needed in the discussion which follows. The reactions of column 1 of Table I should be numered in sequence. The "as measured" Q values in column 2 are the input values in Mev from which the mass equivalents in milli-mass units are secured by multiplying by the usual conversion factor 1.074.

The mass difference $(2\mathrm{D}^2-\mathrm{He}^4)$ can be obtained from disintegration data alone by the combination in "cycles" of Q values from reactions from which all atoms other than D and He have been eliminated by subtraction of identities.[5] One such cycle utilizing n, H^1, D^2, He^3, He^4, Li^6, and Li^7 yields

$$\begin{aligned}(2\mathrm{D}^2-\mathrm{He}^4)&=0.025522\pm0.000045\mathrm{MU}\\&=(Q_1+Q_4+Q_{10}+Q_{13}-Q_9)\times1.074.\end{aligned}\tag{a}$$

A second cycle involving H^1, D^2, He^4, Li^7, Be^8, and Be^9 yields

$$\begin{aligned}(2\mathrm{D}^2-\mathrm{He}^4)&=0.025536\pm0.000039\mathrm{MU}\\&=(Q_{13}+Q_{21}-Q_{16}-Q_{18})\times1.074.\end{aligned}\tag{b}$$

A third cycle which includes H^1, D^2, He^4, Li^6, Be^8, and Be^9 yields

$$\begin{aligned}(2\mathrm{D}^2-\mathrm{He}^4)&=0.025431\pm0.000042\mathrm{MU}\\&=(Q_{11}+Q_{19}-Q_{16}-Q_{18})\times1.074.\end{aligned}\tag{c}$$

These three cycles are sufficient for the argument. If (a), (b), and (c) are combined, the result would be represented approximately by 0.02550±0.000023.

In the case of the four measurements of the $(2\mathrm{D}^2-\mathrm{He}^4)$ doublet by four different instruments, the values (in MU) are 0.02551 ±0.00008;[6] 0.02561±0.00004;[2] 0.025604±0.000009;[3] 0.025612 ±0.000008.[4] The recent measurements of Nier and Ewald may be weighted equally to give

$$0.025608\pm0.000006=(2\mathrm{D}^2-\mathrm{He}^4).\tag{d}$$

The differences between this value and those given by Eqs. (a), (b), and (c) are 12 to 30 times the probable errors of the Ewald-Nier measurement and 2 to 4 times the probable errors of the combined Q values of Eqs. (a), (b), and (c). Q_{11} and Q_{13} naturally come under suspicion as they represent the only Q values obtained from range-energy measurements, while all other Q values are from electrostatic or magnetic deflection studies.[1] Also, Q_{11} should equal $Q_{10}+Q_{13}$; and the fact that the probable errors do not overlap, $Q_{11}-Q_{10}-Q_{13}>\epsilon_{11}+\epsilon_{10+13}$, has indicated that Q_{13} or Q_{11} or both may be away from their true values by more than the assigned probable errors. Equations (a), (b), and (c), considered with the Ewald-Nier value for $(2\mathrm{D}^2-\mathrm{He}^4)$, Eq. (d), furnish a strong indication that Q_{13} and Q_{11} are *both* too small, where hitherto only one or the other of these values or their difference has been under suspicion. An error of the magnitude indicated above in the mass spectrograph result of Ewald would correspond to an error in the doublet separation of $\frac{1}{2}$ the width of a line or an error in the dispersion of $\sim\frac{1}{4}$ percent. An examination of Ewald's remarkable spectra shows how remote is the possibility of any error of that magnitude. Equivalent possibilities in Nier's excellent work are equally unlikely.

The magnitudes and assigned probable errors of the Q values other than Q_{11} and Q_{13} make it appear that the difficulties lie with Q_{11} and Q_{13} in the disagreement between the mass spectrograph and the disintegration mass scales. The source of the possible errors in Q_{11} and Q_{13} might be in the range energy curve.

However, as is well known, the measurement of Q_{13} for the reaction $\mathrm{Li}^7(p,\alpha)\mathrm{He}^4$ is essentially a measurement of the small range difference between the alpha particles from the Li^7 reaction and the alpha-particles of ThC′. The energy of the latter has been measured with very high precision by Briggs,[7] and Rosenblum and Dupouy.[8] Also, indirect checks of several kinds exist for the ThC′ measurements. The only reasonable change in Q_{11} from the ThC′ experiments is associated with the change in $2e/M$ for alpha-particles to conform to the weight of helium given below. There is no significant change in Q_{13} from this consideration. A detailed survey emphasizes that the $\mathrm{Li}^7(p\alpha)\mathrm{He}^4$ and $\mathrm{Li}^6(d\alpha)\mathrm{He}^4$ reaction Q values should be brought to the same high order of accuracy as is obtained by deflection methods for the other Q values listed in Table I of reference 1.

A mass scale for H^1, D^2, He^4, C^{12}, and O^{16} obtained chiefly from doublet measurements is appended. The input data also includes the important results of Bell and Elliott[9,10] for Q_1 for $\mathrm{H}^1(n\gamma)\mathrm{D}^2$, and Taschek, *et al.*,[11] and others[12] for $n-\mathrm{H}^1$ combined to give $2\mathrm{H}^1-\mathrm{D}^2=1.555\pm0.006\times10^{-3}$ MU. This is averaged with Roberts and Nier's[13] measurement $1.549\pm0.006\times10^{-3}$ MU.

The baic mass-spectrographic doublets are as follows:

$$\begin{aligned}(\mathrm{H}^1)_2-\mathrm{D}^2&=\ 15.52\pm0.04\times10^{-4}\ \mathrm{MU}^{9,11,13}\\(\mathrm{D}^2)_3-\tfrac{1}{2}\mathrm{C}^{12}&=422.28\pm0.19\times10^{-4}\ \mathrm{MU}^{10}\\\mathrm{C}^{12}+(\mathrm{H}^1)_4-0&=364.5\ \ \pm0.22\times10^{-4}\ \mathrm{MU}^{13}.\end{aligned}$$

The corresponding mass scale is as follows:

$$\begin{array}{ccc}n=1.0089785; & \mathrm{H}^1=1.0081386; & \mathrm{D}^2=2.0147252;\\ \quad\ 38 & \qquad 32 & \qquad\qquad 57\end{array}$$

$$\begin{array}{cc}\mathrm{He}^4=4.003842; & \mathrm{C}^{12}=12.003895.\\ \qquad 13 & \qquad\qquad 19\end{array}$$

A complete table of isotopic weights derived from both reaction Q-values and mass spectrograph doublets is in preparation for the Nuclear Science Series of the National Research Council.

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The Perfect Diamagnetism of Free Electrons with Application to Superconductivity

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CALCULATIONS have been made on the Fermi-Dirac statistics of free charged particles in a uniform magnetic field in a *finite* box, and of free particles in a magnetic field which decays exponentially from the applied field at the surface of the box. Particular attention is paid to using $\psi=0$ rather than ψ periodic as the boundary condition on the walls of the box.

Considering the first problem above, it follows from the rigorous application of this boundary condition to a finite cylinder of dimensions R, L_z, with H along the axis, that the total number of particles and the energy $U-MH$ at absolute zero are, for an orbit radius smaller than the dimensions of specimen: $\alpha=kc\hbar/eH$,

$$\begin{aligned}N&=(Vk^3/6\pi^2)[1+(3\pi/8R)\alpha+(2/5R^2)\alpha^2]\\U-MH&=(\hbar^2Vk^5/20m\pi^2)[1+(5\pi/12R)\alpha+(10/21R^2)\alpha^2].\end{aligned}$$

Here, $k=(2m\zeta)^{\frac{1}{2}}/\hbar$, where ζ is the energy of the highest occupied state.

The last two terms in brackets correspond to orbits whose centers are just outside (<orbit radius) the specimen[1] (with periodic boundary conditions, these would be counted in the adjoining "big cell"), and orbits which encircle the origin.[2] These extra terms vanish in the limit $V\rightarrow\infty$, N/V finite, so that they depend on the size and the shape of the specimen. For macroscopic specimens they still make a large contribution to the magnetic moment, shown in Fig. 1.

If one assumes that Fig. 1 gives M as a function of $H_{\mathrm{effective}}$ and uses the relation $H_{\mathrm{eff}}=H_{\mathrm{appl}}+4\pi M(H_{\mathrm{eff}})$ to obtain $H_{\mathrm{appl}}=f(H_{\mathrm{eff}})$, and then $M=f(H_{\mathrm{appl}})$, one obtains Fig. 2, curve (1), which indicates almost perfect diamagnetism up to a critical field and then a triple-valued transition (similar to the condensation of a Van der

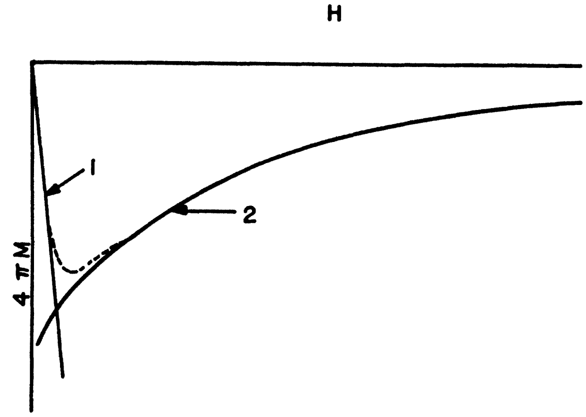

FIG. 1. Magnetization *vs*. field. (1) Orbit radius greater than the specimen dimension (perturbation calculation of plane wave functions). (2) Orbit radius smaller than the specimen dimensions.

Waals gas) to a nondiamagnetic state. This behavior is provisionally identified with the perfectly diamagnetic behavior of a superconductor, which will hold for all specimens of reasonable macroscopic dimensions. If the validity of the above idealized analysis is admitted, the remaining problem is not to explain why some materials *are* perfectly diamagnetic, but rather to explain why most real materials and specimen sizes are *not* perfectly diamagnetic, except a few known superconductors over a very restricted range of field and temperature.

It follows from the above calculation that for fields which are not too large an applied field will be almost completely expelled from the box, leaving some small residual effective field ϵ. But since H must be continuous at the surface of the box, there must be a small layer there where the field rises rapidly from the small internal effective value ϵ to the external applied value. We can take as our second problem the limiting case in which the field is expelled completely except for a thin magnetic layer to provide continuity of H. The solution of this problem is given by curve (2) in Fig. 2. The wave functions are of two classes: (1) plane waves slightly modulated in amplitude and phase at the boundary, and (2) unidirectional waves which creep along the boundary. These unidirectional waves give the major contribution to the curve (2) of Fig. 2. There are no creeping quantum states if $H<\hbar c/ke\lambda^3$. λ is the penetration depth.

Evidently a completely consistent solution of the problem under discussion would be intermediate between the two cases above, but it seems evident that strong diamagnetism for not too large fields will certainly dominate the situation. Self-consistency can be achieved in another way. One compares $\mathbf{J}=-(c/4\pi\lambda^2)\mathbf{A}$ (London's equation) where $\mathbf{A}$ is the given vector potential with the quantum mechanical current

$$\mathbf{J}=(\hbar e/2mi)(\psi^*\nabla\psi-\psi\nabla\psi^*)-(e^2/mc)\mathbf{A}\psi\psi^*,$$

using the ψ's calculated in the presence of this $\mathbf{A}$. If the criterion for the absence of the creeping states is met, then the quantum mechanical current, $\mathbf{J}$ is of order $-\mathbf{A}$. This supports the original conjecture of London.[3]

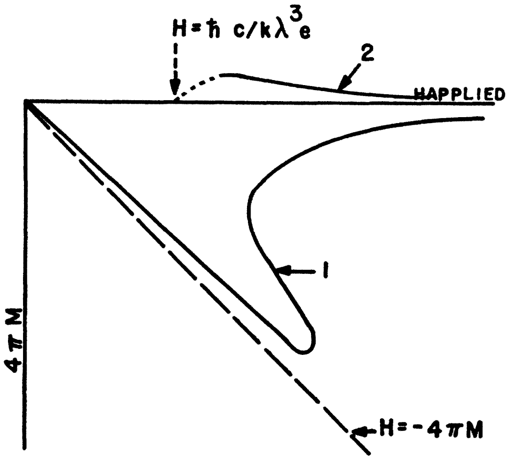

FIG. 2. Magnetization *vs*. applied field (1) H_{interior} uniform. (2) $H_{\text{interior}}=H_{\text{applied}}\exp(-y/\lambda)$, $y=0$ on surface.

It is a pleasure to acknowledge the benefit of numerous discussions of this problem with R. D. Meyers and the members of the cryogenics group of the Naval Research Laboratory.

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On the Determination of Reduced Widths from the One-Level Dispersion Formula

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THE intention of this note is to point out that a common approximation used in applying the one-level nuclear dispersion formula to the determination of reduced level widths may be unjustified in the case of broad levels, which are numerous in light nuclei.

The one-level dispersion formula as given by Eq. (57) of the paper of Wigner and Eisenbud[1] for the λth energy level is

$$\sigma^{ss'}=\frac{\pi}{(2js+1)k_s^2}\Sigma_{ii'}\frac{(2J+1)\Gamma_{\lambda si}\Gamma_{\lambda s'i'}}{(E_\lambda+\Delta_\lambda-E)^2+\frac{1}{4}\Gamma_\lambda^2},\tag{1}$$

and according to Eqs. (58a) and (58b), in their notation,

$$\tfrac{1}{2}\Gamma_{\lambda si}=(B_{si}\gamma_{\lambda si})^2/(1+C_{si}^2),\quad \Gamma_\lambda=\Sigma_{si}\,\Gamma_{\lambda si};$$
$$\Delta_\lambda=\Sigma\Delta_{\lambda si};\quad \Delta_{\lambda si}=\tfrac{1}{2}C_{si}\Gamma_{\lambda si};$$

i is the relative orbital momentum of the pair of particles s; B and C are related to the value and derivative of the extra nuclear wave functions at the nuclear radius a_s. The common approximation in applying these formulas to the determination of reduced level widths, $\gamma_{\lambda si}^2$, is to neglect the variation of the level shift, Δ_λ, with respect to the energy of the incident particle, with the result that Γ_λ is interpreted as the observed level width. It is not hard to calculate this variation, and it will be shown to be important in some cases. In the notation of Yost, Wheeler, and Breit[2] the regular solution to the wave equation in the region external to the nucleus is $F(x)$ and the irregular solution is $G(x)$, the argument x being equal to kr; then

$$\Delta_{\lambda si}=-(\gamma_{\lambda si}^2/a_s)(g_{si}+i),\quad \text{where}\tag{2}$$
$$g_{si}=[d\ln(F_{si}^2+G_{si}^2)^{\frac{1}{2}}/d\ln x]_{x=ka_s}.$$

By expanding Δ_λ linearly with respect to energy about the resonance, usually a good approximation, the one-level formula may be written as

$$\sigma^{ss'}=\frac{\pi}{(2js+1)k_s^2}\Sigma_{ii'}\frac{(2J+1)\Gamma'_{\lambda si}\Gamma'_{\lambda s'i'}}{(E-E_r)^2+\frac{1}{4}\Gamma_\lambda'^2},\tag{3}$$

where the primed widths, which may be called the observed widths, are

$$\Gamma'_{\lambda si}=\Gamma_{\lambda si}[1+\Sigma_{si}\,(\gamma_{\lambda si}^2/a_s)(dg_{si}/dE)_{E=E_r}]^{-1};\tag{4}$$

E_r is the observed resonance energy and satisfies the equation

$$E_\lambda+\Delta_\lambda(E_r)-E_r=0.\tag{5}$$

The same consideration applies to the dispersion theory of Feshbach, Peaslee, and Weisskopf.[3] All of the Γ's in expressions