present form, however, this theory allocates to the cosmic radiation a volume (radius $\sim 10^{17}$ cm) which seems to be too large to avoid nuclear collisions of primary nuclei with intraplanetary matter unless condition (ϵ) is satisfied and an appreciable fraction of the radiation is absorbed in meteoric material.

We wish to express our gratitude to Dr. John Spence of Eastman Kodak Company for preparing special plates, to Miss K. Haile and Mr. R. Richard who carried out most of the scanning, to Mr. T. Putnam for the careful development tests and the microphotographs.

PHYSICAL REVIEW VOLUME 80, NUMBER 6 DECEMBER 15, 1950

Double Transmission and Depolarization of Neutrons

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(Received August 2, 1950)

Slow neutrons from the chain reacting pile, polarized by passage through highly magnetized iron, have been analyzed by transmission through additional iron (the "double transmission" effect). The large double transmission effects expected from the polarization cross section of iron were obtained only when the magnetic field between polarizer and analyzer was adjusted carefully to obtain non-adiabatic transitions of the neutrons relative to the field. The general methods developed have been used to measure the change in polarization of a neutron beam with passage through unmagnetized iron. The rate of depolarization is a measure of the domain size in the iron and domain sizes have been measured in this way for a number of Armco iron samples of different metallurgical treatment.

I. INTRODUCTION

NUMEROUS investigations[1–13] of the polarization of slow neutrons have been made since Bloch[14] first pointed out the possibility of producing such polarization. This phenomenon arises from the presence of an appreciable magnetic interaction between a slow neutron and a paramagnetic atom and in addition the presence of interference between the nuclear and magnetically scattered neutron waves in the case of a ferromagnetic scatterer. The scattering cross section, σ, per atom in a ferromagnet can be written in the form

$$\sigma=\sigma_0\pm p, \qquad (1)$$

where p is the term arising from interference and σ_0 is the sum of the cross sections due to the nuclear interaction alone and the magnetic interaction alone. The double sign of the interference term refers to the two cases of neutron spin parallel and antiparallel to the atomic spins in the ferromagnet. In the case of transmission through a nearly saturated ferromagnet, these two neutron spin states are attenuated to a different extent, corresponding to the difference in the scattering cross sections for these states. Thus an initially unpolarized beam is changed to a polarized one in which there is an excess of neutrons with one sign of spin component, along the field direction in the ferromagnet, over those with spin component of opposite sign. If the ferromagnet is far from saturation, strong depolarizing effects are present also, which nullify the polarizing effects in the magnetic domains.

The first studies of neutron polarization used a polarizer-analyzer arrangement[1–4] in which the intensity of the beam transmitted by two magnetized blocks of iron was measured before and after reversing one of the fields and these intensities compared with the intensity with the blocks unmagnetized. The dependence of the transmission of the analyzer block on its state of magnetization is empirically a measure of the polarization of the incident beam and this effect furnishes a direct way of studying neutron polarization. The intensity differences to be expected from the postulated nature of the magnetic interaction can be understood as follows. With I_+ and I_- as the intensities of neutrons in the two spin states relative to the field, the intensity I transmitted by a saturated iron slab of thickness d is given by

$$\begin{aligned} I\equiv I_++I_-&=\tfrac{1}{2}I_0\exp[-Nd(\sigma_t-p)]\\ &\quad+\tfrac{1}{2}I_0\exp[-Nd(\sigma_t+p)]\\ &=I_0\exp(-N\sigma_td)\cdot\cosh(Npd), \end{aligned} \qquad (2)$$

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where N is the atomic density of iron, σ_t the total cross section for unmagnetized iron, and I_0 the incident (unpolarized) intensity. The transmitted intensity, I, is greater than $I_0 \exp(-N\sigma_t d)$, the intensity transmitted in the unmagnetized case. In the real case of near-saturation compared with zero magnetization, the difference is smaller because of depolarizing effects. With two blocks magnetized parallel, the second augments the difference between the magnetized and unmagnetized intensity. When the two blocks are magnetized antiparallel, the neutrons in the spin state for which the cross section is σ_t+p in the polarizer are attenuated with cross section σ_t-p in the analyzer, and conversely. Thus the second block counteracts the changes in polarization and intensity produced by the first and results in an intensity nearer to that for zero magnetization than with parallel fields.

In the early experiments[1–4] polarization effects were obtained by this method but they were small, and when zero field measurements were included they were inconsistent; i.e., the antiparallel intensities were uniformly higher than those calculated from the parallel and zero field intensities. It was recognized that depolarization within the blocks and between them was the likely explanation of the inconsistency, but the lack of a developed theory of depolarization within the blocks and the poor geometry required by the low intensity sources prevented the solution of the difficulties. Other early methods involving transmission-scattering combination suffered from the same difficulties with the intensity problem accentuated. Emphasis was placed increasingly upon measurements of the intensity change, due to magnetization, in a single iron slab or a series of slabs which would be magnetized only parallel.[3, 5, 7, 8] Subsequently, Halpern and Holstein[15] developed the theory of neutron polarization including the depolarization within the material and the "single transmission effect," E_1, defined as the fractional increase[16] in transmitted intensity produced by magnetizing a ferromagnetic material, has since been the basis of all measurements of the polarization cross section, p.[9–13]

The high intensity thermal neutron beams available from present chain reacting piles provide the resource necessary for a fresh examination of neutron polarization by the polarizer-analyzer method or "double transmission effect." In addition to the intrinsic desirability of a direct and quantitatively satisfactory detection of neutron polarization, this effect provides an independent measurement of the polarization cross section, p. At the time we began this investigation, there was a large discrepancy between the value of p measured experimentally at this Laboratory[12] and the smaller values calculated theoretically,[17, 18] and, to a lesser extent with the recent measurements of other investigators.[9–11] It was felt to be important to show by a direct measurement of polarization that the high degree of polarization predicted by the high p value could be produced. Since the present work was begun, however, Steinberger and Wick[19] have obtained a theoretical value of p in agreement with that of reference 12, and the difference in experimental values has been accounted for as due to differences in the neutron spectra used.[13]

In addition to the study of the double transmission effect, we have applied the techniques developed for double transmission studies to the measurement of magnetic domain sizes in unmagnetized specimens of iron. The depolarization which occurs in a ferromagnet which is not saturated is related theoretically[15] to the linear dimensions of the magnetic domains in the specimen. The fields of the domains deviate in direction from the average so that many irregular precessions of the neutron spin are produced by the components of the domain fields normal to the spin or moment. In the unmagnetized state a ferromagnet has no polarizing effect, the depolarizing effect is isolated, and a measurement of the change in polarization it produces can be related directly to the average linear dimension of the domains of the specimen. We have made such measurements to get information on the range of domain sizes that occur from several treatments of the specimens.

II. THEORY OF DOUBLE TRANSMISSION AND DEPOLARIZATION

For the quantitative description of the double transmission effect we have used the formulas derived by Halpern and Holstein.[15]

$$I=\exp[-(N\sigma_t+\tfrac{1}{2}q)d]\cdot\{I_0[\cosh(d/D)+\tfrac{1}{2}qD\sinh(d/D)]+S_0wD\sinh(d/D)\}, \quad (3)$$

$$S=\exp[-(N\sigma_t+\tfrac{1}{2}q)d]\cdot\{S_0[\cosh(d/D)-\tfrac{1}{2}qD\sinh(d/D)]+I_0wD\sinh(d/D)\}, \quad (4)$$

where I_0, I, N, σ_t, and d have been previously defined, $w=Np$ is the polarization coefficient, q is the depolarization coefficient, and $D=(w^2+\frac{1}{4}q^2)^{-\frac{1}{2}}$. $S=IP=I_+-I_-$, where P is the polarization and $I_\pm$, are the intensities of neutrons with the two orientations relative to the field, and S_0 is the initial value of S.

Equation (3), with the initial polarization zero ($S_0=0$), gives the single transmission effect $E_1(d)$,

$$E_1(d)=[I-I_0\exp(-N\sigma_t d)]/I_0\exp(-N\sigma_t d). \quad (5)$$

The intensities for the parallel and antiparallel cases in a double transmission measurement are obtained by

[15] O. Halpern and T. Holstein, Phys. Rev. **59**, 960 (1941).

[16] The theory of magnetic scattering predicts the existence of a small negative transmission effect, due to purely magnetic scattering, superposed upon the positive effect from interference of nuclear and magnetic scattering. Estimates of its magnitude indicate that its effect on our measurements is negligible. See: F. Bloch and M. Hamermesh, Phys. Rev. **61**, 203 (1942); M. Hamermesh, Argonne National Laboratory Report, ANL-4208, (1948, unpublished); O. Halpern, Phys. Rev. **76**, 1130 (1949).

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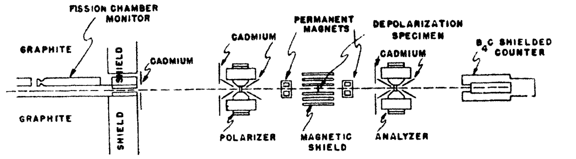

FIG. 1. Apparatus for measurement of double transmission effect and of depolarization of neutrons in magnetic materials.

successive application of Eqs. (3) and (4) for the two blocks. The I and S obtained for the first block from (3) and (4) with $S_0=0$ serves as I_0 and S_0 in (3) for the intensity after the second block, with the sign of S_0 being reversed for the antiparallel case. When w has the same value in polarizer and analyzer, and q likewise, these intensities may be simply expressed in terms of the single transmission effect, E_1, for these values of w and q.

$$I_{\text{par}}=I_0\exp[-N\sigma_t(d_1+d_2)]\cdot[1+E_1(d_1+d_2)],$$
$$I_{\text{anti}}=I_0\exp[-N\sigma_t(d_1+d_2)]\cdot[2(1+E_1(d_1))\times(1+E_1(d_2))-(1+E_1(d_1+d_2))]. \quad (6)$$

It is evident that I_{par} and I_{anti} are symmetric with respect to the value

$$I_0\exp[-N\sigma_t(d_1+d_2)]\cdot[1+E_1(d_1)][1+E_1(d_2)], \quad (7)$$

which corresponds to the final intensity when complete depolarization without loss of intensity occurs between polarizer and analyzer, and also, of course, to the average of the two intensities, I_{par} and I_{anti}. The differences of I_{par} and I_{anti} from (7) represent the effects of the polarization of the beam upon the transmission of the analyzer.

As the measure of the double transmission effect, E_2, we have taken the ratio of the difference of the intensities, measured with fields parallel and antiparallel, to their average,

$$E_2(d_1,d_2)\equiv 2(I_{\text{par}}-I_{\text{anti}})/(I_{\text{par}}+I_{\text{anti}}), \quad (8)$$

so that, from (6) and (7),

$$E_2(d_1,d_2)=2[(1+E_1(d_1+d_2))-(1+E_1(d_1))\times(1+E_1(d_2))]/[1+E_1(d_1)][1+E_1(d_2)]. \quad (9)$$

Previously,[15] the double transmission effect has been defined as the ratio of the difference of intensities with fields parallel and antiparallel to the intensity with both fields zero. While this definition gives a simpler formula, the last member of (9) reducing to the numerator, the inclusion of the zero field measurements brings in the complications of magnetic small angle scattering,[20] especially so when beams of small cross section are used. The effect of this extraneous process can be eliminated only by taking considerable pains. Since other considerations demanded that beams of small cross section be used, it was simpler to use measurements made at high fields, where the small angle scattering is absent, in defining E_2. The formula (9) is convenient for comparison of E_2 measurements with the values to be expected from E_1 measurements on the same material at the same magnetization. Also, it seems to be a simple form for computations when E_2 measurements are being used as an independent measurement of p.

The intensity observed after transmission through a block of magnetized iron depends on both the polarization and the intensity of the incident beam. Thus Eq. (3) has a term proportional to S_0 and one proportional to I_0. In depolarization measurements, the polarization of the beam striking the analyzer is altered by a foil of unmagnetized iron and the corresponding intensity measured. The small additional change produced by scattering and capture in the foil cancels out in the ratios E_p of counts with field to those without. The part associated with the polarization is obtained by subtraction of the ratio obtained with complete depolarization $E_p(\infty)$. The resulting polarization dependent part is studied as a function of depolarizing foil thickness. Formulas relating this dependence to the average linear dimension or size of the magnetic domains in the foil have been given by Halpern and Holstein.[15] These are

$$S=S_0\,\Pi_i[1-\tfrac{1}{3}4\sin^2(gB_i\delta_i/2v)], \quad (10)$$

with the limiting cases

$$S=S_0\exp(-g^2B^2\bar{\delta}d/3v^2),\quad gB_i\delta_i/2v\ll 1, \quad (10a)$$
$$S=S_0\exp[-(\ln 3)d/\bar{\delta}],\quad gB_i\delta_i/2v>1, \quad (10b)$$

where S_0 and S have been defined previously as I_0P_0 and IP, or twice the initial and final spin current densities, g is the gyromagnetic ratio of the neutron, B_i and δ_i the magnetic induction and neutron path length, respectively, in the ith domain along the neutron path, v the neutron velocity, and d the foil thickness.

Equation (10) gives an approximately exponential dependence of S or polarization on thickness for which the half-thickness is given by

$$d_{\frac{1}{2}}=-\frac{0.693\bar{\delta}}{\ln[1-\langle\tfrac{1}{3}4\sin^2(gB_i\delta_i/2v)\rangle_{\text{Av}}]}. \quad (11)$$

The limiting forms for $d_{\frac{1}{2}}$ corresponding to (10a) and (10b) are

$$d_{\frac{1}{2}}=2.079v^2/g^2B^2\bar{\delta},\quad gB_i\delta_i/2v\ll 1, \quad (11a)$$

and

$$d_{\frac{1}{2}}=0.631\bar{\delta},\quad gB_i\delta_i/2v>1. \quad (11b)$$

The average of $\sin^2(gB_i\delta_i/2v)$ has been taken as the linear average over the interval 0 to $\sqrt{3}(gB\bar{\delta}/2v)$ with $B=4\pi M=21{,}600$ gauss (from the saturation magnetization in iron) and $g=1.83\times 10^4$ gauss^{-1} sec.$^{-1}$. We consider this a suitable and consistent evaluation in the intermediate range of domain sizes since Halpern and Holstein[15] state that their formulas give results

[20] Hughes, Burgy, Heller, and Wallace, Phys. Rev. 75, 565 (1949).

correct only as to order of magnitude for values of $\bar{\delta}$, due to uncertainties as to the distribution of domain sizes, demagnetizing fields, and the constancy of the induction B_i within a domain. In view of these uncertainties the effect of "hardening" of the polarization due to the velocity dependence of $d_{\frac{1}{2}}$ has not been included. Instead, the velocity 1500 m/sec. has been used, which corresponds to the average of the exponent in Eq. (10a) over the calculated spectrum for the depolarization work. A plot of $d_{\frac{1}{2}}$ *vs.* $\bar{\delta}$ on these assumptions is given in Fig. 5. The limiting form (11b) is used for $\bar{\delta}>1.4\times10^{-3}$ cm, thus omitting small oscillations given by (11) which are not significant.

III. EXPERIMENTAL ARRANGEMENT AND PROCEDURE

The apparatus used in this work is shown schematically in plan view in Fig. 1. The neutron beam emerges from the thermal column of the Argonne heavy water reactor through a hole in a Cd covered wooden block and passes successively through a steel block between the poles of a polarizing magnet, a magnetic shield, a steel block between the poles of an analyzing magnet, and then passes into a BF_3 proportional counter. The magnets were of identical construction, with 12.7-cm diameter poles of Armco iron coned at 60° to 2.54-cm diameter faces with a 1.27-cm gap. Each magnet was energized by two coils, powered by separate current regulated supplies. Several fans near the magnets provided sufficient cooling for the power absorption of $\frac{1}{4}$ kw per coil. The polarizer and analyzer blocks were of cold-rolled steel, $1.27\times1.27\times d$ cm, cut from the same material that Hughes, Wallace, and Holtzman[12] (hereafter referred to as HWH) used in measurements of the single transmission effect. The blocks were magnetized with a field of 12,000 gauss perpendicular to the beam along the direction of rolling, with the beam direction parallel to the plane of rolling. The field strength was measured at the surfaces of the blocks with a small search coil and a fluxmeter.

The magnetic shield consisted of three coaxial iron cylinders of wall thickness about 1 and 20 cm long, spaced with pieces of brass and Cd, the outer cylinder being an iron pipe with the other two of soft iron. The inner cylinder consisted of two 10-cm lengths so that depolarizing specimens could be mounted at the center of the magnetic shield. The magnetic field at the center of the shield was measured as 0.08 ± 0.08 gauss normal to the shield axis.

The beam was defined by 1-cm square holes in 1-mm Cd diaphragms before and after each steel block, and by a somewhat larger hole at the thermal column which varied in size to obtain the desired neutron intensity, depending upon the block thicknesses. Each magnet was further shielded by a sheet of 1-mm Cd which completely covered the side toward the thermal column, but which had holes sufficiently large and so located that the beam definition by the Cd diaphragms at the steel blocks was not interfered with. The distance of the polarizer block from the thermal column shield was 1.76 m, the distance between the two blocks was 1.65 m with the magnetic shield centered between them, while the center of the counter was 24 cm behind the analyzer block. On the surface of the table supporting the electromagnets and magnetic shield, there were placed two Alnico permanent horseshoe magnets to control the field along the beam outside the magnetic shield.

The counter that we used is the same one that was used by HWH, 20.3 cm in length, 3.74 cm in diameter, filled with 20 cm Hg of B^{10} enriched BF_3 and placed so that the beam traversed the length of the counter. A careful investigation of the effective neutron velocity spectrum with this counter under these conditions by HWH had showed it to be a Maxwell distribution with a most probable velocity of 2250 m/sec. The counter was shielded by a thick surrounding layer of boron carbide. The beam was monitored with a fission chamber inside the thermal column to one side of the beam passing through the double transmission apparatus.

The procedure for measuring the double transmission effect consisted in magnetizing the blocks parallel or antiparallel with predetermined currents, carefully adjusting the permanent magnets so that a compass indicated that the magnetic field at each end of the magnetic shield was accurately perpendicular to the beam and directed the same as the field in the nearer steel block. This resulted also in these field directions in the vicinity of the beam being maintained, approximately, the rest of the way to the respective blocks, and that the field became moderately strong (≈ 50 gauss) a short distance away from the shield and remained strong the rest of the way to the nearer magnetized block. The necessity for these careful adjustments in the attainment of the double transmission effect will be described in Section IV. After taking a counting rate measurement relative to the monitor, the analyzer currents would be reversed, the permanent magnets adjusted for this case and another counting rate measured. Intensities with the magnet currents turned off were taken also, (it was found unnecessary to demagnetize by any special procedure) and frequent background counts were taken. Background measurements, amounting to about three percent of the total counting rate, were taken by placing a 1-mm sheet of Cd over the hole in the Cd shield for the analyzer.

In making depolarization measurements, one or more layers of a certain type of iron would be inserted, normal to the beam, between the two halves of the inner cylinder of the magnetic shield and the transmitted intensity measured as a function of block magnetization as for double transmission measurements. Blocks 2 cm thick were used as polarizer and analyzer. In most cases, only intensities with fields parallel and with no field were measured. This procedure undoubtedly introduces a magnetic small-angle scattering effect through the use of zero current intensity values. How-

TABLE I. Double transmission effect, E_2, and intensity ratios relative to unmagnetized state for parallel (E_{1P}) and antiparallel (E_{1A}) magnetization.

Polarizer thickness (cm) Analyzer thickness (cm)	2.00 2.00	2.00 1.50	2.00 1.00	2.00 0.50	1.50 1.50	1.00 1.50	0.50 1.50	1.00 1.00
E_2, percent	22.10±0.18	18.30±0.30	13.53±0.59	7.32±0.48	15.01±0.40	10.71±0.53	5.95±0.45	7.49±0.78
E_{1P}, percent	33.99±0.24	27.50±0.32	22.51±1.01	17.66±0.53	20.90±0.89	16.11±0.85	11.15±1.03	11.58±0.81
E_{1A}, percent	7.33±0.23	6.12±0.26	7.00±0.87	9.35±0.46	4.02±0.73	4.31±0.83	4.73±0.88	3.53±0.63

ever, the effect is to introduce a constant factor into the observed ratios E_P, which does not affect the slopes of the curves from which $d_{\frac{1}{2}}$ values are obtained.

IV. MEASUREMENTS AND CONCLUSIONS

A. Adiabatic Change of Polarization and the Double Transmission Effect

The observations made on the effect of altering the field between the electromagnets in double transmission were essentially qualitative in nature. No attempt was made to adjust the field to allow quantitative calculations; only the order of magnitude of the field strength was estimated from the oscillation frequency of a compass needle. With 2-cm steel blocks in the polarizer and analyzer magnets, and with the magnetic shield removed, it was found that as long as the field was kept of the order of 100 gauss and slowly varying in direction in the intermediate region, the intensity observed would be independent of reversal of the analyzer field direction, though rotations of the field direction along the beam through 180°, 360°, or more, took place. The intensity in these cases was the same as that observed with the polarizer and analyzer fields parallel and a strong parallel intermediate field. These results are those which one would expect, since for a polarized neutron to change its spin state relative to the field, the field must change direction within a distance small relative to that traveled by the neutron in a Larmor period. For a thermal neutron, this Larmor distance is approximately $80/H$ cm, with H in gauss, so that appreciable changes in the field direction within a distance small compared to 1 cm would be required to effect noticeable depolarization. With the slowly varying field direction that was arranged, the polarized neutrons followed the field direction adiabatically and entered the analyzer with the same orientation relative to the field as in the standard parallel arrangement, whether the analyzer field was parallel to the polarizer field or antiparallel. When the permanent magnets, which were used to control the field in the intermediate region, were removed, the field dropped to a value of the order of one gauss over most of the intermediate region and was irregular in direction. Under these conditions the observed intensity corresponded to complete depolarization, which corresponds to the Larmor precession distance being large relative to the distances over which the field changed direction. It was also found possible, by careful field adjustment, to obtain the full double transmission effect without the use of the magnetic shield. In the parallel fields case the intermediate field was also kept parallel; in the antiparallel case a crossover point was arranged, so that the field as seen by a neutron kept its direction as the neutron left the polarizer but approached zero at a point halfway to the analyzer, then reversed its direction and increased in strength as the analyzer was approached.

These observations bear out the reasons advanced for the failure of previous attempts to obtain double transmission effects of the order expected from measurements of the single transmission effect. With the low intensity sources then available, beams of large cross section and small magnet separations had to be used. Under these conditions the field between the magnets could not be reversed in a manner which prevented most of the neutrons adiabatically reversing their polarization with it.

The measurements of the double transmission effect were made with eight combinations of polarizer and analyzer block thicknesses. The resulting values of E_2 and of the ratios, $E_{1P} \equiv (I_{\text{par}}/I_0)-1$, and $E_{1A} \equiv (I_{\text{anti}}/I_0)-1$, where I_0 is the zero current reading, are given in Table I. The standard deviations listed are the observed root-mean-square deviations of individual measurements and agree well with those calculated from counting statistics. The values of E_2 are also plotted in Fig. 2 as the three functions; (1) $E_2(d, d)$, polarizer and analyzer thicknesses equal; (2) $E_2(2, d)$ polarizer thick-

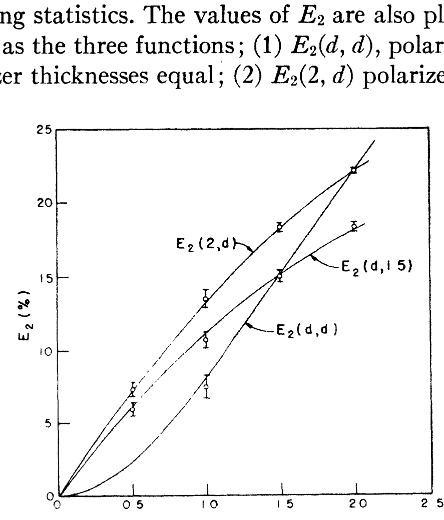

FIG. 2. The double transmission effect as a function of block thickness of polarizer and analyzer. The solid lines are theoretical based on an effective polarization cross section, p, of 2.8 barns and a depolarization coefficient, q, of 0.10 cm^{-1}.

TABLE II. Results of domain size measurements on Armco specimens of different metallurgical treatments.

Specimen	Final treatment		Thickness ($\times 10^3$ cm)	Grain size, Δ ($\times 10^3$ cm)		$d_{\frac{1}{2}}$ ($\times 10^3$ cm)	$\bar{\delta}$ ($\times 10^4$ cm)	$\bar{\delta}/\Delta$
A-1	0%		1.3	0.5		1.9 (2.0)	1.6 (1.5)	
A-2	50%	cold reduction after annealing	1.5	0.5		1.8	1.7	
A-3	97%		1.3	1.2		3.9	0.77	
A-4	98.5%		0.4	0.5		11	0.26	
B-1	36%		2.5	2.2		1.6	1.9; 25	
B-2	68%		2.5	1.5	before final treatment	1.7	1.7	
B-3	84%		2.5	1.4		2.0	1.5	
B-4	92%		2.5	1.3		2.4	1.2	
B-5	96%	cold reduction after anneal and quench	2.5	1.3		2.8 (0.89)	1.1 (3.3)	
B-6	98%		2.5	0.9		2.5	1.2	
B-7	99%		2.5	1.3		4.0	0.75	
B-8	99.5%		2.5	5.7		3.8	0.80	
B-9	99.75%		1.3	5.7		9.9 (5.8)	0.30 (0.52)	
D-1			2.5	2.5		1.1	2.7; 18	0.11; 0.72
D-2	Annealed at 700°C		5.1	3.3		1.5	2.0; 24	0.061; 0.73
D-3			7.6	4.8		2.8	1.1; 44	0.023; 0.92
E-1			2.5	2.4		0.94	3.2; 14	0.13; 0.58
E-2	Annealed at 750°C		5.1	4.4	after final treatment	1.5	2.0; 24	0.045; 0.55
E-3			7.6	4.4		2.4	1.2; 39	0.027; 0.89
F-1			2.5	2.5		1.4	2.1; 23	0.084; 0.92
F-2	Annealed at 800°C		5.1	4.1		1.3	2.3; 21	0.056; 0.51
F-3			7.6	5.7		2.9	1.0; 46	0.018; 0.81

ness constant and equal to 2 cm; (3) $E(d, 1.5)$, analyzer thickness constant and equal to 1.5 cm.

In order to obtain a value of the polarization cross section from the data, the "hardening" (change in velocity distribution with d) of the beam must be considered. The hardening was calculated from the measurements by HWH of the single transmission effect E_1 and of the total cross section of iron as a function of neutron velocity, with the exception of the values for neutron velocities <980 m/sec. where a value of $E_1=0.2$ percent, derived from more recent measurements,[21] were used. These values of E_1 were averaged over the velocity distribution calculated from the total cross section for a series of d's covering the range of the observations. Assuming the validity of the usual quadratic approximation $E_1=\frac{1}{2}(Npd)^2f(x)$ for the thickness $d=1.32$ cm used by HWH, and their results that the depolarization function $f(x)$ is independent of neutron velocity for the cold-rolled steel used, these averages give numbers proportional to $\langle p^2\rangle_{\text{Av}}$ for the corresponding thicknesses. The square roots of their ratios to the value for the incident Maxwell spectrum (p_0) give the desired hardening function, $R=[\langle p^2\rangle_{\text{Av}}/\langle p_0^2\rangle_{\text{Av}}]^{\frac{1}{2}}$, plotted in Fig. 3. We have not used the usual quadratic approximation further in our analysis, since the condition for its validity, $Npd\ll 1$, is not fulfilled under our conditions, $Np\approx\frac{1}{4}$ cm^{-1}. Expansions of the formulas (3), (5), and (9) show that terms in p^4 become appreciable at our larger thicknesses. However, we have assumed that our root-mean-square averages adequately describe the hardening effect on our measurements. From our values of E_2, we obtain as most probable values, $p_0=2.80\pm 0.15$ barns and $q=0.10\pm 0.05$ cm^{-1}. The curves in Fig. 2 have been calculated from these values. [For calculating $E_2(d_1, d_2)$ from (9), it should be noted that

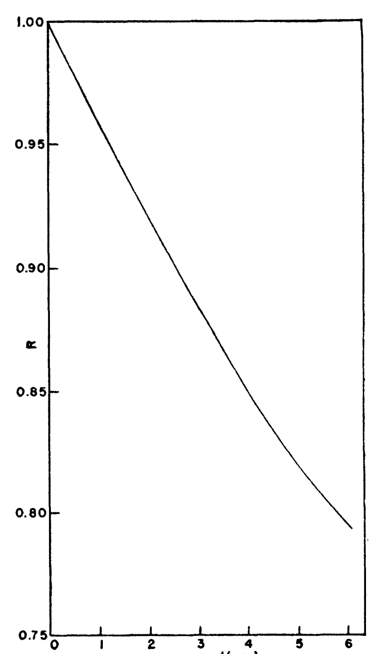

FIG. 3. The theoretical ratio of the r.m.s. value of p to that for a Maxwell distribution. The decrease in p is caused by the hardening of the beam with increasing d.

[21] Burgy, Hughes, and Woolf, Phys. Rev. **76**, 188 (1949).

the value of $E_1(d_2)$ must be computed using the hardening correction on p for the thickness d_1+d_2.]

These results are in disagreement with the measurements of HWH with the same neutron spectrum using the single transmission effect. They obtained $p_0=3.15 \pm 0.10$ barns and $q=0.493$ cm^{-1}, that is, a somewhat higher polarization cross section and a much larger depolarization due to incomplete saturation. The most likely systematic error which may have entered into our measurements is incomplete "piping" of the neutrons from the polarizer field into the analyzer field in the antiparallel case. Another possibility of error is that our electromagnets are less likely to provide a uniform field in the steel blocks than did the magnet used by HWH. On the other hand, if our work is correct, the systematic error most likely to have been present in their work is incomplete elimination of small-angle scattering effects. However, in connection with this possibility it should be mentioned that HWH had evidence that these effects had been eliminated and further studies[20] have confirmed the adequacy of their method of elimination. Despite the disagreement, the original purpose of checking that p was about three times the previous theoretical estimates by direct analysis of the polarization, has been fulfilled.

In the depolarization work, in which 2-cm polarizer and analyzer thicknesses were used, the polarization of the beam incident upon the depolarizing foils is 38 percent as calculated from the values of p and q obtained from the double transmission work. The depolarization studies were made on Armco iron after a few preliminary measurements. A wide range of domain sizes was sought by cold-working and annealing the Armco specimens. To obtain small domain sizes, specimens were prepared having cold reduction in thickness ranging from 0 to $99\frac{3}{4}$ percent. To obtain large domains, growth of large grains was induced by cold-work followed by annealing. Several annealing temperatures

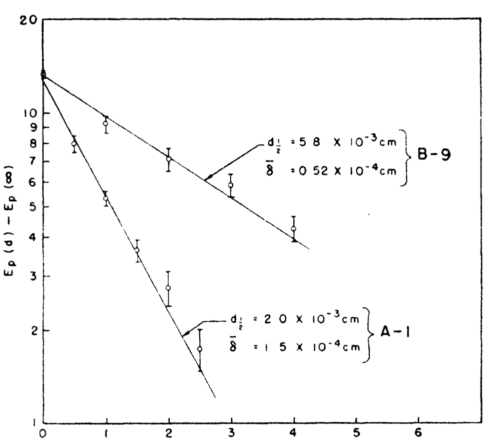

FIG. 4. Decrease of neutron polarization with distance in thin sheets of two Armco samples with different metallurgical treatment (see Table II) and the domain sizes, $\bar{\delta}$, inferred from the rate of depolarization.

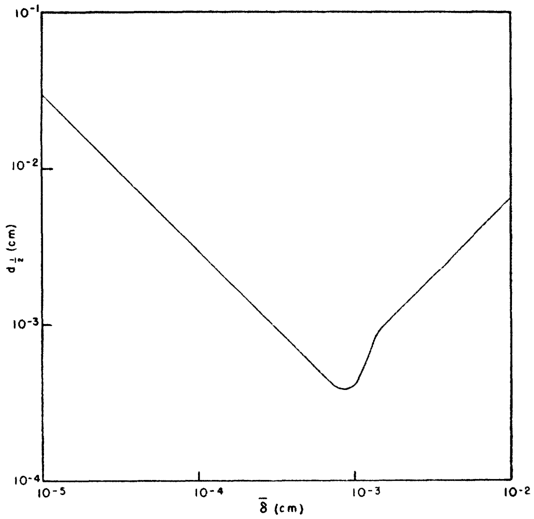

FIG. 5. The theoretical relationship between the half-thickness for depolarization, $d_{\frac{1}{2}}$, and the mean domain size, $\bar{\delta}$.

near the Curie point were used. It was desired to have the specimen thicknesses in the vicinity of 0.025 mm after treatment. Consequently the preparation involved an initial reduction of the Armco stock to a suitable intermediate thickness, followed by annealing to recrystallize the material and free it of strains. The Armco specimens fall into five groups according to their treatment. Those in group A were prepared from 0.0185-in. sheet, and, after the initial reductions in thickness, were annealed at 700°C for 15 min. The final cold reductions are listed in Table II. The specimens in group B were made from 1-in. round stock. Slabs of 0.200-in. thickness were cut from the stock, given an initial cold reduction, annealed at 925°C and water-quenched. The final cold reductions in thickness are listed in Table II. The purpose of the water quench given specimens in group B after annealing was to produce a more uniform grain structure than had been obtained in group A. The average grain sizes in the specimens of groups A and B were measured before giving them the final cold reduction and are listed in Table II. Grain sizes could not be measured after the final reductions. The specimens in groups D, E, and F were rolled from 0.0185 in. to a thickness about 10 percent greater than the planned final thickness, annealed at 710°C, reduced to their final gauge and further annealed 24 hr. The final annealing temperatures were 700°C for group D and 750°C for group E, both below the Curie point, and 800°C for group F, which is above the Curie point. All three annealing temperatures leave the specimens in the alpha-phase. The final thicknesses for groups D, E, and F are given in Table II, together with the grain sizes measured after the final anneal. Standard metallographic techniques were used in all grain size measurements.

To obtain the ratio $E_P(\infty)$ (of count with parallel fields to count without) corresponding to complete de-

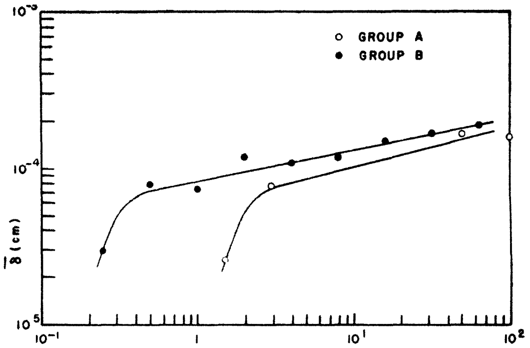

FIG. 6. Relation between domain sizes in groups A and B and the ratio of thickness of specimen after final cold reduction to thickness before, in percent.

polarization of the beam incident upon the analyzer, a foil of steel shim stock, 0.266 mm in thickness was used as the depolarizer. This gave essentially complete depolarization, as checked by the measured half-thickness for depolarization in shim stock and by the double transmission effect with this specimen in place, no difference between parallel and antiparallel intensities being found. Plots of $E_P(d)-E_P(\infty)$ were made for each type of specimen and the half-thickness $d_{\frac{1}{2}}$ for depolarization determined. Curves for two of the specimens are given in Fig. 4.

The $d_{\frac{1}{2}}$ values and corresponding $\bar{\delta}$-values obtained for the specimens are listed in Table II. Figure 5 shows that $\bar{\delta}$ is a two-valued function of $d_{\frac{1}{2}}$, and this results in an ambiguity in the values obtained for $\bar{\delta}$ in ten of the specimens examined. In the remaining twelve a single value is obtained for $\bar{\delta}$, since the larger of the two values given by Fig. 5 is greater than the foil thickness. Where two values of $\bar{\delta}$ are not excluded by this consideration, both are listed in Table II. The values of $d_{\frac{1}{2}}$ and $\bar{\delta}$ given in parentheses are the results of repeat measurements made about six months after the initial studies. An inspection of the results for groups A and B show a considerable decrease in domain size with increase in the severity of cold-working the material. This is shown graphically in Fig. 6 where domain size is plotted against the ratio of sample thickness after the final reduction to thickness before reduction in percent. The cold-working of the iron samples produces strain and breakup of the crystal grains, so that the observed decrease in domain size with increase in cold-work shows that as the size of the crystal grains becomes small, it controls the size of the domains. In groups D, E, and F large-size grains were grown in an effort to obtain large-sized domains, and variations due to difference in the annealing temperature in the vicinity of the Curie point were sought. The measurements show that annealing above the Curie point (group F) gave essentially the same results as annealing below it (groups D, E). The domain sizes of specimens in these three groups are given two values from Fig. 5. Choice of the smaller $\bar{\delta}$-values would give an unlikely inverse relation between domain size and grain size. On the other hand, choice of the larger $\bar{\delta}$-values in these three groups gives an approximate proportionality between domain size and grain size ($\bar{\delta}\approx 75$ percent of grain size). On the assumption of the latter relation the domain sizes of the specimens examined vary in size by a factor of 180, from 0.26×10^{-4} cm (A-4) to 46×10^{-4} cm (F-3). On three of the specimens (A-1, B-5, B-9), the measurements were repeated after a six-month interval (values given in parentheses in Table II). For the two which had had a cold-work treatment the later values of $\bar{\delta}$ were larger than those obtained initially. Thus B-5 and B-9, which had undergone cold reduction of 96 and $99\frac{3}{4}$ percent, respectively, showed increases in $\bar{\delta}$ by factors of 3 and 1.7, respectively. On the other hand, specimen A-1, which had not undergone cold-working, showed an insignificant change. We interpret the changes in domain size noted in the cold-worked samples as being due to grain growth from recrystallization and release of strain. As part of these later measurements, a test was made to see if extreme cold-rolling had produced preferred orientations of the magnetization of the domains. The specimens in group B had been worked by rolling in a single direction. If this resulted in a clustering of easy axes of magnetization about the rolling direction, the depolarization observed when the neutron polarization was parallel to this direction would be much less than when perpendicular. Measurements were made on the strongly worked sample B-9, and no difference was found between the parallel and perpendicular orientations of the neutron polarization relative to the rolling direction. A further test of the effect of extreme cold-rolling on magnetic domains was made by comparing the depolarization, obtained by sending the neutron beam through a specimen at a 60° angle with the normal to the specimen surface, to the depolarization at normal incidence. With the strongly worked specimen B-9 ($99\frac{3}{4}$ percent reduction) the values $d_{\frac{1}{2}}=4.7\times 10^{-3}$ cm, $\bar{\delta}=0.64\times 10^{-4}$ cm were obtained at 60° incidence and $d_{\frac{1}{2}}=5.8\times 10^{-3}$ cm, $\bar{\delta}=0.52\times 10^{-4}$ cm at normal incidence. Thus it would appear that the domains in this specimen have greater dimensions in the plane of rolling than they have normal to this plane. However, this variation of $\bar{\delta}$ with direction relative to the plane of rolling is not present in all strongly worked specimens as a similar comparison with specimen A-3, which had undergone 97 percent reduction, gave no difference.

We wish to acknowledge the extensive help of S. Hugh Paine, of the Metallurgy Division, Argonne National Laboratory, who prepared the Armco specimens for our domain size measurements and made the grain size determinations in them.