establish the amount of this effect, Mr. C. Eggler kindly performed a subsidiary measurement in which he compared the specific saturated activity for irradiations in air and in graphite of a standard indium foil (92.3 mg/cm²) with that of a very thin indium deposit (0.423 mg/cm²; negligible self absorption) on an iron foil finding

$$\frac{(A \text{ thin})}{(A \text{ stan})}\Big/\frac{(A \text{ thin})}{(A \text{ stan})}=0.95.$$
$$\text{(air)} \qquad \text{(graphite)}$$

If all of the neutrons striking the boron counter-indium foil assembly were monodirectional, proceeding from the primary scatterer only, the activity induced in the indium foils would be, as is easily computed, 1.29 times greater for the arrangement chosen, in which the normal to the indium foils made an angle of 55° with the line from the foils to the primary scatterer, than the activity if the neutrons were isotropic. The degree of isotropy was easily checked by observation of the counting rate in a boron counter which was sheathed in cadmium over all of its surface except one end; it was found that the counting rate changed in a gradual manner from a maximum when the counter window pointed directly toward the primary scatterer to a minimum, equal to one-fourth the maximum, when the window pointed away from the primary scatterer. We thus conclude that most of the neutrons striking the indium foil assembly have been scattered in by objects other than the primary graphite scatterer. We also note that the unshielded boron counter responds in approximately the same manner as do the indium foils to deviations from isotropy, since the arrangement kept the axis of the counter parallel to the planes in which the foils were mounted; but the effect in the counter is smaller from the circumstance that neutrons passing perpendicularly through the counter along a diameter experience a fractional reduction in intensity of 0.05, whereas for neutrons passing perpendicularly through a standard indium foil the reduction is 0.092. This study leads us to estimate the effect of nonisotropy in the neutron atmosphere as producing a correction of about 5 percent.

These two corrections then lead us to write for the detection cross section of the counter: 10.6 ± 1 cm².

Adopting for the mean cross section of a nitrogen molecule for absorption of thermal neutrons the figure $2\times1.7\times10^{-24}$ cm², we may say that the number of counts is the same as the number of neutrons absorbed in a mass of air of 10.6 cm²/(0.055 cm²/gm air) = 193 gm air (for single unshielded counter).

This air mass equivalent of the counter is derived from calibration for thermal neutrons. As the cross sections of both boron and nitrogen follow the $1/v$ law, the same air mass equivalent will apply for unshielded counters to slow neutrons of all energies.

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$(n,2n)$ and (n,p) Cross Sections*

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Cross sections of several $(n,2n)$ and (n,p) reactions were measured by bombarding samples with a known flux of neutrons and measuring the induced beta-activity, making elaborate corrections for self-absorption of the beta-rays. Using the known energy spectrum of the incident neutrons, calculations were made of the cross sections to be expected from Weisskopf's statistical theory, and the agreement was satisfactory. The results indicate that level densities in odd-odd nuclei are greater than those in even-even nuclei by a factor of 3 ± 1. Applications of the method to shielding and the use of threshold detectors are discussed.

I. INTRODUCTION

An interesting result of nuclear statistical theories is the prediction of cross sections of nuclear reactions which predominantly proceed through energy states of excitation sufficiently high that resonances are very closely spaced and can thus be meaningfully averaged over. $(n,2n)$ and (n,p) reactions are particularly simple examples of these, since the incident particle faces no Coulomb barrier, whence the cross section for the formation of the compound nucleus is πr^2 (r is the nuclear radius). The cross section for an $(n,2n)$ reaction is then πr^2 times the probability that the compound nucleus will decay by emission of two neutrons which, considering neutron emission as much more probable than any other mode of decay, is just the probability that the first neutron is emitted with low enough energy to leave emission of a second neutron energetically possible. Considering the first neutron to be emitted with an energy spectrum approximating a Maxwell distribution with temperature, T, gives[1]

$$\sigma(n,2n)=\pi r^2[1-(1+\Delta E/T)\exp(-\Delta E/T)], \quad (1)$$

where $\Delta E=E-B$, E=energy of incident neutron, and B=threshold of the reaction.

In the case of (n,p) reactions, emission of a proton is impeded by the coulomb barrier, causing it to be much less probable than emission of a neutron (inelastic scattering) with which it competes, whence

$$\sigma(n,p)=\pi r^2 f_p/(f_n+f_p)\simeq\pi r^2 f_p/f_n, \quad (2)$$

where f_p and f_n are the relative probabilities of the compound nucleus decaying by emission of a proton and neutron. Methods of calculating these quantities are given by Weisskopf.[1]

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[1a] V. Weisskopf, *Lecture Series in Nuclear Physics* (U. S. Govt. Printing Office, Washington, D. C., 1947). [b] V. Weisskopf and Ewing, Phys. Rev. **57**, 472 (1940). [c] V. Weisskopf and J. M. Blatt, M.I.T. Technical Report No. 42 (May 1, 1950), unpublished.

TABLE Ia. (n,2n) reactions studied.

Original nucleus	Material used	Half-life	Method	Other activities observed
N^{14}	NH_4NO_3 Urea	10.1 min	1	None
F^{19}	LiF	112 min	1	None
P^{31}	Ca_3P_2 Red phos.	2.55 min	1–2	2.5-hr Ca(n,γ); 170-min P(n,p)
K^{39}	K_2CO_3 KCOOH	7.5 min	3	12.4 hr (n,γ); 38.5-min (n,α)
Ti^{46}	Ti metal powder	3.08 hr	1–2	57-hr (n,p); 44-min (n,p)
Cr^{50}	Cr_2O_3	42 min	2	3.9 min (n,p)
Fe^{54}	Fe metal	8.9 min	2	2.59-hr (n,p); (8.9-min not observed)
Ni^{58}	Ni metal	36 hr	(1)–3	2.6-hr (n,γ); 72-day (n,p)
Cu^{63}	$Cu^{63}O$[a]	10.1 min	2	5-min (n,γ); 12.8-hr (n,γ)
Zn^{64}	Zn metal	38.3 min	3	13.8-hr (n,γ); 5M(n,p); 57-min (n,γ)
Ga^{69}	Ga_2O_3	68 min	2	20-min (n,γ); 14.1-hr (n,γ)
As^{75}	As_2O_3	16 days	(1)–2	26.8-hr (n,γ)
Rb^{85}	Rb_2CO_3	6.5 hr	2	17.8-hr (n,γ); 19.5-day (n,γ) (6.5 hr not observed)
Zr^{90}	ZrO_2	78 hr	3	62-hr (n,p); 17-hr (n,γ)
Ru^{96}	Ru metal powder	20 min	3	Long; (20 min not observed)
Ag^{107}	Ag metal	24.5 min	1–2	2.3-min (n,γ); 13-hr (n,p); ~40 day ???
Sb^{121}	Sb_2O_3	16 min	1	2.8-day (n,γ)
I^{127}	NH_4I	13 days	(1)–2	25-min (n,γ)
Pr^{141}	Pr_6O_{11}	3.5 min	2	Long

[a] Obtained from Isotope Division, AEC, Oak Ridge, Tennessee.
Methods designated as 1, 2, and 3 described in Sec. II. Parenthesis indicates method was not carried out thoroughly.

TABLE Ib. (n,p) reactions studied.

Original nucleus	Material used	Half-life	Method	Other activities observed
O^{16}	Sugar	8 sec	3	2.1-min (n,2n); long
Mg^{24}	Mg metal	14.8 hr	1	10.2-min (n,γ)
Al^{27}	Al metal	10.2 min	1	2.4-min (n,γ)
Si^{28}	H_2SiO_3	2.4 min	1	170-min (n,γ); 6.7-min (n,p)
Si^{29}	H_2SiO_3	6.7 min	1	170-min (n,γ); 2.4-min (n,p)
P^{31}	NH_4PO_4 Red phos.	170 min	1–2	2.5-min (n,2n)
S^{32}	Flower of sulfur	14.3 day	1–2	None
Ca^{42}	CaOH	12.4 hr	(1)–2	2.5-hr (n,γ)
Ti^{48}	Ti metal powder	44 hr	1	3.08-hr (n,2n); 57-min (n,p)
Ti^{49}	Ti metal powder	57 min	1–2	3.08-hr (n,2n); 44-hr (n,p)
Cr^{52}	Cr_2O_3	3.9 min	(1)–2	42-min (n,2n); long
Fe^{56}	Fe metal	2.59 hr	(1)–2	None
Ni^{58}	Ni metal	72 day	(1)–3	2.6 hr (n,γ); 36-hr (n,2n)
Cu^{65}	Cu metal	2.6 hr	(2)–3	10.1-min (n,2n); 5-min (n,γ); 12.8 hr (n,γ)
Zn^{66}	Zn metal	5 min	1	57 min (n,γ); 13.8 hr (n,γ); 38 min (n,2n)
Ge^{70}	GeO_2	20 min	2	89 min (n,γ)
Se^{76}	Se metal powder	26.8 hr	2	None
Se^{77}	Se metal powder	40 hr	3	26.8-hr (n,p); (40 hr not observed)
Rb^{87}	Rb_2CO_3	74 min	3	17.8-min (n,γ); long (74-min not observed)
Sr^{88}	SrC_2O_4	17.8 min	1	2.7-hr (n,γ)
Zr^{90}	ZrO_2	62 hr	3	78-hr (n,2n)
Mo^{97}	Mo metal	75 min	3	14.6-min (n,γ); long (75-min not observed)
Ru^{101}	Ru metal powder	14 min	2	Long; (14-min not observed)
Ag^{109}	Ag metal	13 hr	2	2.3-min (n,γ); 24.5-min (n,2n); 40 day ???
Ba^{138}	BaOH	33 min	3	85-min (n,γ); long (33 min. not observed)
La^{139}	La_2O_3	85 min	3	40-hr (n,γ); (85-min doubtfully observed)

II. EXPERIMENTAL PROCEDURE

Incident Neutrons

The cross sections of several (n,2n) and (n,p) reactions, as given in Table I, were obtained by irradiating samples with neutrons and measuring the induced beta-activity. The neutrons were obtained by bombarding an internal thick beryllium target with 15-Mev deuterons in the University of Pittsburgh cyclotron. Their energy distribution was determined by observing recoil protons in photographic emulsions[2] and in a triple coincidence anticoincidence counter telescope,[3] and is shown in Fig. 1. The beam intensity was monitored by the 24.5-min activity due to $Ag^{107}(n,2n)Ag^{106}$, except for a few long-lived activities which were monitored by the 14.8-hour activity from $Mg^{24}(n,p)Na^{24}$.

The absolute intensity was determined from the known cross section[4] of $S^{32}(n,p)P^{32}$, which gives results shown[5] to be in agreement with determinations based on the cross sections of uranium and thorium fission, and the reaction $P^{31}(n,p)Si^{31}$. A confirmation in the high energy region is discussed in Sec. IV.

[2] B. Cohen, Carnegie Inst. Tech. Report No. 4 (May 20, 1950), unpublished.
[3] Falk, Carnegie Inst. Tech. Report No. 5 (June 1, 1950), unpublished.
[4] E. D. Klema and A. O. Hanson, Phys. Rev. **73**, 106 (1948).
[5] Allen, Nechaj, Sun, and Jennings, Phys. Rev. **76**, 188 (1949).

Correction for Self-Absorption of Beta-Rays

The samples, which were in the form of $1\frac{1}{4}$ in. diameter pills, were counted with a thin windowed end counter. By insertion of various thicknesses of aluminum between the pills and the counter, it was found that beta-rays were absorbed according to

$$N \sim e^{-y}, \tag{3}$$

where N is the counting rate with a thickness of aluminum, t (in mg/cm^2), interposed, and $y=t/\alpha$. Here, α is a constant which was found to be independent of absorber material. For calculation of self-absorption, Eq. (3) must be integrated through the thickness of the pill, giving the percentage transmission, T, as

$$T=(1-e^{-y})/y. \tag{4}$$

Equations (3) and (4) form the basis of two methods of determining α: (1) by measuring relative activities with various thickness of aluminum between the pill

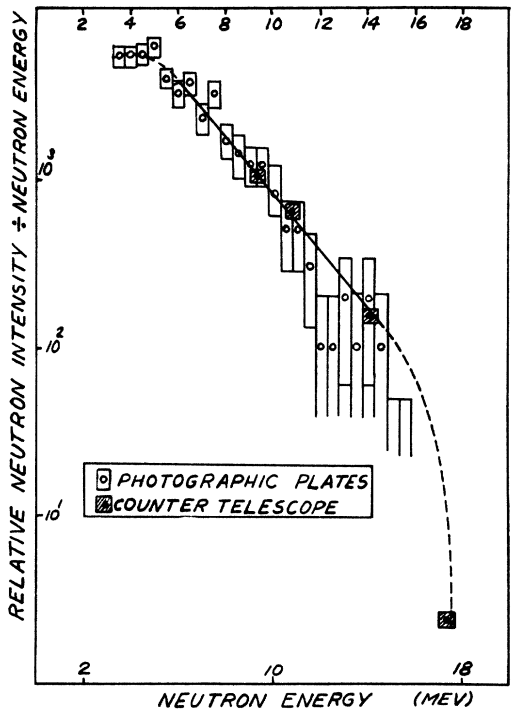

FIG. 1. Energy distribution of incident neutrons (from references 2 and 3).

and window (aluminum absorption), and (2) by measuring the relative activities from pills of various thickness (thin pill). Determinations of α by the two methods are shown plotted against the maximum beta-energies of the activities in Fig. 2.

The following methods were then used for determining cross sections:

Method 1. The best value of α was obtained by averaging the values from thin pill and aluminum absorption determinations, giving some weight to line A of Fig. 2. The corrected activity was then found by an extrapolation of the thin pill activities to zero thickness by use of Eq. (4).

Method 2. As can be shown from Eq. (4) (see reference 2), the ratio of the observed to the corrected activity of a thick pill ($t \gg \alpha$) is dependent only on α, which was determined by aluminum absorption. Corrections were applied for the change in pill to counter distance, and for deviations from Eq. (3). This method was used where sufficiently thin pills ($t \ll \alpha$) were difficult to prepare or could not be given sufficient activity for good counting statistics.

Method 3. In cases where determinations of activities were not sufficiently accurate (due to large corrections for background activities) for a determination of α, it was obtained from line A of Fig. 2, and used to correct thin pill or thick pill activities.

Other Corrections

Studies were made of the effect of backscattering from the pans on which the samples were supported, and of beta-absorption in air and in the counter window, and corrections for these (seldom more than 10 percent) were applied.

III. (n, p) CROSS SECTIONS

The experimentally observed (n,p) cross sections in the cyclotron beam are listed in Table II and plotted[6] *vs* atomic number in Fig. 3. The first conclusion obtainable from Table II is that even-even elements have larger (n,p) cross sections than even-odd and odd-even ones. This can be understood in view of the fact that the density of levels in odd-odd nuclei (the result of an (n,p) reaction in an even-even element) is larger than in even-even nuclei (the result of an $(n\text{-}n)$ reaction) owing to the Pauli exclusion principle. Analysis of the data gives the difference as a factor of 3 ± 1, which is in agreement with the factor of 4 predicted by Weisskopf in an earlier paper.[1b] For this reason, the observed (n, p) cross sections of even-even nuclei were divided by 4 in Fig. 3.

The lines in Fig. 3 represent theoretical calculations based on various values of the parameter r_0 (used in calculating nuclear radii by $r=r_0A^{\frac{1}{3}}\times10^{-13}$ cm) and of nuclear temperatures. Proton capture cross sections [used in calculating f_p in Eq. (2)] are taken from reference 1c.

Curve A represents $r_0=1.3$ and nuclear temperatures from reference 1a; Curve B uses the same temperatures and $r_0=1.5$; and curves C and D were calculated with $r_0=1.3$ and temperatures 25 percent lower and higher, respectively. From these curves, the predictions of any other assumptions can be estimated. For example, curve E represents $r_0=1.3$ and Weisskopf's most recent estimation of nuclear temperatures.[1c] (Curve F was obtained from an old calculation of penetration factors[1b]

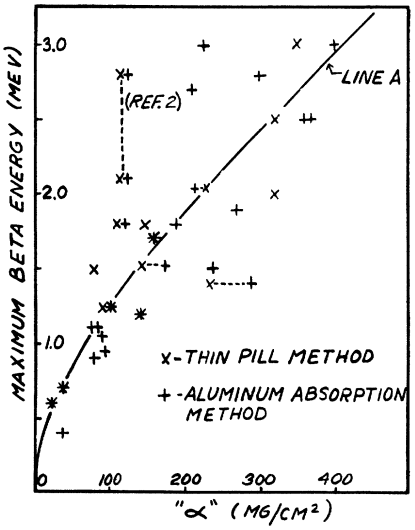

FIG. 2. Determinations of α by thin pill and aluminum absorption methods.

[6] The nucleus O^{16} is omitted from Fig. 3 because it has a very low atomic number and the (n,p) reaction has a high threshold; P^{31} and S^{32} are also omitted because excitation functions for (n,p) reactions in these nuclei have been measured and found to exhibit resonances which explain the large value of the total cross section. Clark Goodman, Editor, *The Science and Engineering of Nuclear Power* (Addison-Wesley Press, Cambridge, 1948–49).

TABLE II. Observed (n,p) cross sections.[a]

(1) Element	(2) Atomic number	(3) Threshold (Mev)	(4) Observed cross section (mb)	(5) Max. error (%)
O^{16}	8	9.3	8	200
Mg^{24}	12	2.1	39	10
Al^{27}	13	2.1	25	10
Si^{28}	14	3.9	45	12
Si^{29}	14	1.8	36	15
P^{31}	15	1.1	120	10
S^{32}	16	1.0	285	5
Ca^{42}	20	2.9	120	10
Ti^{48}	22	1.3	23	15
Ti^{49}	22	1.1	6.5	10
Cr^{52}	24	2.8	15	10
Fe^{56}	26	2.9	18.5	8
Cu^{65}	29	2.3	3.2	40
Zn^{66}	30	3.5	11.0	10
Ge^{70}	32	1.8	14.5	15
Se^{76}	34	2.3	16.5	10
Se^{77}	34	0.0	<8[b]	...
Rb^{87}	37	3.3	<2[b]	...
Sr^{88}	38	4.3	0.93	8
Zr^{90}	40	1.8	3.1	30
Mo^{97}	42	1.5	<1[b]	...
Ru^{101}	44	0.9	<0.8[b]	...
Ag^{109}	47	0.4	2.0	30
Ba^{138}	56	3.1	<0.2[b]	...
La^{139}	57	2.1	<0.2[b]	...

[a] From neutron energy spectrum shown in Fig. 1.
[b] Activity not observed.

and is included because Wäffler's data[7] are compared with it. His values of $\sigma(\text{obs})/\sigma(\text{cal})$ are in substantial agreement with the data of Table II if $\sigma(\text{cal})$ is taken from curve F.)

All curves in Fig. 3 are calculated for zero threshold. The effect of thresholds can be taken into account in one of two ways:

(1) Assuming that level densities depend on the excitation energy above the average of ground states of nuclei of similar mass,[8] the correction is very small and the circles can be compared directly with the curves in Fig. 3.

(2) Assuming that level densities depend on the excitation energy above the ground state of the particular nucleus involved introduces a large correction factor which depends sensitively on the threshold. This calculation is shown by correcting the observed values by the inverse of these correction factors, giving the crosses in Fig. 3, which can then be compared with the curves.

One might expect the first assumption to be valid for highly excited levels—these are important in (n,p) reactions in light elements—and the second to be valid for levels near the ground state which are the major contributors in heavy elements. From Fig. 3, however, it appears that the first assumption can have only limited validity for energies as high as 5 Mev (these are important contributors) for elements of atomic number ~ 30. It might also be noted that curve E agrees exceptionally well with the calculation from assumption (2).

From Fig. 3 it can be concluded that even the best statistical theory can predict only trends in (n,p) cross sections, with considerable variations to be expected; but the general basis for calculating (n,p) cross sections, and their dependence on atomic number can probably be considered to be satisfactorily verified.

IV. $(n,2n)$ CROSS SECTIONS

The observed and calculated $(n,2n)$ cross sections are listed in Table III. The errors correspond to about two standard deviations and were estimated from a detailed study of the factors involved. The calculated values for columns (5) and (6) were obtained using temperatures from reference 1a (T_1) and reference 1c (T_2) and were corrected for K-capture from curves given by Trigg.[9]

The Cu^{63} cross section is in substantial agreement with the result obtained by integration of the experimental excitation function of Fowler and Slye[10] over the incident neutron spectrum; this serves as a check on the absolute calibration. (Wäffler's[7] calibration is in bad disagreement with these, probably owing to an error in the measurement of the neutron flux.) The calculations using T_2 seem to give better agreement with the experimental data than those using T_1, but the excitation function for $(n,2n)$ in Cu^{63} is in better agreement with

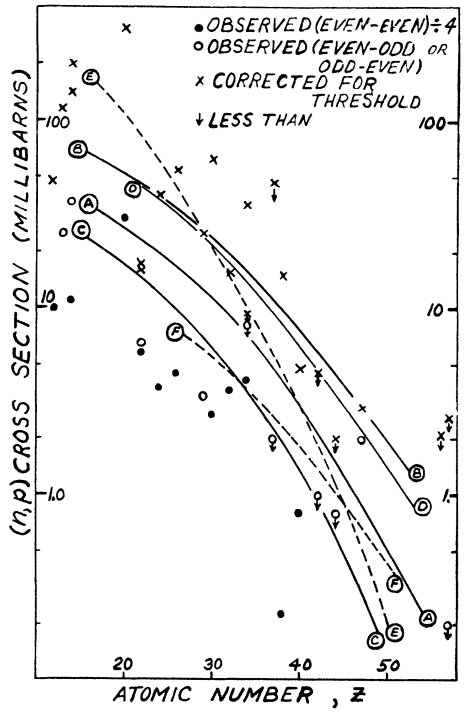

FIG. 3. (n,p) cross sections in cyclotron beam. Lines show calculated values according to various assumptions described in text.

[7] H. Wäffler, Helv. Phys. Acta **23**, 239 (1950).
[8] H. Bethe, Revs. Modern Phys. **9**, 90 (1937).
[9] G. L. Trigg, private communication.
[10] J. L. Fowler and J. M. Slye, Phys. **77**, 787 (1950).

the latter. When the latter are used, there is a tendency for calculated cross sections to be too large. This has also been reported by Taschek[11] and possible explanations have been advanced.[2,12] With the exception of the four lightest elements, for which the theory fails completely, only Zn^{64} and Pr^{141}, show disagreement beyond the expected error. (The excessive activity in Zr was probably due to impurities, as that element is very difficult to purify.) A possible explanation for Zn^{64} was given in reference 2, and the small observed cross section of Pr^{141} can probably be explained by lack of knowledge of the decay scheme of Pr^{140}.

With due consideration for these points, the data from Table III and reference 7 (when corrected for K-capture and the calibration error) give satisfactory support to the validity of Eq. (1).

TABLE III. $(n,2n)$ cross sections.

(1) Element	(2) Threshold (Mev)	(3) σ(obs) (mb)	(4) Error in (3)(±)	(5) σ(obs)/σ(cal) (T_1)	(6) σ(obs)/σ(cal) (T_2)	(7) Percent error in (5) and (6)
N^{14}	10.6	0.255	0.02	0.04	0.2	50
F^{19}	10.4	2.27	0.23	0.24	0.8	50
P^{31}	12.3	14.4	1.5	3.3	10	50
K^{39}	13.2	0.113	0.02	0.045	0.10	60
Ti^{46}	12.3[a]	≥7.7	0.8	≥1.07	≥2.0	50
Cr^{50}	13.4	1.07	0.15	0.31	0.9	70
Fe^{54}	13.9	<2.0	...	<1.0	<3	80
Ni^{58}	11.7	2.9	0.7	0.57	0.9	55
Cu^{63}	10.9	19.6	2.0	0.67	1.2	45
Zn^{64}	11.8	1.6	0.4	0.14	0.25	55
Ga^{69}	10.5[b]	25.8	4.0	0.69	1.2	50
As^{75}	10.3	50	10	1.3[d]	2[e]	60
Rb^{85}	11.5[b]	<4.0[c]	...	...[e]	...[e]	...
Zr^{90}	12.0	14.0	2.0	2.1	3.0	50
Ru^{96}	13.0[b]	<3.1[c]	...	<1.0	<1.0	90
Ag^{107}	9.6	42	3	0.97[f]	1.0[f]	40
Sb^{121}	9.25	40	4	0.99	1.0	40
I^{127}	9.45	31	3	1.9[g]	1.9[g]	60
Pr^{141}	9.4	39	4	0.49	0.5	40

[a] Threshold calculated from masses in L. Rosenfeld, *Nuclear Forces* (Interscience Publishers, Inc., New York, 1948–49).
[b] Thresholds estimated.
[c] Decay scheme of product nucleus not well investigated.
[d] Decays by $\beta+$ and $\beta-$. Branching ratio estimated.
[e] Maximum beta-energy not known, so that K-capture correction could not be made.
[f] Only one of two isomers observed. Branching ratio taken from D. J. Tendam and H. L. Bradt, Phys. Rev. 72, 1118 (1947).
[g] K-capture correction inaccurate, as β-decay is forbidden.

[11] Gittings, Ogle, and Phillips, privately communicated by R. F. Taschek.
[12] V. Weisskopf, in Symposium on Nuclear Shell Structure, New York Meeting of Am. Phys. Soc. (February 4, 1950).

TABLE IV. Favorable threshold detectors.

Approximate threshold (Mev)	Approximate half-lives (minutes) 3	30	300	3000
9.5	Pr	Ag		I
10.5	N	N	F	AS
11.5			Ni	
12.5	P			
13.5		Cr		
16	O			
21		C		

V. APPLICATIONS

Shielding

Since $(n,2n)$ cross sections are an indirect measure of the energy distribution of inelastically scattered neutrons, columns (5) and (6) of Table III, can be taken as a rough index of inelastic scattering efficiency (aside from the factor πr^2). Phosphorous seems to be especially efficient, while nitrogen and potassium are hopelessly inefficient. A useful method of shielding neutrons of energies above 3 Mev might be based on the large (n,p) cross section in sulfur.

Selection of Threshold Detectors

Since almost all high energy neutron threshold detectors employ $(n,2n)$ reactions, Tables Ia and III can be considered as a detailed study of threshold detectors giving suitable chemical compounds to use (often not a simple matter), the half-life of the activity, the other activities that must be corrected for, the threshold, and the cross section. An approximate absolute calibration can be arrived at by correcting for self-absorption by Eq. (4) with an approximate value for "α" obtainable from Fig. 2.

Table IV lists some favorable threshold detectors (i.e., those which have no seriously interfering activities) by threshold and activity half-life. It may be noted from Table IV that, for most purposes, several other detectors are more favorable than is copper, which has been used quite commonly.

The author would like to acknowledge the extensive help of Professor E. Creutz and L. Wolfenstein in carrying out these experiments and calculations, and the cooperation of Professor A. J. Allen and the operating personnel of the University of Pittsburgh cyclotron in the use of that instrument.