

FIG. 1. Magnetization *vs.* field. (1) Orbit radius greater than the specimen dimension (perturbation calculation of plane wave functions). (2) Orbit radius smaller than the specimen dimensions.

Waals gas) to a nondiamagnetic state. This behavior is provisionally identified with the perfectly diamagnetic behavior of a superconductor, which will hold for all specimens of reasonable macroscopic dimensions. If the validity of the above idealized analysis is admitted, the remaining problem is not to explain why some materials *are* perfectly diamagnetic, but rather to explain why most real materials and specimen sizes are *not* perfectly diamagnetic, except a few known superconductors over a very restricted range of field and temperature.

It follows from the above calculation that for fields which are not too large an applied field will be almost completely expelled from the box, leaving some small residual effective field ϵ . But since H must be continuous at the surface of the box, there must be a small layer there where the field rises rapidly from the small internal effective value ϵ to the external applied value. We can take as our second problem the limiting case in which the field is expelled completely except for a thin magnetic layer to provide continuity of H . The solution of this problem is given by curve (2) in Fig. 2. The wave functions are of two classes: (1) plane waves slightly modulated in amplitude and phase at the boundary, and (2) unidirectional waves which creep along the boundary. These unidirectional waves give the major contribution to the curve (2) of Fig. 2. There are no creeping quantum states if $H < \hbar c/k\epsilon\lambda^3$. λ is the penetration depth.

Evidently a completely consistent solution of the problem under

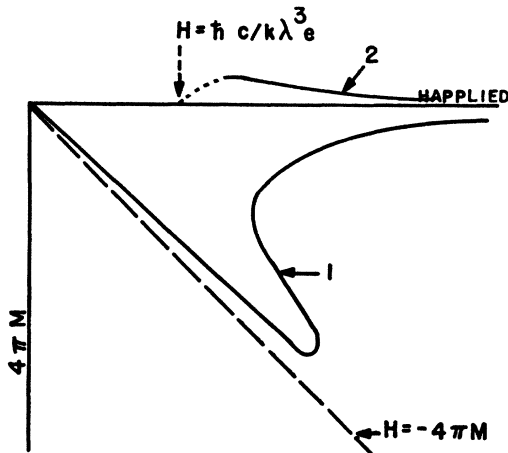


FIG. 2. Magnetization *vs.* applied field (1) H_{interior} uniform. (2) $H_{\text{interior}} = H_{\text{applied}} \exp(-y/\lambda)$, $y=0$ on surface.

discussion would be intermediate between the two cases above, but it seems evident that strong diamagnetism for not too large fields will certainly dominate the situation. Self-consistency can be achieved in another way. One compares $\mathbf{J} = -(c/4\pi\lambda^2)\mathbf{A}$ (London's equation) where \mathbf{A} is the given vector potential with the quantum mechanical current

$$\mathbf{J} = (\hbar e/2mi)(\psi^*\nabla\psi - \psi\nabla\psi^*) - (e^2/mc)\mathbf{A}\psi\psi^*,$$

using the ψ 's calculated in the presence of this \mathbf{A} . If the criterion for the absence of the creeping states is met, then the quantum mechanical current, \mathbf{J} is of order $-\mathbf{A}$. This supports the original conjecture of London.³

It is a pleasure to acknowledge the benefit of numerous discussions of this problem with R. D. Meyers and the members of the cryogenics group of the Naval Research Laboratory.

¹ L. Landau, *Z. Physik* **64**, 629 (1930).

² C. G. Darwin, *Proc. Camb. Phil. Soc.* **27**, 37 (1931).

³ F. London, *Nature* **140**, 793, 834 (1937).

On the Determination of Reduced Widths from the One-Level Dispersion Formula

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THE intention of this note is to point out that a common approximation used in applying the one-level nuclear dispersion formula to the determination of reduced level widths may be unjustified in the case of broad levels, which are numerous in light nuclei.

The one-level dispersion formula as given by Eq. (57) of the paper of Wigner and Eisenbud¹ for the λ th energy level is

$$\sigma^{ss'} = \frac{\pi}{(2j_s+1)\hbar_s^2} \sum_{ii'} \frac{(2J+1)\Gamma_{\lambda si}\Gamma_{\lambda s'i'}}{(E_\lambda + \Delta_\lambda - E)^2 + \frac{1}{4}\Gamma_\lambda^2}, \quad (1)$$

and according to Eqs. (58a) and (58b), in their notation,

$$\begin{aligned} \frac{1}{2}\Gamma_{\lambda si} &= (B_{si}\gamma_{\lambda si})^2/(1+C_{si}^2), & \Gamma_\lambda &= \sum_{si} \Gamma_{\lambda si}; \\ \Delta_\lambda &= \sum \Delta_{\lambda si}; & \Delta_{\lambda si} &= \frac{1}{2}C_{si}\Gamma_{\lambda si}; \end{aligned}$$

i is the relative orbital momentum of the pair of particles s ; B and C are related to the value and derivative of the extra nuclear wave functions at the nuclear radius a_s . The common approximation in applying these formulas to the determination of reduced level widths, $\gamma_{\lambda si}^2$, is to neglect the variation of the level shift, Δ_λ , with respect to the energy of the incident particle, with the result that Γ_λ is interpreted as the observed level width. It is not hard to calculate this variation, and it will be shown to be important in some cases. In the notation of Yost, Wheeler, and Breit² the regular solution to the wave equation in the region external to the nucleus is $F(x)$ and the irregular solution is $G(x)$, the argument x being equal to kr ; then

$$\begin{aligned} \Delta_{\lambda si} &= -(\gamma_{\lambda si}^2/a_s)(g_{si}+i), & \text{where} \\ g_{si} &= [d \ln(F_{si}^2 + G_{si}^2)/d \ln x]_{x=ka_s}. \end{aligned} \quad (2)$$

By expanding Δ_λ linearly with respect to energy about the resonance, usually a good approximation, the one-level formula may be written as

$$\sigma^{ss'} = \frac{\pi}{(2j_s+1)\hbar_s^2} \sum_{ii'} \frac{(2J+1)\Gamma'_{\lambda si}\Gamma'_{\lambda s'i'}}{(E-E_r)^2 + \frac{1}{4}\Gamma_\lambda'^2}, \quad (3)$$

where the primed widths, which may be called the observed widths, are

$$\Gamma'_{\lambda si} = \Gamma_{\lambda si} [1 + \sum_{si} (\gamma_{\lambda si}^2/a_s)(dg_{si}/dE)E - E_r]^{-1}; \quad (4)$$

E_r is the observed resonance energy and satisfies the equation

$$E_\lambda + \Delta_\lambda(E_r) - E_r = 0. \quad (5)$$

The same consideration applies to the dispersion theory of Feshbach, Peaslee, and Weisskopf.³ All of the Γ 's in expressions

(42) and (43) of reference 3 should be primed; in the notation of their paper, with the exception of g which is defined in Eq. (2) above; and with F^2+G^2 now equal to the $|v|^2$ of their paper, we have

$$\begin{aligned}\Gamma_{\pi}^{\prime(r)} &= -(2x/|v|^2)[(df_0/dE) - (dg/dE)]^{-1}E = E_r, \\ \Gamma_a^{\prime(r)} &= -2h[(df_0/dE) - (dg/dE)]^{-1}E = E_r;\end{aligned}\quad (6)$$

$\alpha^{-1}f_0$ is the real part of the logarithmic derivative of the wave function at the nuclear surface. The unprimed Γ 's of expressions (41) and (12) remain unchanged.

An example where this correction is important is the 456-kev resonance in the $C^{12}(p\gamma)$ reaction. The quantity dg/dE , which can be obtained from Breit's tables,^{2,4} is here actually larger than df_0/dE ; the sign of dg/dE is such that Γ is about $2\frac{1}{2}$ times larger than the observed width, Γ' . With this correction the reduced width, γ^2 , becomes about equal to \hbar^2/Ma , M is the reduced mass (the exact value of γ^2 is sensitive to the value of the nuclear radius that is used). This reduced width \hbar^2/Ma is equal to that given by Eq. (64) of reference 1 for a square well interaction and likewise the limiting value given by Eq. (44) of this same reference. This is an indication that the s -wave $C^{12}+p$ interaction can be treated as a simple one-body problem. The reduced width of the low energy neutron s -wave scattering by C^{12} is also equally large. This width is determined by fitting a one-level dispersion curve, including s -potential scattering, to the neutron scattering data up to 2 Mev, where scattering from higher partial waves becomes important. The scattering length in the low energy region is positive, and the bound state associated with it is the 3.10-Mev level of C^{13} . The correspondence of this bound level with the 456-kev resonance level of N^{13} has been pointed out.⁵ In showing this correspondence it is necessary to consider this correction given by Eq. (4). A detailed report on the s -wave proton and neutron interaction with C^{12} will be forthcoming.

In applying the one-level dispersion theory at energies considerably off resonance, formulas (1) above or (38) and (39) of reference 3 should be used rather than the approximate expression (3); that is, f_0 may be a linear function of E (basic assumption of the one-level formula) over a wider range of energies than g .

In the case of s -wave neutron reactions, g is zero so that there is no correction. For higher partial waves dg/dE can be calculated from the penetrability factors $|v|^2$ given by Eqs. (45a) of reference 3; in the case of p neutrons $E dg/dE = (ka)^2[1 + (ka)^2]^{-2}$. For example, if the resonance at 1.2 Mev in the scattering of neutrons by He^4 is p -wave, then $dg/dE = 0.16$ Mev⁻¹. Supposing $\gamma^2 = \hbar^2/Ma$, a being taken as $2.5 \cdot 10^{-13}$ cm, then the observed width at resonance would be $\Gamma' = 0.76$ Mev. Were γ^2 infinite, the observed width would be only 1.2 Mev.

An exact criterion for the application of this correction cannot be given. It is probably not negligible for resonances for which $\gamma^2 \gtrsim \hbar^2/10Ma$ (s -neutron resonances, of course, being excepted).

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¹ E. P. Wigner and L. Eisenbud, Phys. Rev. 72, 29 (1947).

² Vost, Wheeler, and Breit, Phys. Rev. 49, 174 (1936).

³ Feshbach, Peaslee, and Weisskopf, Phys. Rev. 71, 145 (1947).

⁴ Bloch, Hull, Broyles, Bouricius, Freeman, and Breit, *Coulomb Functions for Reactions of Protons and Alpha-Particle with the Lighter Nuclei*, Yale University (unpublished).

⁵ R. G. Thomas, Phys. Rev. 80, 136, 138 (1950), and J. B. Ehrman, private communication.

The Hall Coefficient of Semiconductors

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IN a recent letter Johnson and Lark-Horowitz¹ have given an expression for the Hall coefficient of a semi-conductor which takes into account both thermal and impurity scattering of electrons in the conduction band. This theory is based on the following assumptions. (1) The mean free path for thermal scattering,

l_1 , is independent of the velocity of the electrons, and the mean free path for impurity scattering, l_2 , is proportional to the fourth power of the velocity. The actual mean free path l is thus given by

$$l = l_1 l_2 / (l_1 + l_2). \quad (1)$$

(2) It is assumed that the resistivity ρ can be expressed as the sum of the resistivity ρ_T due to thermal scattering alone plus the resistivity ρ_I due to impurity scattering alone; i.e.,

$$\rho = \rho_T + \rho_I. \quad (2)$$

The Hall coefficient R is then given by the equation

$$R/(1/ne) = (\sqrt{\pi}/48)(\rho/\rho_I)^2 \int_0^\infty x^{3/2} e^{-x} dx / (1 + \theta x^2)^2, \quad (3)$$

where, as a result of Eq. (2), θ is set equal to $\frac{1}{6}[(\rho/\rho_I) - 1]$. This theory, however, appears to be not entirely satisfactory, for assumption (2) is not consistent with Eq. (1), which leads directly to Eq. (3). If the resistivity is determined from Eq. (1), it is easy to show that the Hall coefficient can be expressed as a function of ρ_I/ρ by Eq. (3) and the equation,

$$\rho_I/\rho = \frac{1}{6} \int_0^\infty x^3 e^{-x} dx / (1 + \theta x^2), \quad (4)$$

in terms of the parameter $\theta = \rho_T/6\rho_I$, which is now no longer equal to $\frac{1}{6}[(\rho/\rho_I) - 1]$.

Figure (1) shows $R/(1/ne)$ as a function of ρ_I/ρ as determined

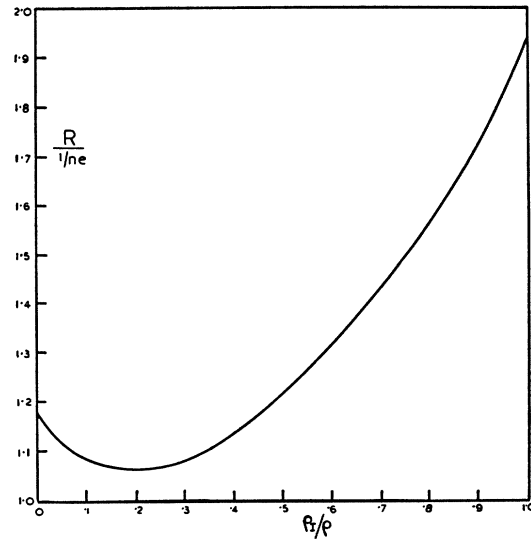


FIG. 1. Variation of the Hall coefficient R with the ratio of the resistance due to impurity scattering to total resistance; n is the number of electrons per cc.

from Eqs. (3) and (4). The integral of Eq. (4) can be expressed in finite terms, but Eq. (3) must be evaluated numerically. It will be noticed that, except at the end points, the values of $R/(1/ne)$ lie higher than those given by Johnson and Lark-Horowitz [Fig. 1, of reference 1]. A consequence of this is that the mobilities as given by these authors in a later letter,² should be generally reduced; e.g., by a factor of 1.8 at $\rho_I/\rho = 0.2$ and by a factor of 2.0 at $\rho_I/\rho = 0.5$. The discrepancy between the original values of the mobilities and the higher values found by Pearson, *et al.*,³ would not, therefore, appear to be removed in the manner suggested.

From Eq. (4) it is easy to find the ratio $\rho/(\rho_T + \rho_I)$. It reaches a maximum value of approximately 1.43 at ρ_I/ρ in the neighborhood of 0.30.

¹ V. A. Johnson and K. Lark-Horowitz, Phys. Rev. 79, 176 (1950).

² V. A. Johnson and K. Lark-Horowitz, Phys. Rev. 79, 409 (1950).

³ G. L. Pearson, Phys. Rev. 76, 179 (1949).