta_1+\cos\theta_1\cos\theta_2+\cos^2\theta_2).$$

The spread for each cause was evaluated separately, those for source and ring by calculating the amounts of Ga^{72} and heavy water enclosed between $\frac{1}{2}°$ angular intervals about the line joining the center of the source and mean circumference of the toroid; that for the detector by measuring the detection sensitivity as a function of the distance from the center of its face, with a neutron howitzer. The howitzer consisted of a gallium source enclosed in a 0.5-cm layer of heavy water which was attached to the bottom of a 35-cm long copper tube

TABLE I. Measured intensity ratios.

Measurement		Counts/min, corrected for all decays	$I_{45°}/I_{90°}$ corrected for solid angle	$I_{45°}/I_{90°}$ corrected for spread at 90°
S.R.[a] 45°	*I*	27,360±180	0.746±0.005	0.739±0.006
S.R. 90°		73,480±350		
S.R. 45°	*II*	23,620±260	0.748±0.007	0.740±0.007
S.R. 90°		63,320±400		
B.R.[b] 45°	*I*	85,200±450	0.817±0.004	0.809±0.005
B.R. 90°		104,630±550		

[a] S.R. = small ring.
[b] B.R. = big ring.

of 2-cm diameter. The tube dipped into a large tank of water, so that in all directions but the axis of the tube the neutrons were slowed down and absorbed. The ethyl iodide containers, covered in cadmium, were placed over this 2-cm opening and irradiated for an hour in each position. The three distributions were then combined, giving a curve of angular spread against the relative weight of over-all neutron efficiency. The curve derived in this manner corresponds to a single source in the $90°+\alpha$ position and *one* detector only; for both detectors two such distributions are added with their peaks symmetrically placed at angle α on either side of the average of 90°. The final distribution is used to supply the correction factor β in the expression

$$I_{90°}=(a+\beta b)$$

by evaluation of the formula,

$$\begin{aligned} I_{90°}&=a+b-\tfrac{1}{3}b\{\Sigma w_\gamma[\cos^2(\gamma-\tfrac{1}{2}) \\ &\quad+\cos(\gamma-\tfrac{1}{2})\cos(\gamma+\tfrac{1}{2})+\cos^2(\gamma+\tfrac{1}{2})]/\Sigma w_\gamma\} \\ &=a+b-\tfrac{1}{3}b[1+\cos 1°(\Sigma w_\gamma \cos 2\gamma/\Sigma\omega_\gamma) \\ &\quad+\tfrac{1}{2}\cos 1°+\tfrac{1}{2}\Sigma\omega_\gamma\cos 2\gamma/\Sigma\omega_\gamma], \end{aligned}$$

where w_γ is the weight of an angular spread of $\pm\frac{1}{2}°$ about the angle γ. The corrections are:

Large toroid $\beta=0.9744$;
Small toroid $\beta=0.9813$.

(3) *Scattering of neutrons by heavy water.* The uncorrected measurements overestimate the contribution of σ_m. After the measured intensities at 90° had been corrected as described above, a linear extrapolation was made from the ratios $I_{45°}/I_{90°}$ for each ring, to zero thickness of heavy water. The correction amounts to 10 percent, in agreement with calculations on the scattering of neutrons by D_2O. This is in contradiction with the results of Genevese,[2] who found no variation in $I_{90°}$ for ring thicknesses below 10 mm. It is possible that, at 90°, scattering of neutrons into his detector exactly counterbalances scattering of neutrons out of the beam. If this is so, a large variation in $I_{45°}$ would be expected. Our extrapolation of $I_{45°}/I_{90°}$ avoids this error. Meiners[3] used spheres of 15- and 10-mm diameter and also extrapolated linearly which may be in error for such large spheres.

(4) *Correction for γ-ray momentum.* The value for $I_{45°}/I_{90°}$ in the laboratory system will not be the same as the value in the center-of-mass system.

An exact calculation can be made for σ_m/σ_e from the expression[4] obtained from geometrical considerations.

$$\frac{\sigma_m}{\sigma_e}=\frac{1.5R(1-7\alpha^2/4)-0.75(1+2\alpha^2)}{1-R(1-\frac{3}{4}\alpha^2)}\cdots, \qquad (3)$$

[2] F. Genevese, Phys. Rev. 76, 1288 (1949).
[3] E. P. Meiners, Phys. Rev. 76, 259 (1949).
[4] I. F. E. Hansson and L. Hulthén, Phys. Rev. 76, 1163 (1949), and private communication.

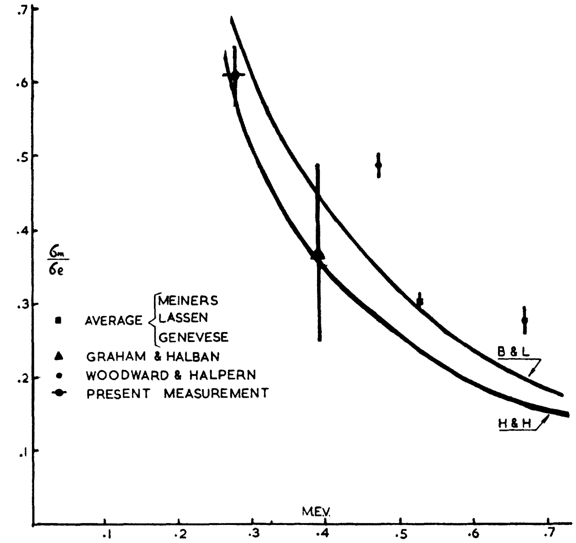

FIG. 2. The dependence of σ_m/σ_e on gamma-ray energy.

where $R=(I_{45°}+I_{135°})/2I_{90°}$, which is the measured ratio of intensities.

$$\alpha=\bar{v}/v,$$

where $\bar{v}$ is the velocity of the mass center in the laboratory system, and v is the velocity of the neutron or proton relative to the mass center. For 2.51 Mev, $\alpha=0.0774$. Table I gives the measured values. Finally σ_m/σ_e can be calculated from Eq. (3). The linearly extrapolated value of $I_{45°}/I_{90°}$ to zero D_2O thickness is $0.654\pm0.015\pm0.011$. This gives by substitution in Eq. (3) the value

$$\sigma_m/\sigma_e=0.61\pm0.04.$$

In Fig. 2 is shown the ratio σ_m/σ_e plotted against the excess energy of the γ-ray producing photo-disintegration over the binding energy of the deuteron. The experimental point at 2.62 Mev (RdTh γ-rays) is the result of Graham and Halban,[5] that at 2.758 Mev (Na^{24} γ-rays) being a weighted average from several authors.[2,3,6–8] The top curve in Fig. 2 was obtained by using the calculations of Bethe and Longmire,[9] but adjusting for the new values of the effective range in the triplet state, (1.71×10^{-13} cm), the binding energy (2.231 Mev), and the γ-ray energies (2.758, 2.618, 2.504 Mev). The other curve is from the calculations of Hansson and Hulthén.[4]

The present value of σ_m/σ_e falls very close to the Hansson and Hulthén curve, but is not in contradiction

[5] G. A. R. Graham and H. Halban, Revs. Modern Phys. 17, 297 (1945).
[6] N. O. Lassen, Phys. Rev. 74, 1533 (1948): 75, 1099 (1949).
[7] W. M. Woodward and I. Halpern, Phys. Rev. 76, 107 (1949).
[8] B. Hamermesh and A. Wattenberg, Phys. Rev. 76, 1408 (1949).
[9] H. A. Bethe and C. Longmire, Phys. Rev. 77, 647 (1950).

with that of Bethe and Longmire. The latter agrees better with the weighted average of the measurements at 2.76 Mev, which has a higher accuracy. The results of Halpern and Woodward[7] are difficult to fit to either curve. It is evident that one should try to obtain at energies of 2.504 and 2.618 Mev results with the same precision as that already obtained for 2.76 Mev. It is unlikely that the final curve of the ratio σ_m/σ_e against energy will lie significantly above the Bethe-Longmire curve. Although the experimental ratio σ_m/σ_e is not in contradiction with the Bethe-Longmire theory, it should be noted that the experimental total cross section[10] $\sigma_r=\sigma_m+\sigma_e$ lies well above the theoretical values.

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[10] G. R. Bishop, C. H. Collie, H. Halban, A. Hedgsarv, K. Siegbahn, S. DuToit, and R. Wilson, Phys. Rev. 80, 211 (1950).

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Fine Structure of the Hydrogen Atom.* Part II

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In the first paper of this series, the shift of the $2^2S_{\frac{1}{2}}$ level of hydrogen was determined to be 1000 Mc/sec. A new apparatus differing from the original one in details, but not in principle, has been built in order to improve the accuracy of the above result. This provides a greater yield of metastable hydrogen atoms, a more homogeneous magnetic field, and more accurate means of measurement of magnetic field and frequency. With these improvements, preliminary measurements of considerably increased accuracy have been made on both hydrogen and deuterium. The transitions observed were $2^2S_{\frac{1}{2}}$, $m=\frac{1}{2}$, to $2^2S_{\frac{1}{2}}$, $m=-\frac{1}{2}$, as well as to $2^2P_{\frac{1}{2}}$, $m=\frac{1}{2}$ and $m=-\frac{1}{2}$. The first transition permits observation of the hyperfine structure of $2^2S_{\frac{1}{2}}$, as well as an accurate calibration of magnetic field. Hyperfine structure was also resolved for the last transition in hydrogen. There was no observable difference between the level shifts for hydrogen and deuterium which may be taken as 1062±5 Mc/sec. Later papers of this series will deal with the numerous experimental and theoretical corrections necessary to obtain a level shift accurate to 1 Mc/sec.

F. NEW APPARATUS

32. Introduction

THE apparatus described[59] in Part I was improvised during the exploratory work necessary to establish the formation and detection of metastable hydrogen atoms. It was geometrically inconvenient and not well suited for precise measurements. In addition, the magnetic field was found to be excessively inhomogeneous, the magnet too small, and the pumping speed inadequate, and no cold traps were provided.

A second apparatus has been built with extensive improvements in the above respects, and in the auxiliary equipment relating to magnetic field, radiofrequency, and power, etc. With this, it has been possible to obtain results having an internal consistency close to 1 Mc/sec. In order to determine the relative positions of the $2^2S_{\frac{1}{2}}$, $2^2P_{\frac{1}{2}}$, and $2\,^2P_{\frac{3}{2}}$ levels, however, it is necessary to apply many experimental and theoretical corrections to the raw data. Fortunately, it is possible to establish empirically that these corrections can lead to a change of only a few megacycles per second in the results. In this paper, accordingly, we will give an account of the work up to about a year ago, when it was possible to quote a result[60] with a limit of error of ±5 Mc/sec. It is planned to devote Part III of this series to a discussion of the corrections and Part IV to a determination of the precise level shifts from the data.

33. General Features

A cross section of the apparatus in a horizontal plane is shown in Fig. 27 and has the same general features as those shown in Fig. 19 of Part I. Source of atomic hydrogen *a*, electron bombarder *c*, *d*, *e* and detector *j*, *k* are now in separately pumped chambers. The source chamber is pumped by the main pump (DPI-MC 275) and the other chambers by differential pumps (DPI-GF 20W) emptying into the source chamber. There is a

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[59] Part I of this series appeared in Phys. Rev. 79, 549 (1950). Frequent references to this paper are made. Sections, figures, equations, and footnotes of Part II are numbered consecutively after those of Part I. The designation of states by letters α, β, *a*, *b*, *c*, *d*, *e*, *f* is explained in Fig. 14.

[60] R. C. Retherford and W. E. Lamb, Jr., Phys. Rev. 75, 1325 (1949).