because of the odd nature of the last integrand; the ambiguity arises in the exchange of orders of integration, indicated by $(=)$. It may then be suggested that this procedure, if generally followed, would eliminate all quadratic divergences from quantum electrodynamics, rendering them not only finite but zero. This rule can be criticized in the same way as above; namely, it depends [like (9a)] on a particular algebraic property of the divergent function, and would almost certainly not be true if a physically correct convergent function stood in its place. That is, a convergent theory would be expected to replace divergences by finite, but nonzero quantities; and if numerical results are obtained with the present theory by setting divergences equal to zero, then in the event of a convergent theory the calculation must be repeated to rediscover the originally divergent terms. Finally, it is clear that the situation with (14) is even less satisfactory than with (9a); for (9a) holds for all divergent functions (but presumably not for convergent functions, if they existed), while (14) must be specially modified to suit each type of divergent function. For example, the prescription (14) for use with $\delta(k^2)$ does not hold for the corresponding Feynman function $1/k^2$; in this case one has

$$\int d^4k\frac{1}{k^2}=-\frac{i}{2}\int d^4k\int_{-\infty}^{\infty}\exp(i\alpha k^2)\frac{\alpha}{|\alpha|}$$

$$(=)-\frac{i}{2}\int_{-\infty}^{\infty}\frac{\alpha}{|\alpha|}d\alpha\int d^4k\exp(i\alpha k^2)=\frac{\pi^2}{2}\int_{-\infty}^{\infty}\frac{d\alpha}{|\alpha|^2} \quad (15)$$

the last expression being quadratically divergent, as expected.

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Formative Time Lags of Spark Breakdown in Air in Uniform Fields at Low Overvoltages*

L. H. FISHER AND B. BEDERSON†
Department of Physics, College of Engineering, New York University, University Heights, New York
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An attempt has been made to establish the region of validity of the streamer and Townsend mechanisms of spark breakdown in air by measurements of the formative time lag of the breakdown process.

Formative time lags were measured in a uniform field for overvoltages of a few percent down to as close to threshold as possible. Such measurements have been carried out as a function of pressure (atmospheric to a few cm of Hg) and plate separation (0.3 to 1.4 cm). For pressures greater than 200 mm Hg, the formative time lags very close to threshold are of the order of 100 μsec and longer. These times are orders of magnitude longer than those previously reported. For all pressures studied, the time lags decrease as the percent overvoltage (percent o.v.) increases, until at about two percent above threshold, the formative times are of the order of 1 μsec. The time lag *vs* percent o.v. curve is independent of pressure from atmospheric down to 200 mm Hg. At a given percent o.v., the time lags increase linearly with plate separation. Changing the approach voltage from two to four kv below breakdown does not affect the results appreciably. The number of initiating electrons at the cathode has been varied by a factor of about seven, and this again does not materially alter the results.

The long time lags and their dependence on pressure and percent o.v. cannot be explained by secondary emission of electrons from the cathode by positive ion bombardment. The proper explanation is the enhancement of field intensified ionization due to field distortion acting in conjunction with a photoelectric secondary process. The experiment demands an extension of the streamer concept of breakdown, and makes very questionable the role of positive ion bombardment of the cathode in spark breakdown for the pressures and plate separations studied. No transition region for change of streamer to Townsend breakdown was found.

I. INTRODUCTION

THE theory of spark breakdown has undergone radical changes in the last ten years.[1-4] Until 1946 it was commonly believed that in air,[5] the Townsend mechanism of breakdown is valid below values of the product of pressure[6] (p) and plate separation (δ) of 200 mm×cm, and that above this value of $p\delta$, the breakdown proceeds by the streamer mechanism. How-

* Supported by the ONR and the Research Corporation. For preliminary reports of this work, see L. H. Fisher and B. Bederson, Brookhaven Gas Discharge Conference, October, 1948; Phys. Rev. **75**, 1324, 1615 (1949); Pittsburgh Gaseous Electronics Conference, November, 1949; Phys. Rev. **78**, 331 (1950).

† Now at Massachusetts Institute of Technology, Cambridge, Massachusetts.

[1] L. B. Loeb, *Fundamental Processes of Electrical Discharge Through Gases* (John Wiley and Sons, Inc., New York, 1939).

[2] J. M. Meek, Phys. Rev. **57**, 722 (1940).

[3] L. B. Loeb and J. M. Meek, *The Mechanism of the Electric Spark* (Stanford University Press, Stanford University, California, 1941).

[4] H. Raether, Arch. Elektrotech. **34**, 49 (1940); Z. Physik **117**, 375, 524 (1941); Ergeb. exakt. Naturw. **22**, 73 (1949).

[5] Unless otherwise noted, all statements in this paper refer to air.

[6] Raether takes the value of this product as 1000 mm×cm, see reference 4.

ever,[7] on the basis of an analysis of Meek's equation for calculating sparking potentials, it was pointed out that the transition of Townsend to streamer mechanism may depend not on the product $p\delta$ but rather on the values of p and δ separately. It appeared impossible to study the transition by means of sparking potentials. A more hopeful approach seemed to lie in the study of formative time lags of spark breakdown.[8,9]

The formative time lag is defined as the time necessary for a potential difference to be maintained across a gap before it breaks down, provided a primary source of ionization is present. From mobility measurements it is known that at the sparking potential the positive ion requires about 18 μsec to cross a one-centimeter gap at atmospheric pressure. In 1936, Schade[10] measured formative time lags for the glow discharge in neon at very low pressures. He found time lags between 10 μsec and 0.1 sec. Schade could not measure times shorter than 10 μsec with his equipment, a circumstance of great importance in the subsequent theoretical development of the field. But even earlier, and continuing up to the present[11] there had been observed a formative time lag of spark breakdown in air near atmospheric pressure so short that the positive ions formed in the gap could not possibly have had time to cross the gap. These times have been found to be of the order of 0.1 μsec, some observers reporting formative times as short as 10^{-3} μsec. Thus, the Townsend theory was shown to be inadequate at pressures near atmospheric and the streamer theory of Loeb and Meek,[2,3] and Raether[4] was developed for this pressure region. The general impression, with the enormous prestige of the Townsend theory and the confirming work of Schade, was that the Townsend theory applies at small values of $p\delta$ (where the time lags were found to be long), and that the streamer theory applies at large $p\delta$ (where the time lags were found to be short).

The work reported here was undertaken to determine the transition region.

II. APPARATUS AND EXPERIMENTAL PROCEDURE

The ionization chamber used in this study was previously employed by Sanders[12] and Posin[13] for measurements of the first Townsend coefficient in air and nitrogen respectively. The chamber was later modified to its present form[14] when it was used to measure sparking potentials in air. The chamber was originally constructed with the aid of a grant in support of Sanders' research by the National Research Council. A description of the chamber may be found in the original papers.[12,14] The levelling and polishing of the electrodes, the measurement of electrode separation, the admitting and drying of the air, and the measurement of pressure were carried out as described previously.[14] All pressures have been corrected to 22°C.

In order to avoid the measurement of statistical time lags, the cathode was illuminated with ultraviolet light from a dc quartz mercury arc. The light was focused by a quartz lens and passed through a quartz window in the side of the chamber. To determine the photoelectric current i_0 emitted at the cathode, the gas amplified current i was measured with a voltage across the gap of about 10 percent below the sparking potential V_s. An electrometer was used to measure the resulting potential drop V across a resistor R inserted in series with the gap. Thus $V=i_0e^{\alpha\delta}R$, where α is the first Townsend coefficient as given by Sanders.[12] Hence the number of electrons n_0 leaving the cathode per second is given by

$$n_0=V/(1.6\times10^{-19}Re^{\alpha\delta}),\qquad(1)$$

where all electrical quantities are in practical units. The irradiation was adjusted so that at least several electrons were emitted each microsecond, and was large enough in nearly all cases to prevent statistical variations in the time lags yet small enough to prevent distortion of the electric field. The ultraviolet lamp was on continuously during a set of measurements. No elaborate provisions were made for stabilizing the dc voltage for the mercury arc, since the experimental data were found to be very insensitive to small changes in n_0.

In order to determine the formative time lags, an approach voltage $V_0(V_0<V_s)$ is applied across the gap. Then, at some time, an additional voltage V_1 (the pulse voltage) is applied, where $V_0+V_1>V_s$. The time that V_0+V_1 must be maintained before the gap breaks down is the formative time lag.

TABLE I. Resistances in megohms and capacity in microfarads (unless otherwise noted.)

R_1	0.7 Ω	R_{10}	20
R_2	0.1	R_{11}	2000 Ω
R_3	1	R_{12}	25 Ω
R_4	20	R_{13}	1
R_5	1	R_{14}	1
R_6	100	R_{15}	1
R_7	1000 Ω	R_{16}	1
R_8	1000 Ω	R_{17}	1
R_9	3300 Ω		
C_1	0.25	C_7	0.01
C_2	0.25	C_8	0.01
C_3	0.25	C_9	50 mmfd
C_4	0.25	C_{10}	16
C_5	0.25	C_{11}	1000 mmfd
C_6	0.25	C_{12}	1000 mmfd

[7] L. H. Fisher, Phys. Rev. **69**, 530 (1946); L. B. Loeb, Proc. Phys. Soc. (London) **60**, 561 (1948).
[8] For a detailed description of the background leading to this research, see L. H. Fisher, Elec. Eng. **69**, 613 (1950).
[9] A somewhat parallel experimental program was undertaken by B. Gänger, Arch. Elektrotech. **39**, 508 (1949).
[10] R. Schade, Z. Physik **104**, 487 (1937).
[11] See for example P. O. Pedersen, Ann. Physik **71**, 317 (1923); W. Rogowski, Arch. Elektrotech. **20**, 99 (1928); H. J. White, Phys. Rev. **49**, 507 (1936); R. R. Wilson, Phys. Rev. **50**, 1082 (1936); M. Newman, Phys. Rev. **52**, 652 (1937); R. C. Fletcher, Phys. Rev. **76**, 1501 (1949). The results of these and other authors have been summarized in reference 1, p. 441 and by R. Strigel, *Elektrische Stossfestigkeit* (Verlag. Julius Springer, Berlin, 1939).
[12] F. H. Sanders, Phys. Rev. **41**, 667 (1932).
[13] D. Q. Posin, Phys. Rev. **50**, 650 (1936).
[14] L. H. Fisher, Phys. Rev. **72**, 423 (1947).

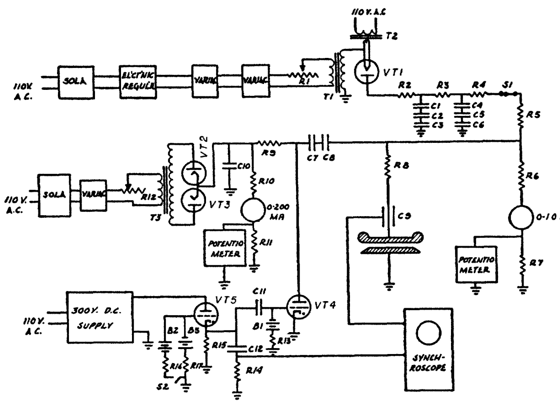

FIG. 1. Circuit diagram. Values of circuit elements are given in Table I.

The circuit diagram of the electronic apparatus is shown in Fig. 1. The values of the circuit elements are given in Table I.

The V_0 power supply is shown in the upper section of the diagram. V_0 is measured by R_6, a bank of five 20-megohm high accuracy wire wound resistors. The absolute value of R_6 is not known, but comparison of sparking potential measurements with those made at Berkeley[14] in the same chamber with precision Taylor resistors indicates that the value of R_6 is known to within one percent. (The present experiment requires relative rather than absolute accuracy.) The ripple in V_0 was measured as 0.1 v at 20 kv. The voltage regulation is satisfactory, the voltage remaining fixed to within several volts over many minutes.

The V_1 power supply (shown in the center section of Fig. 1) is capable of delivering from 0 to 5 kv, with a maximum ripple of about 0.2 v. The trigger circuit is shown in the bottom section of the diagram.

The delay between the application of the sweep trigger pulse and the start of the sweep is about 0.1 μsec. When VT_5 fires, VT_4 also fires forcing the potential of the high voltage electrode of the spark gap to fall by an amount equal to the value of V_1 minus the voltage drop in VT_4 while firing. The voltage across the gap becomes essentially equal to V_0+V_1 in a time determined by the time constant of R_8 and the gap capacity. The capacity of the gap (with the low voltage electrode connected to the chamber) has been measured as 190 mmfd at $\delta=1$ cm. Thus, the time constant for the rise of the pulse (assuming VT_4 breaks down instantaneously) is 0.19 μsec; with $\delta=0.3$ cm (the smallest electrode separation studied) the time constant is 0.4 μsec. Actually, the time of breakdown of VT_4 is of the order of 0.5 μsec, so that the time for the pulse to reach its full value across the electrodes ranges from about 0.6 to 1.0 μsec, depending on electrode separation. Because of the long times observed in the experiment this rise time can be neglected. The time constant of the pulse decay circuit is 0.2 sec. Since the present equipment was not used to measure times longer than 100 μsec, the pulse decay time is unimportant.

A signal from the pulse across the chamber is picked up on the vertical deflection plates of the synchroscope. The delay of several tenths of a microsecond in the breakdown of VT_4 is always sufficient to make the start of the pulse visible. When the spark gap breaks down, a sharp break is seen on the synchroscope trace. Normally, the spark signal has too high an amplitude to be seen. A typical trace is shown in Fig. 2. The dotted part of the signal is due to the spark current and can be seen only when V_s is small.

The synchroscope has four sweeps; by using the various sweeps, times from about 0.5 μsec to 100 μsec can be measured. The sweeps were calibrated against a crystal oscillator.

After the chamber was filled with air, approximately fifty sparks were passed with the V_0 supply in order to season the plates.[8,14] After these sparks passed, V_s assumed a value which at any given time was very definite, but which increased slightly, perhaps by a few tenths of one percent, as the measurements were carried out. This small gradual rise is probably due to the warming up of R_6. At any given time, V_s was reproducible to within 2 or 3 v.

After determination of V_s, a value of V_0 exactly 2 kv below V_s was applied. As V_s changed, V_0 was adjusted so as to maintain V_s-V_0 at 2 kv. (This procedure was justified by the consistent results obtained.) V_1 was then applied, and the time lag was read by visual observation.

The overvoltage ΔV is defined by $\Delta V=V_0+V_1-V_s-V'$, where V' is a correction due to the loss of part of V_1 in the circuit. The percent o.v. is defined as $100\,\Delta V/V_s$.

At a selected value of ΔV and p, about 10 measurements of the formative time lag were made. These measurements constitute a run. Runs were made for various percent o.v. from about two to as close to zero as possible. The entire procedure thus far described constitutes a series, and such series were carried out at various pressures and plate distances. Measurements were then carried out to study the effect of increasing the ultraviolet illumination; the effect of changing V_0 such that V_s-V_0 was 4 kv was also determined.

Before each measurement in a run, both V_0 and V_1 were checked by potentiometer. V_s was determined at the start of a run, several times during the run, and at the end of the run. In calculating the data, it was

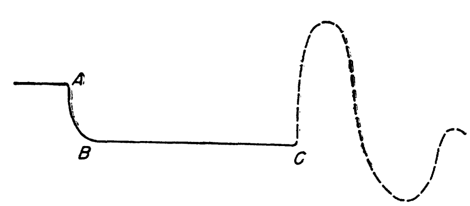

FIG. 2. Typical synchroscope trace. A—Start of pulse; AB—pulse rise; C—beginning of spark; BC—time lag in inches.

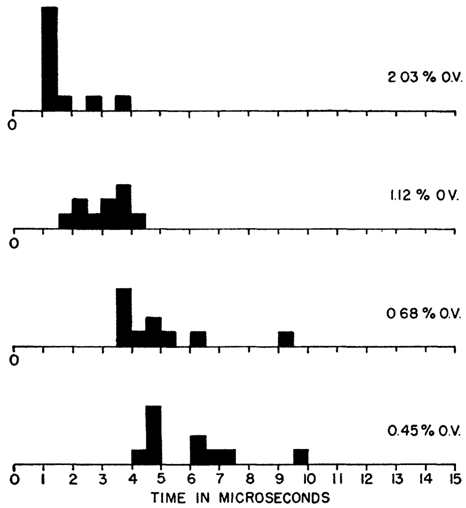

FIG. 3. Typical time lag distribution for four runs. $p=532$ mm Hg, $\delta=1$ cm. One square block represents one measurement.

assumed that if V_s increased, it increased linearly with the number of sparks passed.

Correction for the losses in V_1 due to the drop in VT_4 and to capacity division was made by assuming that for a given V_0, the lowest value of V_1 which gives consistent failures corresponds to $\Delta V=0$. The value of V' thus obtained agrees to within a few percent with the losses as calculated from the known steady-state voltage drop in VT_4 and the measured capacities of the circuit.

III. EXPERIMENTAL RESULTS

All time lags reported represent the average of the individual measurements of a run. The minimum time lag in a run was usually about half of the average time lag, while the maximum time lag in a run was about twice the average time lag. The spread in the time lags in a run increases approximately linearly with the average time lag of the run. Figure 3 shows a typical distribution for four runs at various percent o.v. When the average time lag is of the order of tens of microseconds, the spread is as large as the time lag itself. However, in not a single case was an abnormally short time lag observed. That is, no single measurement gave a time lag which was less than about 40 percent of the average time lag. Thus the measurements represent formative time lags. The fluctuations of the formative time lags cannot be explained by the lack of initiating primary electrons. A part, but not all, of the fluctuations are due to the electronic errors. The other part of the fluctuation is probably inherent in the statistical nature of the spark.

The time lags as a function of percent o.v. for $\delta=1$ cm are plotted in Fig. 4 for four values of the pressure. The term "low illumination" signifies that the ultraviolet light used to illuminate the cathode was filtered through several thicknesses of copper screen to reduce its intensity. The primary current obtained when the screens were used varied from about 1.3 to about 6 electrons/μsec. The term "2000-volt pulse" signifies that V_0 was set at exactly 2 kv below V_s. The principal feature of Fig. 4 is that for all values of the pressures studied, the time lags for very low percent o.v. are quite long and indeed may increase without limit as the percent o.v. approaches zero. As the percent o.v. is increased, the average time lags decrease extremely rapidly, and at about two percent o.v., the time lags are of the order of one microsecond, in agreement with the results of previous investigators. The measurements below two percent o.v. represent essentially a previously unexplored region.

An additional feature of the curve is the lack of dependence of the time lags on pressure to within the experimental accuracy of the apparatus.[15] For the four values of pressure for which the lags are plotted in Fig. 4, the time lags for a given percent o.v. are approximately the same. If pressures below several hundred mm Hg were included in the graph, the time lags would follow the curve of Fig. 4 until the percent o.v. reaches

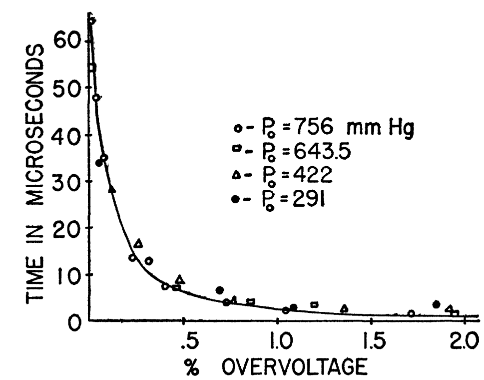

FIG. 4. Time lag *vs* percent o.v., $\delta=1$ cm. Low illumination, 2000-volt pulse

a few tenths of one percent. As the percent o.v. is decreased below this value, the time lags do not increase as rapidly as the curve in Fig. 4. The departure from the curve at small percent o.v. becomes more pronounced as the pressure is lowered. These shorter time lags for low pressures at overvoltages of a fraction of a percent may be instrumental. (There is an error in V_1 of several volts and for low pressures this error becomes an appreciable percentage of V_s, while for high pressures this error is only a few hundredths of a percent of V_s.)

Figure 5 contains the data of Fig. 4 with the addition of data for $\delta=1.4$, 0.6, and 0.3 cm. Here again, there is no dependence of the time lags on pressure. It is seen, however, that the average time lags for a given percent o.v. increase with increasing gap separation. Figure 6 shows three sections of Fig. 5 for 0.05, 0.1, and 0.2 percent o.v. It is seen that for a given percent o.v., the

[15] It is interesting to note that Gänger (reference 9) who studied formative time lags in air in the range from about four to several hundred percent o.v., also found no marked pressure dependence over the range of one atmosphere to 25 mm of Hg.

time lags increase linearly with increasing plate separation.

To determine the possible effects of the illumination and of V_0 on the time lags, additional data were taken at $\delta = 1$ cm and at various pressures. First, the copper screens were removed from the path of the ultraviolet light. The primary current then ranged from 18 to 20 electrons/μsec. The time lag curve is identical with that shown in Fig. 4 to within the experimental error. It can thus be stated that for an average primary current of between 3 and 20 electrons/μsec, the time lags are independent of the illumination. Second, the effect of V_0 was investigated at low illumination by changing V_0 from two to four kv below breakdown, and again to within the experimental accuracy, the data are identical with those shown in Fig. 4.

To determine the reproducibility of the measurements, duplicate data were obtained at various pressures for $\delta = 1$ cm (low illumination, $V_s - V_0 = 2$ kv)

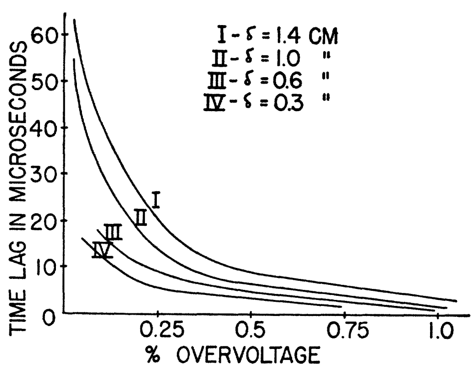

FIG. 5. Time lag *vs* percent o.v. for four values of plate separation. Curves represent average of data taken at all pressures between atmospheric and about 200 mm Hg. Low illumination, 2000-volt pulse.

after the entire experiment was completed. The resulting data again coincide with the results shown in Fig. 4.

There is an error introduced because of the electronic apparatus, such as the ripple in V_0 and V_1, the uncertainty in the magnitude of V_1, and the rise time and decay of V_1. An upper limit of 5 v can be set for the cumulative error of all these factors. Since this error is essentially independent of V_0, the accuracy of the data depends on the pressure. Thus at atmospheric pressure and $\delta = 1$ cm, the lower limit of quantitatively reliable data is 0.017 percent o.v., while for a pressure of 20 mm Hg, this lower limit is about 0.3 percent o.v.

IV. DISCUSSION OF RESULTS

The existence of long time lags at low percent o.v. indicates that (for the experimental conditions studied) the motion of positive ions plays an important role in the breakdown process. The fact that the time lags decrease continuously and smoothly as the percent o.v. is increased indicates that the role of the positive ions gradually decreases in importance as the field strength

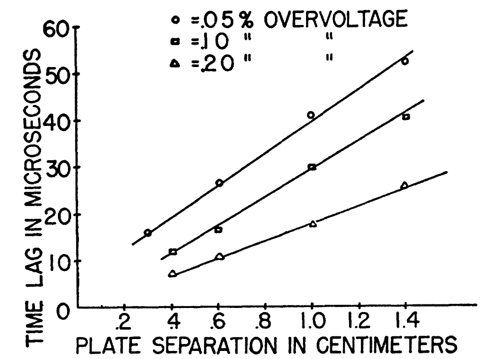

FIG. 6. Time lag *vs* plate separation for three values of percent o.v. Curves represent average of data taken at all pressures between atmospheric and about 200 mm Hg. Low illumination, 2000-volt pulse.

increases, until finally the time lags become so short that the positive ions may be considered to remain stationary throughout the formative time of breakdown.

One can try to account for the results on the basis of the Townsend discharge mechanism, in which secondary electrons are liberated from the cathode by positive ion bombardment. However, for the Townsend mechanism, even at very high overvoltages the time lags should still be in the microsecond region. Since the time lags decrease much more rapidly with increasing percent o.v. than can be explained by the variation of the positive ion velocity, the role of the positive ions is not the emission of secondary electrons from the cathode.

The second possibility for explaining the role of the positive ions in producing the spark is based on the fact that the positive ions aid the spark formation by producing a distorted field in the gap.[16] Electrons produced in the cathode region subsequent to the first avalanche multiply more intensely than in the undistorted field provided α/p increases more rapidly than linearly with E/p, the ratio of field strength to pressure. These newly created electrons may be produced by a secondary mechanism (the most probable ones being photoionization in the gas and photoelectric emission at the cathode) or by the constant primary electron current produced by illumination of the cathode. As the positive ions, originally created in the anode region, move toward the cathode, the field strength in the cathode region increases more and more rapidly with time. Thus, owing to the curvature of the α/p *vs* E/p curve, the quantity $\exp(\int_0^\delta \alpha dx)$, which represents the number of ion pairs produced by an electron originating near the cathode, increases with time. If conditions in the gap are not satisfactory for the formation of a spark when the first primary electron has crossed the gap, it is possible that as the positive ions created in the first avalanche approach the cathode, subsequent avalanches may produce the charge densities necessary for the develop-

[16] For a discussion of the effect of space charge falsification of the second Townsend coefficient, see Varney, White, Loeb, and Posin, Phys. Rev. 48, 818 (1935).

ment of a streamer. With such a mechanism, times corresponding to many transit times of a positive ion may occur before the spark develops. As the percent o.v. is increased, the enhanced field necessary for the production of the spark requires a shorter distance of travel of the positive ions and so the time lag decreases. As the percent o.v. is increased further, no motion of the positive ions is required to produce a spark, and a streamer emanates from the anode. With even higher overvoltages, adequate distortion fields for the production of a streamer can be built up even before the primary electron crosses the entire gap. Thus, the distortion field provides a smooth transition from formative times long compared to the positive ion transit time to times comparable to the electron transit time.

The method whereby sufficient initiating electrons are supplied to maintain the pre-spark current is now considered further. It can be shown[17] that if, under the assumption of a primary current with no secondary mechanism, the field strength in the cathode region as a function of time is derived for times less than the transit time of a positive ion (assuming no diffusion and a parabolic dependence of α/p on E/p), and if one assumes that a spark occurs when this distortion field reaches a critical value, fair agreement with the experimental data is obtained. However, if such a mechanism is valid, it is necessary to assume that all the primary electrons are emitted from a small region on the cathode. This is unlikely, since the spark rarely occurred at the same cathode spot twice in succession. Furthermore, no dependence of the time lag on illumination was observed, whereas such a mechanism would depend strongly on the magnitude of i_0. Better agreement with the experimental data is obtained by assuming a secondary mechanism involving photo-emission at the cathode by photons produced in the avalanche.[17]

It is believed, therefore, that two processes acting together produce the observed long time lags at low overvoltages. These are (1) a suitable secondary mechanism (most probably photo-emission at the cathode) to maintain and perhaps increase the pre-spark current and (2) space charge distortion due to the large number of positive ions in the gap. It should be noted that such a space charge mechanism can explain the long time lags obtained for low as well as high pressures. Indeed, in view of the fact that even at the lowest pressures studied (several cm of Hg) the time lags for overvoltages of a few percent are as short as those obtained for the higher pressures, it is clear that the classical Townsend theory, hitherto assumed valid for the product $p\delta$ less than several hundred mm Hg, is inadequate.

The search for the transition from the streamer to the Townsend mechanism of spark breakdown has revealed no simple dichotomy in mechanisms as envisaged by Loeb and Meek, and by Raether; the mechanism of the spark depends much more strongly on the percent o.v. than on the value of p and δ. These experiments are being continued with other gases.[18]

The authors thank Dr. H. J. White of the Research Corporation and Mr. M. Menes for their aid in developing the electronic circuit, and Professor L. B. Loeb of the University of California for his interest in the problem and for the loan of the chamber used in the research.

[17] To be published.

[18] G. A. Kachickas and L. H. Fisher, Phys. Rev. **79**, 232 (1950).