In our theory, the wave functions, Ψ_k, of the superconducting electrons are taken to be linear combinations of Bloch functions, ψ_k, which have energies near the Fermi surface. One consequence of a very small effective mass of the superconducting electrons is that the wave functions are not altered very much by an applied field, so that the London equations follow. We wish to point out here as a second consequence that the wave functions must extend over distances of at least 10^{-4} cm, in accord with Pippard's results. A small effective mass corresponds to the fact that it takes considerable energy to confine the wave function to a small region, as would be required to change the concentration near the surface. A localized superconducting electron can be described by a wave function of the form $U(x)\Psi_k(x)$, where $U(x)$ represents the envelope and Ψ_k extends through the crystal with uniform amplitude. The effective mass can be used to estimate the energy change which comes from the variation of $U(x)$. To confine the wave function to a distance $\pi/\Delta k$ requires an energy of the order of

$$\Delta E = \hbar^2(\Delta k)^2/2m_s,$$

where m_s is the effective mass. We have shown that $m_s \sim 10^{-4}$ m, so that $\Delta E \sim 5\times10^{-16}$ erg for $\Delta k \sim 10^4$ cm^{-1}. This corresponds to thermal energy for a few degrees absolute and is the order of the magnetic energy, $H_c^2/8\pi$, per superconducting electron.

In case the size of the wave function is limited by the dimensions of the crystal, as it is in thin films and in colloidal particles, one can take $U(x)=$constant. The Bloch functions of the normal metal from which the linear combinations are formed are localized and satisfy the boundary conditions at the surface. No additional energy is required to localize the electrons in the superconducting state. As the dimensions are made smaller there are fewer terms in the sum over the Bloch states. One would not expect much change in transition temperature as long as the number of terms is sufficient to give a good approximation to the integral obtained as a limit for large volumes.

There is some question concerning the wavelengths of the vibrational waves which interact most strongly with the electrons. The low energies involved in the transition suggest that perhaps long wavelengths are most important. The fact that there is little change in T_c for dimensions of 5×10^{-6} cm shows that the important wavelengths are no more than, say, one-tenth of this, or less than 5×10^{-7} cm. If *all* wavelengths are involved, there are around 10 terms in the sum for a volume of 10^{-18} cm^3, so that one might, in any case, expect changes in T_c for dimensions of $\sim10^{-6}$ cm or less.[7] A measurement of the way in which T_c changes with film thickness or particle sizes hould provide information about the important vibrational wavelengths.

The application of the effective mass concept to the calculation of the surface energy between normal and superconducting phases will be given in a subsequent communication.

[1] A. B. Pippard, Proc. Roy. Soc. (London) **A203**, 210 (1950).
[2] A. I. Shalkinov, Nature **142**, 74 (1938).
[3] E. T. S. Appleyard and A. D. Misener, Nature **142**, 474 (1938); Appleyard, Bristow, and Misener, Proc. Roy. Soc. (London) **A172**, 540 (1939).
[4] D. Shoenberg, Proc. Roy. Soc. (London) **A175**, 49 (1940).
[5] J. Bardeen, Phys. Rev. **79**, 167 (1950); **80**, 567 (1950); **81**, 469 (1951). H. Fröhlich, Phys. Rev. **79**, 845 (1950) has developed a theory along somewhat similar lines.
[6] F. London, Proc. Roy. Soc. (London) **A152**, 24 (1935); Phys. Rev. **74**, 562 (1948).
[7] J. Bardeen, Phys. Rev. **79**, 167 (1950). The estimate given there of $\sim10^{-6}$ cm for the size of the wave function is incorrect, because no account was taken of the increase in the number of interactions with increase in volume.

Magic Numbers and the Missing Elements Technetium and Promethium

HANS E. SUESS
Institute for Nuclear Studies, University of Chicago, Chicago, Illinois
January 15, 1951

IN order to find an explanation for the absence of β-stable isotopes of the elements Tc and Pm,[1-3] a more general abnormality in the region following the closing of the 50- and 82-neutron shell should be taken into consideration. This abnormality can be recognized from the differences of the binding energies of pairs of isobars as found from β-decay schemes. These show that

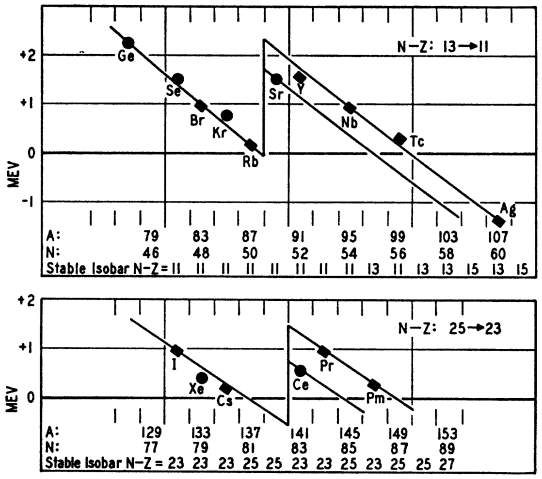

FIG. 1. Differences in the binding energies of the ground states of isobars as derived from beta-decay schemes.

in these regions the binding energies of the even Z, odd N nuclei (in their ground states) do not lie on the same energy surface as those of the odd Z, even N nuclei.

To illustrate this, the β-decay energies for nuclei with a given isotopic number $I=N-Z$ may be plotted *versus* $A=N+Z$. Figure 1 shows such plots for $I=13$ and $I=23$ as examples. Following the general rule of the equivalence of odd N and odd Z nuclei with respect to their binding energies, both kinds of odd mass numbered nuclei with $N<50$ or 82 lie close to a common line intersecting the zero line of decay energy at a point where the isobar stability changes to the next higher value of I. After the filling of the 50- and 82-neutron shell, an upward shift in the β^- decay energies occurs equivalent to the drop in the binding energy of the last neutron. This shift is somewhat larger, however, for the odd Z than for the odd N nuclei, indicating that the drop is not equal for paired and for unpaired neutrons. The general trend in this region can be represented by two different lines: the lower one representing the trend in the decay energies of the odd N, the higher one that of the odd Z nuclei. Thus, for a given I, the isobars with odd numbers of neutrons become stable at a lower mass number than those with an odd number of protons. This difference is large enough to cause the β-instability of all nuclei with a certain odd number of protons, incidentally those of $Z=43$ and 61.

In Fig. 1 the value for Zr^{93} is omitted, because it seems uncertain whether the low decay energy as reported by Steinberg and

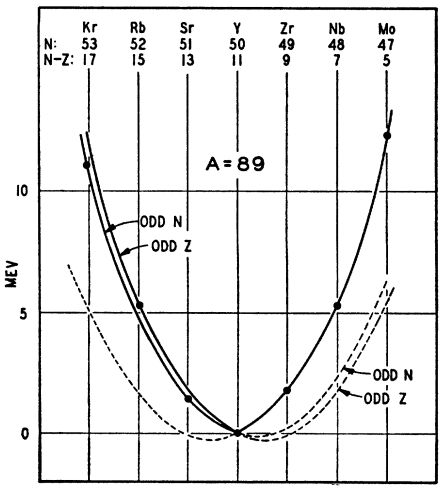

FIG. 2. Energy parabolas for mass number 89, illustrating the proposed split in the energy surface for odd N and odd Z nuclei in the region $N>50$.

Glendenin[4] (0.06 Mev) leads to the ground state or to an excited state of Nb^{93}. Further evidence for the proposed rule can be obtained from the behavior of nuclei with isotopic numbers other than those shown in Fig. 1. One finds that in all cases of nuclei with $N>50>Z$ and N somewhat larger than 82, the β^- decay energies of the odd Z nuclei lie systematically higher than those of the odd N nuclei. It may be noted that the unexpected instability and comparatively high decay energy of In^{115} fit well into this picture. A similar, but presumably smaller, "split" seems to occur at $N>126$, though the lack of β-decay data makes it impossible to predict the β-instability of the isotopes of At (85) or Fr (87) from β-decay systematics. In the region of $N>28$ and $Z<40$ a similar abnormality seems to hold, but here the β^- energies of the odd N nuclei seem to be systematically higher than those of the odd Z nuclei.

The proposed picture can be described in other words by saying that the line of maximum stability (the Gamow line) is shifted towards a region of smaller neutron excess when the number of neutrons exceeds 50 or 82 but that this shift is larger for the odd Z than for the odd N nuclei, so that in the region of $N>50>Z$ and for N somewhat larger than 82 the Gamow line is split in two lines referring to odd Z and odd N nuclei, respectively (Fig. 2).

The numerical values for the lowering of the binding energies of the last neutron at $N=50$ and 82 as derived from Fig. 1 are as follows (in Mev):

N:	50	82
paired neutron:	2.4	2.0
unpaired neutron:	1.8	1.3.

These figures are in good agreement with the first estimate made by Mayer[2] of about 2 Mev for all cases.

There are several possible ways of explaining these abnormalities, but it seems premature to the writer to give any one of them final preference. Thanks are due to Maria Mayer for extensive discussions of these possibilities.

[1] H. Jensen, Naturwissenschaften 26, 381 (1938).
[2] M. G. Mayer, Phys. Rev. 74, 235 (1948).
[3] L. Kowarski, Phys. Rev. 78, 477 (1950).
[4] E. R. Steinberg and L. E. Glendenin, Phys. Rev. 78, 624 (1950).

On the Radioactivity of Hf^{181} and Hf^{175}

ARNE HEDGRAN AND SIGVARD THULIN
Nobel Institute of Physics, Stockholm, Sweden
January 15, 1951

SINCE the interpretation of the measurements on n-irradiated Hf has been shown to be complicated because of the presence of different activities,[1,2] we have separated electromagnetically the Hf isotopes of a pile-irradiated sample. Besides the known activity of Hf^{181} there was also one corresponding to mass number 175, amounting to about 2.5 percent of the total activity as measured by a G-M counter with a 2.2 mg/cm² window.

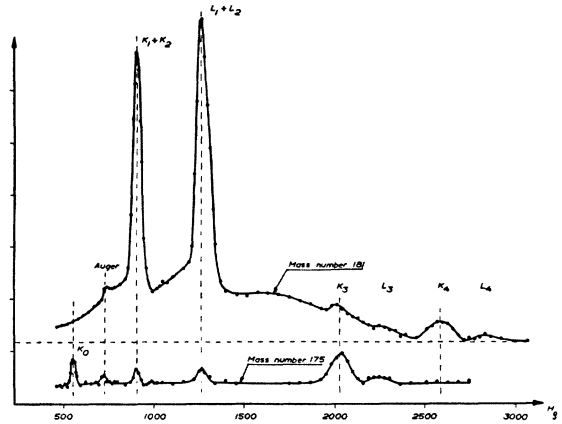

FIG. 1. β-spectra of the separated mass numbers 181 and 175 isotopes. The intensities are not drawn to the same scale.

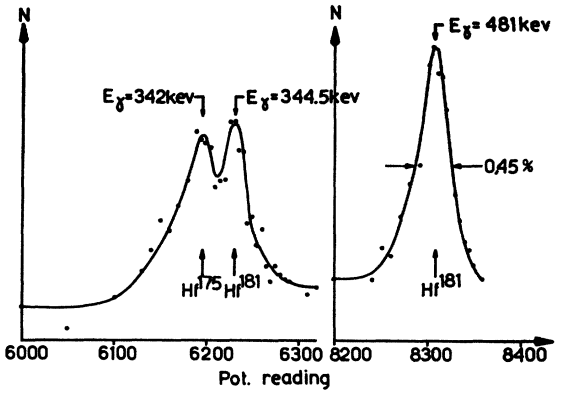

FIG. 2. The high energy photo-lines of unseparated Hf.

The separated isotopes, collected on 2.5 mg Al/cm², were used as samples for β-spectrometer investigations, the results of which are shown in Fig. 1. The letters K_1, $K_2\cdots L_1$, $L_2\cdots$ indicate the K and L conversion lines of the well-known γ-rays of energies 132, 135, 345, and 481 kev, respectively. Because of the presence of small amounts of the strong mass 181 activity in mass 175 source this spectrum also contains the two intense lines of Hf^{181}, but this will not affect the interpretation of the measurements.

The activity of mass number 175 is no doubt due to Hf^{175} which decays by K-capture to Lu^{175}, having two K conversion lines of energies 26 and 279 kev. A comparison of the two β-spectra shows that there must be two different γ-lines of about 345 kev, one in each isotope. Thus our measurements confirm the results of Cork *et al.*[1]

In order to resolve the two 345-kev γ-rays in an unseparated sample, it was investigated in a high resolution β-spectrometer. Figure 2 shows on the left the photo-electron lines from a 0.7 mg/cm² uranium converter. The energies of the corresponding γ-rays are 342.2 and 344.9 kev, and they are of almost the same intensity.[3] On the right of Fig. 2 the high energy photo-line is shown for comparison.

Since only about half of the 345-kev radiation belongs to Ta^{181}, it would appear that the intensity of the 135-kev radiation was too high to be accounted for by the term scheme and the intensity data given by Chu and Wiedenbeck.[4] Therefore, some measurements were made to determine the intensity ratios in the decay.

From the β-spectrum in Fig. 1. we find $(NK_1+NK_2)/N_\beta=0.32$ and $(NL_1+NL_2+N_M)/N_\beta=0.36$. These values are considerably less than those given in reference 4.

From a measurement in the high resolution β-spectrometer we find $NK_1:NK_2=2:1$, and $NK_2:NL_2\approx 8:1$. From the secondary electron spectrum (Fig. 2) we estimate, taking account of the variation of the photo cross section, the intensity ratio $\gamma_{345}:\gamma_{481}\approx 1:8$.

Measurements of the photo-electron lines in gold of the unconverted 132- and 135-kev radiations indicate an intensity ratio of roughly 5:1. From this we conclude that the 135-kev ray is very highly converted, with almost all of the conversion taking place in the K shell. This means that the intensity of this radiation is much lower than one would expect from previous β-spectrometric investigations. We estimate it to be about 20 percent of the intensity of the 132-kev radiation. Although our determination of the intensity of the 345-kev radiation belonging to the decay of Hf^{181} ($\approx$12 percent of the 481-kev radiation) is still somewhat lower than this, we cannot place any significance on this difference because of the uncertainty of some of our intensity comparisons, and we conclude that in spite of the complexity shown in the activity of n-irradiated Hf the term scheme given originally by Chu and Wiedenbeck is probably correct.

[1] Cork, Stoddard, Rutledge, Branyan, and Le Blanc, Phys. Rev. 78, 299 (1950).
[2] M. Deutsch and A. Hedgran, Phys. Rev. 79, 400 (1950).
[3] All measurements were made at the same time, about one month after an irradiation of 4 weeks duration in the Harwell pile.
[4] K. Chu and M. Wiedenbeck, Phys. Rev. 75, 226 (1949).