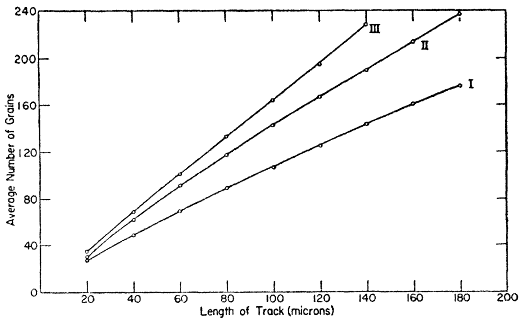

FIG. 1. Average number of grains in a given length of track as measured from the end of the track. Curve I is for protons, curve II is for deuterons, and curve III is for α-particles.

for the α-tracks were greater than the actual measured ranges. The discrepancy was as much as ten percent. In determining the predicted ranges, use was made of the range-energy curve for α-particles as given by Lattes.[4] The curve was verified below 10 Mev but above this value serious departure was found.

The error was assumed to have come from one of two sources. Either the cyclotron beam did not contain 20-Mev α-particles but rather 17.5-Mev α-particles, or the range-energy curve was in error above 10 Mev when applied to Ilford E-1 nuclear emulsions. The first source was eliminated by two considerations; first, it was improbable that a cyclotron producing 10-Mev deuterons would produce 17.5-Mev α-particles, and second, the elastically scattered α-particle tracks from C^{12} and O^{16} at angles greater than 90° to the cyclotron beam were correctly predicted assuming 20-Mev α-particles and using the lower part of the range-energy curve. The curve may be modified as shown in Fig. 2. Curve I is that given by Lattes and curve II is that derived from the Lattes' proton range-energy curve which this author has found to apply to Ilford E-1 nuclear emulsions when various targets were bombarded with 5-Mev protons from the Washington University cyclotron. By assuming that within the experimental limits an α-particle and a proton of the same velocity has the same range, i.e., an α-particle with four times the energy of a proton, loses energy four times as fast due to its double charge, one arrives at curve II. With the modified curve, the ranges of the elastically scattered α-particles are accurately predicted.

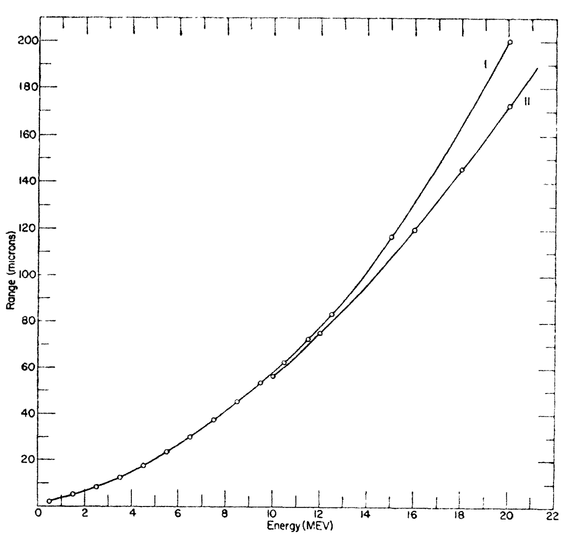

FIG. 2. Alpha-particle range-energy curve. Curve I is that obtained by Lattes (see reference 4). Curve II is a modified curve as applied to Ilford E-1 nuclear emulsions.

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[1] T. R. Wilkins and H. J. St. Helens, Phys. Rev. **54**, 783 (1938).
[2] M. J. Wilson and W. Venselow, Phys. Rev. **75**, 1144 (1949).
[3] Results to be published.
[4] Lattes, Fowler, and Cuer, Proc. Phys. Soc. London **59**, 883 (1947).

The Probable Energy Loss of Electrons in Matter*

S. D. WARSHAW† AND JOSES J. L. CHEN
University of Southern California, Los Angeles, California
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ALTHOUGH the energy degradation of electrons passing through matter is of interest in many experiments, there have been no good measurements of this quantity since the early work of White and Millington.[1] While the average energy loss can be calculated by more or less familiar formulas,[2] Christy has recently emphasized[3] that the use of the *average* rather than the *most probable* energy loss for charged particles can lead to as much as 20 to 30 percent error in the case of moderately energetic electrons. The most probable loss has been calculated most recently by Landau[4] who obtained the energy loss distribution in the form of a universal numerical function, and thereby an expression for the most probable loss. In the result there is a characteristic non-linear dependence of the probable loss on the thickness of the material traversed. This is also true of a classical expression[5] in which, however, the velocity dependence is slightly different and which has a different range of validity. Landau's expression is

$$\Delta E_{\mathrm{prob}} = \xi[\ln(\xi/\epsilon)+0.37], \tag{1}$$

where

$$\xi = nZ(2\pi e^4/mc^2\beta^2) \quad \text{and} \quad \epsilon = [I^2(1-\beta^2)/2mc^2\beta^2]\exp(\beta^2),$$

and n is the number of atoms, with Z electrons each, per cm^2 of the foil material, β is the velocity of the incident electron in units of the velocity of light c, and m is the electronic mass. The mean excitation potential I is given by Mano.[6] The classical result[5] differs from this only by the introduction of a factor $2(k/K)^2$ in the argument of the logarithm, where $k=1.123$, $K=2v_e/v$ and $v_e=e^2/\hbar$ cm/sec. For $K \ll 1$ Bohr[7] has shown that the quantum treatment is the more appropriate.

The non-linear dependence of the loss on the thickness of stopping material becomes appreciable when foils thicker than some minimum are used, this minimum being defined by the condition of compound scattering.[8] To satisfy this condition while still keeping the thickness much less than the range of electrons in the material is, however, not difficult. For the β-particles from the K-conversion line in Ba 137 (624 kev) traversing aluminum foils, it is required that the thickness be in the middle portion of the range between 0.2 and 260 mg/cm². Hence a direct measurement of the probable loss with a foil thickness from about 5 to 40 mg/cm² of aluminum may be used to verify Landau's expression. The resulting curve of loss *versus* foil thickness should show a small but observable curvature. Our preliminary measurements were made on the shift of the Ba^{137} K-line for various thicknesses of both aluminum and tin. A double thin lens β-ray spectrometer, which is described elsewhere,[9] was used. The stopping material was mounted on a supporting frame which was then placed immediately in front of the conventional thin source holder. With the spectrometer set for 2.1 percent resolution and with 100 microcuries of Cs^{137} as the source, the transmission curve of the K-conversion line was measured as a function of foil thickness and the maxima of these curves were taken to correspond to the most probable final momentum in each case. A comparison of the peak shifts, on an energy scale, with Landau's expression (Eq. (1)) for the cases of both aluminum and tin, is given in Fig. 1, with the average loss for aluminum, as calculated from the Bethe-Bloch formula,[2] included for contrast. A slight correction for resolution to the apparent peak location has been applied to the experimental values, the correction being about 0.04 to 0.28 kev for aluminum points and 0.26 to 1.02 kev for tin points, in a total loss from 5 to 44 kev. It can be seen that the agreement is excellent; however, other workers[10] report that a similar set of measurements, which appeared while this work was still in progress, show only "reasonable" agreement.

For the resolution used, quantitative measurement of the energy distribution is not possible; however, the line shape as a function of foil thickness (Fig. 2) is in qualitative agreement with Landau's theory.

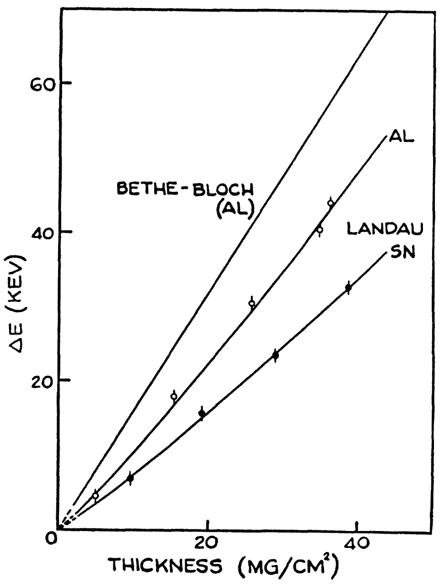

FIG. 1. Probable energy loss *versus* thickness curves for Al(O) and Sn(●) foils. Solid lines are from Landau's theoretical expression. The curve for *average* loss in Al (Bethe-Bloch) is given for comparison.

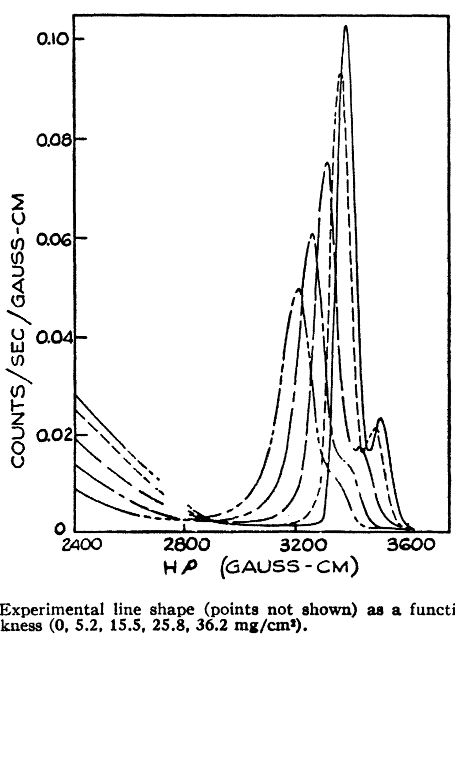

FIG. 2. Experimental line shape (points not shown) as a function of Al thickness (0, 5.2, 15.5, 25.8, 36.2 mg/cm²).

Further work, using lines of different energies, and with other foil materials, is now in progress.

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† Now at the Institute of Radiobiology and Biophysics, University of Chicago, Chicago, Illinois.

[1] P. White and G. Millington, Proc. Roy. Soc. **A120**, 701 (1928).
[2] W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, London, 1944), second edition, p. 219 ff.
[3] Unpublished work.
[4] L. Landau, J. Phys. U.S.S.R. **8**, 201 (1944).
[5] See the discussion in Rutherford, Chadwick, and Ellis, *Radiations from Radioactive Substances* (Cambridge University Press, London, 1930), p. 438 f.
[6] M. G. Mano, Comptes Rendus **197**, 319 (1933).
[7] N. Bohr, Kgl. Danske Vid. Sels. Math.-fys. Medd., Acad. Copenhagen **28**, 8, 78 (1948).
[8] E. J. Williams, Proc. Roy. Soc. **A169**, 531 (1939).
[9] Van Atta, Warshaw, Chen, and Taimuty, Rev. Sci. Inst. (to be published).
[10] Birkhoff, Hayes, and Goudsmit, Phys. Rev. **79**, 199 (1950).

Effect of Finite Range Interactions in the (jj) Coupling Shell Model

DIETER KURATH
University of Chicago, Chicago, Illinois
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RACAH has presented as an argument[1] against the jj coupling model the statement that for identical particles with Majorana interaction the spin of the ground state is not always that of the odd nucleon. The basis of this statement is the fact that the statistical weight of singlet coupling for particles in equivalent orbits is given by the expectation value of the operator:

$$\Sigma S_{ik}=\Sigma(\tfrac{1}{4}-\mathbf{s}_i\cdot\mathbf{s}_k).$$

This expectation value attains a maximum for minimal I consistent with the exclusion principle, and would thus predict $I=\frac{3}{2}$ for three particles or three holes in a shell.

For identical particles, the Majorana operator is

$$\Sigma P_{ik}=\Sigma(-\tfrac{1}{2}-2\mathbf{s}_i\cdot\mathbf{s}_k),$$

so it leads to the same conclusions. If one chooses the interaction potential of a pair of nucleons as

$$V_{12}=P_{12}\{V_0\exp[-(r_{12}/r_0)^2]\}$$

where r_0 is a parameter giving the range of nuclear forces, one sees that Racah's considerations apply to the case of infinite range, r_0. It is known[2] that with spin-orbit coupling, a δ-function radial dependence gives for the spin of the ground state, the spin of the odd nucleon. Since this is the case of zero range, an investigation has been made to see at what range this level crosses the level $I=\frac{3}{2}$.

Calculations were carried out for three particles in the $1d_{5/2}$ and $1f_{7/2}$ shells, these being the first two cases where the spin of $\frac{3}{2}$ differs from that of the odd nucleon. Spin-orbit coupled functions were used whose radial dependence is the harmonic oscillator function with no nodes,

$$R_l(r)=N_l r^l\exp[-(r/r_l)^2],$$

where l refers to the orbital angular momentum. Energy levels were obtained whose separation depends only on the contribution from the unfilled shell and hence on the ratio (r_l/r_0). These contributions to the energy are plotted in Fig. 1 in units of $(-V_0)>0$. In the $1d_{5/2}$ case, the level $\frac{3}{2}$ is lower for $(r_d/r_0)<0.755$. In the $1f_{7/2}$ case, the $\frac{3}{2}$ level is lowest for $(r_f/r_0)<0.738$, $5/2$ is lowest for $0.738<(r_f/r_0)<0.787$, and for greater (r_f/r_0), $7/2$ is the ground state.

In order to see at what range these cross-overs occur, the value of r_l may be correlated with empirical nuclear constants. This is done by calculating the square-well problem to give the nuclear radius as $1.48\times10^{-13}A^{\frac{1}{3}}$, and also the experimental binding energy of the last nucleon. The oscillator wave function is then picked