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On the Tritium HFS and the Anomalous Magnetic Moment of the Triton*

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The hfs of tritium is studied using the model of A. Bohr according to which the electron centers on the proton at small electron-nuclear separations. The hfs effects due to this recentering of the electron are calculated for three theories of the origin of the triton moment anomaly: the spin-orbital moment theory of Avery and Sachs, the phenomenological interaction moment theory of Blanchard, Avery, and Sachs, and the meson exchange moment theory of Villars. In addition the hfs contributions due to relativistic effects of internal nuclear motion are calculated for deuterium, and the result is used to estimate the uncertainty in the tritium hfs coming from this source. It is found that in each case considered the hfs effects may be classified as either "Bohr effect," proportional to the size of the nucleus, or "orbital effect," proportional to the size of the region of centering of the electron. On each of the three theories the total hfs effect is of the order of magnitude of the present experimental uncertainty in the determination of the hfs. While the effect is large enough so that it could be observed with only a small improvement of the experimental precision, it is about the same for each theory considered and would not, therefore, serve as a possible means of distinguishing among the several theoretical accounts of the triton moment anomaly.

I. INTRODUCTION

ACCORDING to the simplest theory of the H^3 nucleus the triton magnetic moment should be equal to, or slightly less than, one proton moment, whereas the experimental value of μ_T is greater than μ_P by about $0.2\mu_0$, μ_0 being the nuclear magneton $e\hbar/2Mc$. A detailed consideration of the matter[1] has shown that one cannot account for the observed triton moment as simply the sum of spin and orbital moments of individual nucleons unless one abandons the assumption that the triton ground state is the predominantly S-state expected on the usual theories of nuclear forces. A number of suggestions have been made as to the source of the excess triton moment, each requiring special assumptions about the nature of the triton ground state and/or the nature of nuclear forces. Consequently, an experiment which can serve as a test of these theories of the triton moment is of some interest in the study of nuclear forces. That the precision determination of the tritium hfs is such an experiment was suggested by a recent theoretical study[2] of the deuterium hfs.

The observed value[3] of the deuterium hfs is somewhat larger than the value calculated for a point deuteron having the observed deuteron moment.[4] A. Bohr has shown[2] that a discrepancy of this sort arises because the hfs contribution of the neutron spin is reduced when the electron is within the nucleus. An essential part of Bohr's demonstration was the assumption that at small electron-nuclear separations the electron wave function centers on the proton position rather than the deuteron center of mass. This recentering leads in addition to a reduction of the proton orbital hfs.[5,6] A detailed calculation shows[5] that the sum of these two theoretical corrections to the hfs is in good agreement with the value of the observed discrepancy for deuterium, so that it is probable that Bohr's assumption is basically correct.

A similar recentering of the electron wave function must occur for tritium, so that some such discrepancy[6a] might be expected for the tritium hfs. In the following, estimates of this discrepancy, which is due to both the Bohr effect and the orbital effect, will be made on the basis of each of several theories of the triton moment.

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[1] R. Avery and R. G. Sachs, Phys. Rev. 74, 1320 (1948).
[2] A. Bohr, Phys. Rev. 73, 1109 (1948).
[3] J. E. Nafe and E. B. Nelson, Phys. Rev. 73, 718 (1948).
[4] Bloch, Levinthal, and Packard, Phys. Rev. 72, 1125 (1947).
[5] F. Low, Phys. Rev. 77, 361 (1950).
[6] E. N. Adams II, Phys. Rev. 77, 755 (1950).
[6a] The hfs discrepancy is defined as the amount by which the hfs differs from that calculated for a point nucleus which has the observed nuclear moment.

These theories will be referred to as: A. Spin and orbital moment (SOM) theory. B. Phenomenological interaction moment (PIM) theory. C. Meson exchange moment (MEM) theory.

Previous estimates[1,7] of the Bohr effect for the tritium hfs have indicated that it is much smaller than that for the deuterium hfs, and is of the order, in fact of the present experimental uncertainty in the determination of the hfs. It was found in the present work that the *total* hfs effect probably exceeds the present experimental uncertainty in only one of the cases considered. While a moderate improvement of the experimental precision would permit this one case, which occurs on the PIM theory, to be definitely distinguished from the others, it appears that because of theoretical uncertainties no further distinctions would be possible.

The hfs effects considered here are so small that it was thought necessary to examine the role of relativistic contributions to the nuclear moment in the hfs of the centered electron. Since the relativistic contributions to the deuteron moment have been discussed previously,[8,9] it was simpler to study the effects in question for the deuterium hfs rather than for the tritium hfs. However, on the basis of the deuterium results one can easily estimate how much the relativistic effects contribute to the uncertainty in the interpretation of the tritium hfs effects. The uncertainty from this source is found to be unimportant.

II. THE ELECTRON WAVE FUNCTION AND THE MAGNETIC FIELD DUE TO THE ELECTRON

Of the theories of the triton moment which are to be considered here, no one of them is complete in the sense that it is a part of a more comprehensive theory which has been shown to give a satisfactory account of the triton binding energy and of the known properties of the deuteron. For this reason it did not appear to be feasible to make such a detailed treatment of the triton wave function as would be required in order to carry through a second-order perturbation method analogous to that used by Low[5] for the calculation of the deuterium hfs effects. Instead, the hfs was computed in first order only, on Bohr's original assumption that for large electron-nuclear separations the electron wave function is the $1s$ Coulomb function centered on the nuclear center of mass, while for electron-nuclear separations smaller than a critical length D, it is the same $1s$ function centered on the proton. As a means of improving the quantitative validity of the results obtained in Bohr's approximation, the value of D was taken as a free parameter which was then fixed by the requirement that the method used here gave the same value for the deuterium hfs effect as is given by Low's more accurate method.

[7] E. Fermi and E. Teller, conference at Pocono Manor (1948).
[8] R. G. Sachs, Phys. Rev. **72**, 91 (1947).
[9] G. Breit and I. Bloch, Phys. Rev. **72**, 135 (1947).

For the present purpose it is convenient to regard the hfs as due to the interaction of the nucleus with the electromagnetic field arising from the electron current. The value of the vector potential at a point $\mathbf{r}$ in the nucleus is, then:

$$\mathbf{A}(\mathbf{r}) = -e\langle \boldsymbol{\alpha}_e / |\mathbf{r}-\mathbf{r}_e|\rangle, \quad (1)$$

in which the angular brackets denote the electronic expectation value.

In evaluating Eq. (1) it is sufficiently accurate to use the well known approximation in which the electron large function is replaced by the nonrelativistic $1s$ Coulomb function. The $1s$ function must be modified so that it centers on the proton for distances smaller than D. Using such a wave function and making use of the relations $r \ll a$, $r \ll D$, $D \ll a$, one finds for $\mathbf{A}(\mathbf{r})$:

$$\begin{aligned}
&\mathbf{A} = \mathbf{A}_1 + \mathbf{A}_2 + \mathbf{A}_3\\
&\mathbf{A}_1(\mathbf{r}) = -(8\pi/3)\mu_B \psi_{(0)}{}^2[\boldsymbol{\sigma}_e \times \mathbf{r}/2](1-2D/a)\\
&\mathbf{A}_2(\mathbf{r}) = -(8\pi/3)\mu_B \psi_{(0)}{}^2[\boldsymbol{\sigma}_e \times (\mathbf{r}-\mathbf{r}_\pi)/2](2D/a) \quad (2)\\
&\mathbf{A}_3(\mathbf{r}) = -(8\pi/3)\mu_B \psi_{(0)}{}^2[-(3|\mathbf{r}-\mathbf{r}_\pi|/2a)\boldsymbol{\sigma}_e \times (\mathbf{r}-\mathbf{r}_\pi)/2],
\end{aligned}$$

in which $\psi_{(0)}$ denotes the value at the origin of the nonrelativistic $1s$ Coulomb function, $\boldsymbol{\sigma}_e$ is twice the expectation value of the electron spin angular momentum, a is the Bohr radius, and the label π is used to denote the proton coordinate. All vectors are measured from the center of mass. The magnetic field derived from the potential $\mathbf{A}(\mathbf{r})$ can be written as the sum of three terms:

$$\begin{aligned}
&\mathbf{H}_1(\mathbf{r}) = -(8\pi/3)\mu_B \psi_{(0)}{}^2 \boldsymbol{\sigma}_e\\
&\mathbf{H}_2(\mathbf{r}) = -(8\pi/3)\mu_B \psi_{(0)}{}^2(-2|\mathbf{r}-\mathbf{r}_\pi|/a)\boldsymbol{\sigma}_e\\
&\mathbf{H}_3(\mathbf{r}) = -(8\pi/3)\mu_B \psi_{(0)}{}^2 \quad (3)\\
&\qquad \times\left[\frac{|\mathbf{r}-\mathbf{r}_\pi|}{4a}\left\{\frac{3(\boldsymbol{\sigma}_e\cdot[\mathbf{r}-\mathbf{r}_\pi])(\mathbf{r}-\mathbf{r}_\pi)}{|\mathbf{r}-\mathbf{r}_\pi|^2} - \boldsymbol{\sigma}_e\right\}\right].
\end{aligned}$$

The notation is *not* intended to imply that $\mathbf{H}_1 = \mathrm{curl}\mathbf{A}_1$.

The interaction of the nucleus with the field provides for the atom an additional energy $E(\mathbf{A})$ given by:

$$E(\mathbf{A}) = -\mu_P\langle \boldsymbol{\sigma}_\pi \cdot \mathbf{H}(\mathbf{r}_\pi)\rangle - \mu_N \sum_\nu \langle \boldsymbol{\sigma}_\nu \cdot \mathbf{H}(\mathbf{r}_\nu)\rangle - \frac{1}{c}\left\langle \int d^3\mathbf{r}(\mathbf{A}\cdot\mathbf{S})\right\rangle, \quad (4)$$

in which the sum of ν is a sum over neutrons, $\mathbf{S}(\mathbf{r})$ is a nuclear operator the expectation value of which gives the density of the nuclear electric current, and the angular brackets denote the nuclear expectation value. The hfs energy is the difference between the values of $E(\mathbf{A})$ for the $F = I + \frac{1}{2}$ and $F = I - \frac{1}{2}$ states of the atom. It is well known that the difference is proportional to the value of $E(\mathbf{A})$ for the $F = I + \frac{1}{2}$ state, so this value of $E(\mathbf{A})$, which is denoted as U, will be referred to as the hfs energy.

III. THE MAGNETIC STRUCTURE EFFECTS ON THE HFS

The terms in Eq. (4) which involve $\mathbf{H}_1$ give the neutron and proton spin hfs as calculated for a point nucleus, while the terms in $\mathbf{H}_2$, $\mathbf{H}_3$ give the corrections to the spin hfs for the finite space extension of the nucleus and for the recentering of the electron wave function. The proton spin hfs is "normal," i.e., the same as it would be for a point nucleus, as is evidenced by the vanishing of $\mathbf{H}_2(\mathbf{r})$ and $\mathbf{H}_3(\mathbf{r})$ at the proton. The neutron hfs is changed by an amount:

$$\delta U_N = -(8\pi/3)\mu_B\mu_N\psi_{(0)}{}^2\sum_\nu\Big\langle(2r_{\pi\nu}/a)(\boldsymbol{\sigma}_e\cdot\boldsymbol{\sigma}_\nu) - (r_{\pi\nu}/4a)\left\{\frac{3(\boldsymbol{\sigma}_e\cdot\mathbf{r}_{\pi\nu})(\boldsymbol{\sigma}_\nu\cdot\mathbf{r}_{\pi\nu})}{r_{\pi\nu}{}^2} - (\boldsymbol{\sigma}_e\cdot\boldsymbol{\sigma}_\nu)\right\}\Big\rangle. \quad (5)$$

The hfs effect given by Eq. (5) is the Bohr effect. Since the first term is by far the larger, the Bohr effect is just a reduction of the neutron hfs by the relative amount $2\bar{r}_{\pi\nu}/a$.

The proton orbital hfs can be obtained from the third term on the right-hand side of Eq. (4) by inserting for the current density operator $\mathbf{S}(\mathbf{r})=(e/M)\delta(\mathbf{r}-\mathbf{r}_\pi)\mathbf{p}_\pi$. Then

$$U_{\mathrm{ORB}} = -\frac{1}{c}\Big\langle\int d^3\mathbf{r}[\mathbf{A}(\mathbf{r})\cdot\mathbf{S}(\mathbf{r})]\Big\rangle = \left(\frac{8\pi}{3}\right)\mu_B\mu_0\psi_{(0)}{}^2\langle(\boldsymbol{\sigma}_e\cdot\mathbf{L}_\pi)\rangle(1-2D/a). \quad (6)$$

The orbital hfs comes entirely from $\mathbf{A}_1$, since $\mathbf{A}_2$ and $\mathbf{A}_3$ vanish at the proton. Except for the factor $(1-2D/a)$, Eq. (6) gives the normal hfs contribution of the orbital moment; from the occurrence of that factor one sees that the orbital hfs is reduced by the relative amount $2D/a$. Such a reduction, which results from the vanishing of the contribution to the orbital hfs from the entire region of centering, is to be expected on semi-classical considerations.[5,6]

It will be seen that there are, in addition, magnetic structure effects on the hfs contributions of exchange moments and of the interaction moments which arise as a consequence of the velocity dependence of the nuclear interactions; these effects may be analogous to either the Bohr effect or the orbital effect. A "Bohr effect" (effect proportional to $\bar{r}_{\pi\nu}/a$) in such a case is just caused by a weakening of the interaction between the electron and the current distribution when the electron is within the distribution. An "orbital effect" (effect proportional to D/a) results if the magnetic moment is not a translational invariant, since, in effect, the recentered electron interacts with the moment of the currents about the proton rather than the moment about the center of mass.

IV. EFFECTS ON THE HFS CONTRIBUTION OF THE TRITON MOMENT ANOMALY

Fermi-Teller Effect

Before discussing the hfs effects which are peculiar to the individual theories a discussion will be given of the Bohr effect for the triton ^2S state, since it must be taken into account in computing the total hfs effect in each of the three cases. The effect is not entirely analogous to the Bohr effect on the deuterium hfs. The ^2S state of the triton is predominantly a space symmetric ^2S_s state in which the neutron spins are paired, and for this state there is no neutron spin hfs and no Bohr effect on the hfs. However, it was pointed out by Fermi and Teller[7] that because of the spin dependence of nuclear forces the ^2S_s state has admixed into it a small amount of ^2S_a state, antisymmetric in the neutron space coordinates, in which the neutron spins are parallel. They showed that this admixture of states has the property that the proton spin is partially aligned with the spin of whichever neutron is closer to it. Although the neutron spins still contribute almost nothing to the total magnetic moment, there is within the nucleus an average neutron spin distribution which is parallel to the proton spin at short distances from the proton and antiparallel to the proton spin at larger distances. The electron penetrates the outer part of the spin distribution more often than it does the inner part, so there is a slight net contribution to the hfs from the inner part. Because the neutron moment is negative, the result is a decrease of the tritium hfs.

On any of the theories to be considered here the ^2S_a admixture must be taken to be very small. It was considered adequate for the present purpose to assume simply that $|^2S_a|^2 \sim 0.01|^2S_s|^2$. On this basis the Fermi-Teller effect was estimated to be:

$$\frac{\delta_{\mathrm{F-T}}U}{U} = -\frac{4}{3}|^2S_a\,^2S_s|\frac{|\mu_{\mathrm{N}}|}{\mu_{\mathrm{T}}}\times\left(\frac{2\bar{r}_{\pi\nu}}{a}\right) \sim -0.08|^2S|^2(2\bar{r}_{\pi\nu}/a). \quad (7)$$

SOM Theory

In order to account for the observed value of the triton moment as simply the sum of spin and orbital moments of individual nucleons (SOM theory) it is necessary to abandon the assumption that the triton ground state is a predominantly S state. Assuming the triton ground state to be an arbitrary admixture of all possible states of $I=\frac{1}{2}$ and even parity, Sachs[10] and Avery and Sachs[1] have investigated the possibility of fitting the observed triton moment on a SOM theory. They found that an additional condition which is imposed on the triton wave function by the observed value of the He^3 moment is extremely restrictive, so

[10] R. G. Sachs, Phys. Rev. 73, 312 (1947).

that there is no possibility of satisfying it except with a wave function which has a P state admixture of from 40 to 100 percent, the remainder of the wave function being ^2S state. According to present ideas it seems unlikely that this is a good description of the triton wave function; however, in order to find out what the hfs effect would be on a SOM theory, the hfs was calculated using the triton wave function which Avery and Sachs found to offer the best prospect of accounting for the observed moments. It was somewhat surprising to find that despite the large P state admixture the orbital moment is very small; the triton moment is increased for the P state admixture chiefly because the neutron spins are no longer paired.

The total hfs effect is given by:

$$(\delta U/U) = -(\mu_0/9\mu_T)[2|^2P|^2 - |^4P|^2](2D/a) - 0.08|^2S|^2(2\bar{r}_{\pi\nu}/a) - (|\mu_N|/9\mu_T) \times [4|^2P|^2 + 8\sqrt{2}|^2P^4P| - 10|^4P|^2](2\bar{r}_{\pi\nu}/a). \quad (8)$$

The size of the effect varies *inversely* as the amount of P state admixture; for the two extreme cases it was found to be:

40 percent P state ($|^2P|^2 \approx 0.18$; $|^4P|^2 = 0.20$) (8a)

$$(\delta U/U) \sim -0.11(2\bar{r}_{\pi\nu}/a) - 0.006(2D/a);$$

100 percent P state ($|^2P|^2 \approx 0.38$; $|^4P|^2 \approx 0.61$) (8b)

$$(\delta U/U) \sim -0.06(2\bar{r}_{\pi\nu}/a) - 0.002(2D/a).$$

PIM Theory

The triton wave function which is required by the SOM theory is quite different from that found by a study of the triton binding energy. In order to account for the triton binding energy using current phenomenological two-body interactions the triton wave function should be taken to be a ^2S function with a few percent of ^4D admixture.[11] If the ^4D admixture is taken to be about four percent, such a wave function also gives agreement[1] with the sum of the observed values of the H^3 and He^3 moments, although not, of course, with the values of the H^3 and He^3 moments individually. The agreement can be understood if it is assumed that both nuclei have, in addition to the nucleon spin and orbital moments, other moments which arise as a consequence of the nucleon-nucleon interaction, the "interaction moments" of the two nuclei being of equal magnitude and opposite sign.

It can be shown that interaction moments of this sort are to be expected, even on a phenomenological theory of nuclear forces, when the nucleon-nucleon interaction involves either explicit velocity dependence[12] (direct $L-S$ coupling) or implicit velocity dependence[13] (space exchange). The existence of an interaction moment is inferred as follows: in the presence of an externally arising electromagnetic field the nuclear Hamiltonian is modified by replacing $\mathbf{p}_j \to \mathbf{p}_j - (e_j/c)\mathbf{A}(\mathbf{r}_j)$ wherever the nucleon momentum $\mathbf{p}_j$ occurs in the Hamiltonian; interactions involving space exchange are included in the scheme by expressing the exchange operator as a power series in the nucleon momenta. As a consequence of this replacement the Hamiltonian includes terms in which the vector potential of the external field occurs together with the nuclear interaction. Such terms represent a magnetic energy depending directly on the nucleon-nucleon interaction, and a nucleus for which this energy does not vanish has a non-zero interaction moment.

For the case of pure space exchange Sachs[13] has derived an explicit expression for the exchange current which gives rise to the interaction moment. His expression may be rewritten in terms of an operator $\mathbf{S}_X(\mathbf{r})$, which is defined so that its nuclear expectation value gives the exchange current density at an arbitrary point $\mathbf{r}$.

$$\mathbf{S}_X(\mathbf{r}) = (ie/\hbar)\sum_\nu \mathbf{r}_{\nu\pi}\left\{\int_0^1 d\alpha\,\delta(\mathbf{r}-\mathbf{r}_\pi - \alpha\mathbf{r}_{\nu\pi})\right\} J_{\pi\nu}P_{\pi\nu}{}^X, \quad (9)$$

in which $J_{\pi\nu}$ describes the spin and space dependence of the neutron-proton interaction. From Eq. (9) the energy of interaction between the exchange currents and the external field may readily be obtained.

In the case of the explicitly velocity dependent interactions the magnetic energy terms may take a variety of forms, according to the precise manner in which the interactions depend on the nucleon momenta. Only two interactions will be considered here, *viz.*, the interactions (5) and (6) of Blanchard, Avery, and Sachs, which were found to lead to suitable interaction moments for H^3 and He^3. Interaction (6) differs from interaction (5) only by a space exchange operator; however, the magnetic energy terms in the two cases differ somewhat more, being given, for our purpose, by:

$$E^{(5)} = (e/\hbar c)\sum_\nu J_{\pi\nu}{}^{(5)}(\boldsymbol{\sigma}_{\pi\nu} \times \mathbf{r}_{\pi\nu} \cdot \mathbf{A}_\pi), \quad (10a)$$

$$E^{(6)} = (e/2\hbar c)\sum_\nu J_{\pi\nu}{}^{(6)}(\boldsymbol{\sigma}_{\pi\nu} \times \mathbf{r}_{\pi\nu} \cdot \mathbf{A}_{\pi\nu}). \quad (10b)$$

The exchange moment which arises from the currents Eq. (9) has been found[14] to contribute only about $0.016\mu_0$ to the triton moment. Thus if the H^3 moment anomaly of $0.2\mu_0$ is to be accounted for on a PIM theory, it must be assumed that the neutron-proton interaction includes an interaction of either type (5) or type (6) and that this interaction has a strength adequate to provide the required interaction moment. (Although a linear combination of interactions (5) and (6) would do just as well for this purpose, only the two pure cases need be considered in computing the hfs effect.)

The hfs contribution of the exchange currents Eq. (9) may be obtained directly from Eq. (4) using the vector potential Eq. (2). The appearance of the δ-function in

[11] H. Feshbach and W. Rarita, Phys. Rev. **75**, 1384 (1949); R. E. Clapp, Phys. Rev. **76**, 873 (1949).
[12] Blanchard, Avery, and Sachs, Phys. Rev. **78**, 292 (1950).
[13] R. G. Sachs, Phys. Rev. **74**, 433 (1948); **76**, 1605 (1949).
[14] R. Avery and E. N. Adams II, Phys. Rev. **75**, 1106 (1949).

the operator $\mathbf{S}_X(\mathbf{r})$ has the result that the exchange current vanishes except on straight line filaments connecting the neutrons to the proton, the current having the direction $\mathbf{r}-\mathbf{r}_\pi$ at points on the filament. Since $\mathbf{A}_2(\mathbf{r})$ and $\mathbf{A}_3(\mathbf{r})$ are perpendicular to $\mathbf{r}-\mathbf{r}_\pi$ at all points, it is clear that the scalar products $\mathbf{A}_2\cdot\mathbf{S}_X$ and $\mathbf{A}_3\cdot\mathbf{S}_X$ vanish, and the entire hfs comes from the term in $\mathbf{A}_1\cdot\mathbf{S}_X$. It follows that the hfs contribution of the exchange moment is reduced by the factor $(1-2D/a)$, which is characteristic of the orbital effect. The hfs contribution of the interaction moment arising from interaction (5) also shows an "orbital effect," as can be seen by using the vector potential Eq. (2) in Eq. (10a) and noting that $\mathbf{A}_2(\mathbf{r})$ and $\mathbf{A}_3(\mathbf{r})$ vanish for $\mathbf{r}=\mathbf{r}_\pi$. In the case of interaction (6), however, all three terms in the vector potential make a nonvanishing contribution to the hfs and it is found that for this interaction moment the hfs effect is a "Bohr effect." From Eq. (10b) it is clear that the "Bohr effect" is in this case proportional to r_0/a rather than $\bar{r}_{\pi\nu}/a$, r_0 being the range of the interaction $J_{\pi\nu}^{(6)}$.

Choosing the interaction moment as $0.25\mu_0$, as is required to fit the triton moment for the case of four percent ^4D state, and taking into account the F-T effect, and the Bohr effect and orbital effect caused by the ^4D state admixture, one finds for the hfs effect: (a) assuming interactions (5) (velocity dependence without space exchange)

$$\frac{\delta U}{U}=-0.27\frac{\mu_0}{\mu_T}\left(\frac{2D}{a}\right)-\frac{2}{3}\frac{|\mu_N|}{\mu_T}|^4D|^2\left(\frac{2\bar{r}_{\pi\nu}}{a}\right)-\frac{1}{3}\frac{\mu_0}{\mu_T}|^4D|^2\left(\frac{2D}{a}\right)-0.08|^2S|^2\left(\frac{2\bar{r}_{\pi\nu}}{a}\right)\sim-0.1(2\bar{r}_{\pi\nu}/a)-0.09(2D/a), \quad (11a)$$

(b) assuming interaction (6) (velocity dependence with space exchange)

$$\frac{\delta U}{U}=-0.25\frac{\mu_0}{\mu_T}\left(\frac{3r_0}{2a}\right)-0.016\frac{\mu_0}{\mu_T}\left(\frac{2D}{a}\right)-\frac{2}{3}\frac{|\mu_N|}{\mu_T}|^4D|^2(2\bar{r}_{\pi\nu}/a)-\frac{1}{3}\frac{\mu_0}{\mu_T}|^4D|^2(2D/a)-0.08|^2S|^2(2\bar{r}_{\pi\nu}/a). \quad (11b)$$

If a mixture of the two interactions is considered, the hfs effect is just the weighted mean.

MEM Theory

If nuclear forces result from the exchange of charged mesons between nuclear particles, then there must exist exchange currents in the nucleus, and these exchange currents can make a nonvanishing contribution to the nuclear magnetic moment. Villars[15] and Thellung and Villars[16] have investigated such exchange moments in special cases and have found that the pseudoscalar theory appears likely to lead to an exchange moment of the right sign and size to account for the triton moment anomaly. Using the pseudoscalar theory Villars[15] found a MEM operator which is equivalent to the operator:

$$\mathbf{M}_X=\mathbf{M}_{X1}+\mathbf{M}_{X2}$$

$$\mathbf{M}_{X1}=-(ie/\hbar c)(f\kappa)^2\times\sum_\nu\frac{1}{\kappa}\left\{\mathbf{r}_{\pi\nu}\frac{(\boldsymbol{\sigma}_\pi\times\boldsymbol{\sigma}_\nu\cdot\mathbf{r}_{\pi\nu})}{r_{\pi\nu}^2}\left(1+\frac{1}{\kappa r_{\pi\nu}}\right)-(\boldsymbol{\sigma}_\pi\times\boldsymbol{\sigma}_\nu)\right\}\epsilon^{-\kappa r_{\pi\nu}}P_{\pi\nu}{}^\sigma P_{\pi\nu}{}^r$$

$$M_{X2}=(ie/\hbar c)\sum_\nu[\mathbf{r}_\pi\times\mathbf{r}_\nu]U_{\pi\nu}P_{\pi\nu}{}^\sigma P_{\pi\nu}{}^r, \quad (12)$$

in which $U_{\pi\nu}$ is the (improper) neutron-proton static interaction of the pseudoscalar theory.

The explicit expression for the current operator which gives rise to the magnetic moment Eq. (12) was used in Eq. (4) together with the vector potential Eq. (2) in order to compute the contribution of the exchange moment to the tritium hfs. It was found that the contribution of the moment $\mathbf{M}_{X1}$ shows a "Bohr effect," while that of $\mathbf{M}_{X2}$ shows an "orbital effect."

For the numerical evaluation of the total hfs effect on the MEM theory the triton wave function was again taken to be 96 percent ^2S state, four percent ^4D state. The moment $\mathbf{M}_{X2}$ vanishes for the triton ^2S state, but it does make a contribution through $S-D$ interference, since $U_{\pi\nu}$ includes a tensor interaction. Because the ^4D state amplitude is small, however, $\mathbf{M}_{X2}$ is much smaller than $\mathbf{M}_{X1}$, which does not vanish in the ^2S state. For the magnitude of $\mathbf{M}_{X2}$ the estimate $0.016\mu_0$ was used.[14] The magnitude of $\mathbf{M}_{X1}$ was then taken to be $0.25\mu_0$ as required for the theory to give the observed triton moment. Equation (12) shows that the exchange currents extend only a distance r_0 from the proton, r_0 being the range of the nuclear interaction. Accordingly, the estimate $\bar{r}_{\pi\nu}\sim r_0$ was used in evaluating the "Bohr effect" for $\mathbf{M}_{X1}$. The total hfs effect, including the F-T effect, and the ^4D state effects, was found to be:

$$\frac{\delta U}{U}=-0.25\frac{\mu_0}{\mu_T}\left(\frac{3}{2}\frac{r_0}{a}\right)-0.016\frac{\mu_0}{\mu_T}\left(\frac{2D}{a}\right)-\frac{2}{3}\frac{|\mu_N|}{\mu_T}|^4D|^2\left(\frac{2\bar{r}_{\pi\nu}}{a}\right)-\frac{1}{3}\frac{\mu_0}{\mu_T}|^4D|^2\left(\frac{2D}{a}\right)-0.08|^2S|^2\left(\frac{2\bar{r}_{\pi\nu}}{a}\right)\sim-0.06\left(\frac{2r_0}{a}\right)-0.01\left(\frac{2D}{a}\right)-0.1\left(\frac{2\bar{r}_{\pi\nu}}{a}\right). \quad (13)$$

[15] F. Villars, Helv. Phys. Acta **20**, 476 (1946).

[16] A. Thellung and F. Villars, Phys. Rev. **73**, 924 (1948).

V. THE HFS EFFECT FOR THE RELATIVISTIC CORRECTION TO THE NUCLEAR MOMENT

In order to give an accurate theoretical interpretation of a nuclear magnetic moment one must take account of certain relativistic corrections to the moment which are a result of the rapid motion of the nucleon under nuclear forces. Even for the deuteron the relativistic correction is of the order of one percent of the total deuteron moment, and it is just because the sign of the relativistic correction is unknown that the amount of ^3D state admixture in the deuteron ground state cannot be fixed from the observed value of the deuteron moment. The hfs contribution of the relativistic correction is of the order of 10^3 times the present experimental uncertainty in the hfs, so it can be seen that if the experimental precision were much improved, it would be necessary to consider the effect which the electron recentering has on the hfs contribution of the relativistic correction. For this reason the hfs contribution of the relativistic correction was considered in some detail.

Because the relativistic corrections have already been discussed extensively for the deuteron, it was simpler to do the detailed calculations for deuterium. The nuclear Hamiltonian which was used is that given by Breit[17] for a neutron-proton interaction which transforms like a scalar. In order to include hfs effects of order $(v/c)^2$ in the nucleon the well-known expression[18] for the magnetic interaction of two Dirac particles was used to obtain the proton Dirac moment hfs and a corresponding approximation[19] to obtain the Pauli moment hfs of the neutron and proton. The method of handling the nuclear wave function was the same as that of Breit and Bloch.[9] The procedure was to calculate the contributions to the hfs which are of order $(v/c)^2$ in the nucleons and to compare these with the relativistic correction to the magnetic moment of the deuteron, which is given by:[20]

$$\Delta_R\mu_D = -\mu_N\langle\boldsymbol{\sigma}_\nu T/6Mc^2\rangle - (\mu_P-\mu_0)\langle\boldsymbol{\sigma}_\pi T/6MC^2\rangle - \mu_0\left\langle\boldsymbol{\sigma}_\pi\frac{[2E_D-T]}{2Mc^2}\right\rangle - \mu_0\left\langle\boldsymbol{\sigma}_\pi\frac{r_{\pi\nu}}{12MC^2}\frac{\partial V_{\pi\nu}}{\partial r_{\pi\nu}}\right\rangle, \quad (14)$$

in which T is the internal kinetic energy of the deuteron, $V_{\pi\nu}$ is the (central) neutron-proton interaction potential, E_D is the energy of the deuteron ground state, and the angular bracket denotes expectation value in the deuteron ground state.

The following effects were found to result from the recentering of the electron:

(a) The hfs contribution of the relativistic correction to the neutron Pauli moment (first term on the right-hand side of Eq. (14) shows a somewhat complicated "Bohr effect."

(b) The hfs contributions of the corrections given by the second and third terms of Eq. (14) are normal, i.e., unaffected by the recentering.

(c) The hfs contributions of the fourth term of Eq. (14) shows an "orbital effect."

In the hfs calculation there arise certain divergent integrals which contribute an uncertainty in the hfs of the relative order $\alpha^2(m/M)\ln(b/a)$ in which b is the "proton radius." These integrals have been discussed in the case of the H^1 hfs[21] and are thought to be small. In any case they should be about the same for H^1, H^2, and H^3, since most of the contribution to such integrals is from distances much smaller than the region of centering. It may be expected, therefore, that in the observed H^1-H^3 hfs ratio these effects just cancel, so they do not need to be considered in estimating the uncertainty in the tritium hfs due to the relativistic correction.

On the basis of the deuterium hfs calculation it is possible to estimate the size of the tritium hfs effect. For this purpose the triton wave function may be taken to be simply a ^2S state. The bulk of the hfs comes from the proton spin moment, so all of the relativistic correction contributes normally to the hfs with the exception of the part corresponding to the fourth term in the deuteron moment correction Eq. (14). This correction to the deuteron moment resembles the Thomas correction to the hydrogen fine structure; it may be thought of as resulting from a precession of the proton spin due to the acceleration of the proton by the nuclear forces. From this point of view the orbital effect for its hfs contribution results because there is no acceleration of the proton relative to the recentered electron.

By the virial theorem the correction in question may be written:

$$-\mu_0\left\langle\boldsymbol{\sigma}_\pi\frac{r_{\pi\nu}}{12MC^2}\frac{\partial V_{\pi\nu}}{\partial r_{\pi\nu}}\right\rangle = -\mu_0\left\langle\boldsymbol{\sigma}_\pi\frac{T}{6MC^2}\right\rangle = -\mu_0\left\langle\boldsymbol{\sigma}_\pi\frac{T_\pi}{3MC^2}\right\rangle. \quad (15)$$

The hfs effect can now be estimated for tritium by assuming Eq. (15) to give the corresponding correction to the triton moment. Before making a numerical estimate, however, it should be noted that if a vector, instead of a scalar potential had been assumed then: (a) the correction given in Eq. (15) would have been of the opposite sign, and (b) there would be an additional correction to the magnetic moment arising from the second-order correction to the Hamiltonian,[9] and the hfs contribution of this correction would also show an

[17] G. Breit, Phys. Rev. **51**, 248 (1937).

[18] G. Breit, Phys. Rev. **34**, 553 (1929).

[19] See the magnetic interaction used by J. Schwinger, Phys. Rev. **78**, 135 (1950).

[20] The correction given by Eq. (14) agrees with that given by Sachs (reference 8) rather than that given by Breit and Bloch (reference 9). Professor Breit has been kind enough to check this part of the calculations and has informed us that Eq. (14) is correct.

[21] Breit and Meyerott, Phys. Rev. **72**, 1023 (1947); Breit, Brown, and Arfken, Phys. Rev. **76**, 1299 (1949).

"orbital effect." Taking into account this last source of uncertainty and using $(T_\pi/Mc^2)\sim 0.01$ it was estimated that the uncertainty in the hfs contribution due to the relativistic correction is of the order of:

$$(\delta_R U/U)=\pm 0.01(\mu_0/\mu_T)(2D/a). \tag{16}$$

VI. RESULTS AND CONCLUSIONS

In order to obtain numerical values of the tritium hfs effects one requires values of r_0, $\bar{r}_{\pi\nu}$ and $(D/\bar{r}_{\pi\nu})$. For the purpose of illustrating the relative magnitudes of the effects expected in the various cases it is sufficient to use the estimates $r_0\sim 1\times 10^{-13}$ cm, $\bar{r}_{\pi\nu}\sim 2\times 10^{-13}$ cm, $(D/\bar{r}_{\pi\nu})\sim 2.5$. This last estimate was obtained from Low's detailed deuterium calculations[5] on the following basis. According to a semiclassical interpretation, the recentering of the electron wave function occurs in that region of space for which the ratio of electron velocity to proton velocity exceeds a certain critical value. This critical ratio, which should be the same for deuterium and tritium, is proportional to $(\bar{r}_{\pi\nu}/D)$. The numerical value of the ratio was determined by requiring that the orbital effect on the deuterium hfs as given by Eq. (6) agree with Low's value. The calculation[22] led to a value $(D/\bar{r}_{\pi\nu})\sim 2.5\pm 1$.

The hfs discrepancy δU is defined as the amount by which the observed tritium hfs differs from that calculated for a point triton having the observed triton moment. With presently attained experimental precision there is no statistically meaningful discrepancy, the data giving:[23,24]

$$(\delta U/U)\approx -[0.06\pm 0.25](2\bar{r}_{\pi\nu}/a). \tag{17}$$

The uncertainty given in Eq. (17) is the sum of the quoted experimental uncertainties in the measurement of the hfs and in the direct measurement of the magnetic moment. In Table I the experimental discrepancy is compared with the discrepancy predicted on each of the theories of the triton moment which were discussed in Sec. IV.

The amount by which the theoretical values in Table I are uncertain is large. The greatest uncertainty comes from the crudeness of the electron wave function which was used to obtain the magnetic field at the nucleus.

TABLE I. The hfs discrepancy $(\delta U/U)$ in units of $(2\bar{r}_{\pi\nu}/a)$.

Observed	-0.06 ± 0.25
SOM: 40 percent P state	-0.1
100 percent P state	-0.06
PIM: velocity dependence—nonexchange	-0.3
exchange	-0.1
MEM	-0.1
Relativistic correction	± 0.01

The other chief source of uncertainty is that, since the radial dependences of the several triton states are unknown, $\bar{r}_{\pi\nu}$ cannot be calculated for each case separately. Other sources of uncertainty seem likely to be relatively unimportant. It has already been seen that the relativistic effects of internal nuclear motion are not large enough to spoil the interpretation of the hfs effects. Effects due to the possible spatial extension of the proton moment, which have been discussed by Bohr and Low for the case of the deuterium hfs, can be ignored in interpreting the tritium hfs, since such effects should be the same for H^1 and H^3 and would not, therefore, affect the tritium-hydrogen hfs ratio. Finally, the hfs uncertainties depending on the cutoff of the Coulomb field at the "proton radius" should be unimportant as was asserted in Sec. V.

Within the present experimental uncertainties the predicted hfs effect is consistent with experiment no matter which theory of the triton moment is considered. In order to observe effects of the size predicted it would be necessary to increase the experimental precision by at least a factor of 2 but preferably by a factor of 4 or 5. Unfortunately, the hfs effect is not theory sensitive, and the size of the effect on any one theory can be calculated only very approximately. Only in the case of the PIM theory (velocity dependence without space exchange) is the effect sufficiently different to be distinctive. While an increase of experimental precision might permit a test of the suitability of that particular theory, it is quite possible that the evidence would remain ambiguous because of the permissability, mentioned in Sec. IV, of a mixture of the two kinds of PIM theory. It is necessary to conclude therefore, that the tritium hfs is likely to furnish only meager information about the magnetic structure of the triton, since it is improbable that the information which could be obtained from the hfs would afford any insight into the process which leads to the existence of the triton magnetic moment anomaly.

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[22] As a check the Bohr effect for the deuterium ^3S state was recomputed taking into account the fact that there is a small, but non-zero, probability for the occurrence of $r_{\pi\nu}>D$. The value for the Bohr effect found by this calculation differs from that given in Eq. (6) by about 15 percent, the correction depending sensitively on the value of D. The value of the correction agrees with that of a similar correction given by Low if $(D/\bar{r}_{\pi\nu})\sim 2$, in agreement with the value found from the orbital effect.

[23] E. B. Nelson and J. E. Nafe, Phys. Rev. **75**, 1194 (1949).

[24] Bloch, Graves, Packard, and Spence, Phys. Rev. **71**, 551 (1947).