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The Conservation of Energy and Momentum in Compton Scattering[*]

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An experiment similar to that of Bothe and Maier-Leibnitz and others has been carried out to verify that in Compton scattering of 2.62-Mev quanta the scattered quantum and recoil electron are emitted without delay and in the relative directions demanded by the conservation laws. Better angular resolution and statistics and fewer experimental uncertainties than in the previous experiments were obtained. The results are in agreement with simultaneity and the conservation of energy and momentum.

I. INTRODUCTION

IN 1936 an attempt was made by Shankland[1] to verify that energy and momentum are conserved in Compton scattering. The negative result he at first obtained stimulated a number of repetitions of this experiment[2] as well as considerable theoretical speculation,[3] particularly over a possible revival of the statistical theory of Bohr, Kramers, and Slater.[4] In these experiments it was hoped to show that when a beam of gamma-quanta is scattered, coincidences between the counters detecting the scattered photon and recoil electron could be obtained only when the counters were in the relative directions from the scattering foil required by the conservation laws.

Although these experiments were carried out with relatively poor angular resolution and statistics, and with a number of uncertainties, difficult to estimate quantitatively, there is no reason to doubt their unanimous conclusion that contrary to Shankland's original result the conservation laws are indeed satisfied.[5] Neither do we know of any further evidence suggesting that energy and momentum are not conserved in this process. Nevertheless it was felt that the technical advances made since 1936, which permitted a considerable reduction in the large experimental uncertainties, justified the repetition of an experiment on so fundamental a point.

II. GENERAL ARRANGEMENT

The general arrangement of this experiment, shown in Fig. 1, is similar to that used by Bothe and Maier-Leibnitz and others. A filtered, collimated beam of gamma-rays from RaTh strikes a Be foil. The recoil electron and scattered photon are detected by two counters whose output pulses are put in coincidence. According to the photon theory and the conservation laws, coincidences should be obtained when and only when the two counters and the beam are coplanar, and when the angles θ and ϕ of the gamma- and electron counters satisfy

$$\cot\phi=(1+\gamma)\tan\tfrac{1}{2}\theta, \tag{1}$$

where γ is the energy of the incident quantum in units of the rest energy of the electron.

The primary difficulty of the experiment lies in the

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[1] R. S. Shankland, Phys. Rev. **49**, 8 (1936).

[2] R. S. Shankland, Phys. Rev. **50**, 571 (1936); **52**, 414 (1937). J. C. Jacobsen, Nature **138**, 25 (1936); W. Bothe and H. Maier-Leibnitz, Zeits. f. Physik **102**, 143 (1936); Phys. Rev. **50**, 187 (1936); G. Bernardini and S. Franchetti, see Bretscher, "*Kernphysik*" (Verlag. Julius Springer, Berlin, 1936).

[3] P. A. M. Dirac, Nature **137**, 298 (1936); R. Peierls, Nature **137**, 904 (1936); E. J. Williams, Nature **137**, 614 (1936); N. Bohr, Nature **138**, 25 (1936); F. Cernuschi, Comptes Rendus **203**, 777 (1936).

[4] Bohr, Kramers, and Slater, Phil. Mag. **47**, 785 (1924); Zeits. f. Physik **24**, 69 (1924).

[5] The conservation laws were also supported by the cloud-chamber experiments of A. H. Compton and A. W. Simon, Phys. Rev. **26**, 289 (1925) and of Crane, Gaerttner, and Turin, Phys. Rev. **50**, 302 (1936). Simultaneity, although not the conservation laws, was verified by the experiments of W. Bothe and H. Geiger, Zeits. f. Physik **26**, 44 (1924), Naturwiss. **13**, 440 (1925); W. E. Burcham and W. B. Lewis, Proc. Camb. Phil. Soc. **32**, 637 (1936); A. Picard and S. Stahel, J. de phys. et rad. **7**, 326 (1936); and R. Hofstadter and J. A. McIntyre, Phys. Rev. **78**, 24 (1950).

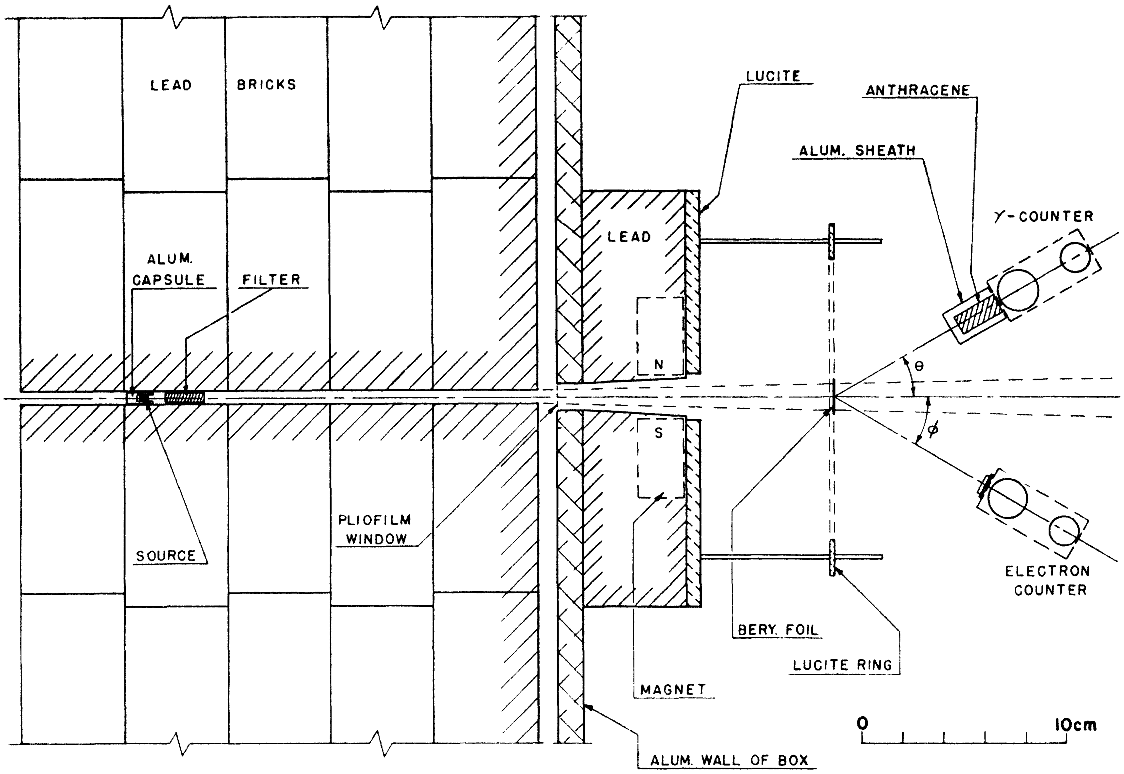

FIG. 1. General arrangement of apparatus.

low coincidence rate. To achieve a rate considerably larger than the background rate without the scattering foil, it is necessary to make the foil thick enough so that further scattering of the recoil electrons in it is appreciable. To minimize this scattering the electrons, and hence also the incident quanta, should have as high an energy as possible. Since there exists no long-lived source of sufficiently high energy monochromatic gamma-rays, corrections must be made for components of other energies in the beam.

It is the aim of the present experiment to show that for a given angle θ of the gamma-counter, the greatest number of coincidences is obtained for that angle ϕ which satisfies Eq. (1), and further that the distribution of coincidences about this angle is accounted for quantitatively by scattering of the recoil electrons in the foil, inhomogeneity of the gamma-radiation, and other reasons to be discussed. Most of the improvements in the present experiment are due to the use of a scintillation counter (instead of a Geiger counter) for detecting the scattered gamma-rays. The higher efficiency and long-term stability of the former, and the use of a stronger source of RaTh than was available to the previous investigators, have permitted us to improve the angular resolution, to reduce the thickness of the foil (and hence the effect of electron scattering in it), and to increase the filtering of the gamma-rays, and at the same time to obtain many more coincidences for each angular position.

The gamma-counter was set at a scattering angle of 30° to which, for the 2.62-Mev quanta of ThC$''$,[6] corresponds a recoil angle of 31.3°. Some advantages of this almost symmetrical arrangement are discussed by Bothe and Maier-Lebnitz.[2] Rather than keep the angle of one counter fixed and vary that of the other about the "correct" direction, we chose, as Bothe and Maier-Leibnitz did, to fix the positions of both counters and move the foil further from or closer to the source than that "correct" position for which the counters were in corresponding directions from the foil. This had the advantage that the background rate without the foil, which depends on the positions of the counters, remained constant, thereby reducing the long counting time.

III. APPARATUS

Gamma-rays from a 195-mg Ra equivalent of RaTh were filtered through 2 cm of lead and collimated in a 3.2-mm diameter hole through a lead shield. The flaring at the end of the collimator was designed to minimize the number of gamma-rays and electrons scattered from the collimator walls which reach the counters. A magnet at the end of the collimator deflected these electrons and reduced the background in the electron counter by

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[6] C. D. Ellis, Proc. Roy. Soc. **A138**, 318 (1934). Recent measurements of Hornyak, Lauritsen, and Rasmussen, Phys. Rev. **76**, 731 (1949) and of J. L. Wolfson, Phys. Rev. **78**, 176 (1950) give values from 2.613 to 2.618 Mev.

40 percent. The beryllium scattering foil, 16 mm square and 13.8 mg/cm^2 thick was supported on a strip of Pliofilm 1 mg/cm^2 thick whose ends were fastened to a light ring of Lucite. Ring and foil could be moved parallel to the beam.

To reduce scattering of the gamma-rays and recoil electrons from the air, the scattering foil and counters were enclosed in a large box, with 0.5-mm thick metal walls, filled with helium. The box walls did not contribute appreciably to the background, while replacing the air with helium reduced the gas scattering by a factor of seven.

The gamma-ray detector was a rectangular parallelopiped of anthracene, 10×18 mm in area and 24 mm thick, in front of a 1*P*21 photo-multiplier. The crystal was covered with an Al foil reflector, while tube and crystal were wrapped with black tape to keep out light. A 3.2-mm thick Al sheath over the crystal kept out scattered electrons. A thicker sheath made no further measurable change in the coincidence rate.

The electron detector was a plate of anthracene, 13×17 mm in area and 2 mm thick, covered by a thin Al window. Between the crystal and 1*P*21 photomultiplier was a 3-mm thick Lucite plate to give the crystal mechanical support. For the electrons which it is required to count (about 1 Mev) the efficiency of this counter was essentially 100 percent. The detectors were mounted on light Al frames, and could be rotated about a vertical axis through the center of the scatterer.

The pulses were amplified in Jordan-Bell-type linear amplifiers,[7] passed through a discriminator and fed into a crystal diode coincidence circuit. The resolving time was set at 0.3 μsec. so that no coincidences would be lost due to the rise times of the linear amplifiers (about 0.17 μsec.).

To determine the absolute number of coincidences to be expected it is essential to know the absolute efficiency of the gamma-detector for quanta of different energies. From the weight and thickness of the anthracene crystal and the Klein-Nishina cross section, the number of recoil electrons produced in the crystal by gamma-rays of any energy can be calculated readily. While there is little doubt that this value is accurate to within about one percent, the uncertainty in the efficiency arises from the fact that only a fraction of these electrons is detected (the "detection efficiency").

For the gamma-counter used, and for gamma-rays from a Co^{60} source calibrated by the Bureau of Standards, it was determined that projecting the integral pulse-height curve back to zero pulse height gave rates which agreed with the calculated ones to within five percent. Some precautions to be observed in such absolute gamma-counting have been given previously.[8] From the pulse-height curve the fraction of recoil electrons detected at various other discriminator biases can therefore be determined, and amounted to 0.92 ±0.03 under the conditions used in the experiment. For these same conditions the absolute efficiency for annihilation radiation from Cu^{64} was measured by gamma-gamma-coincidence counting at 180°, and from the thickness of the crystal the detection efficiency was thereby calculated to be 0.70±0.03. This was in agreement with the value obtained from the integral pulse height curve for this energy, on the assumption that projecting the curve to zero pulse height gives 100 percent detection efficiency. Since this assumption was satisfied for these two energies, the detection efficiencies for other energies were calculated from their integral pulse-height curves.

Corrections were made for the Al sheath over the crystal. For the different positions of the foil the energy of the 2.62-Mev quanta after scattering into the gamma-counter varied from 1.2 to 1.8 Mev, and the detection efficiency from 0.92 to 0.95, while the absolute efficiency varied from 15.4 to 13 percent. In no part of the experiments did the results depend appreciably on the measurement of quanta with energies below that of annihilation radiation, for which the detection efficiencies were known only roughly.

IV. EXPERIMENTAL PROCEDURE

Measurement of the Background

The background coincidence rate without the foil in position is due to purely accidental coincidences, true coincidences due to cosmic rays and room background, and true coincidences from scattered quanta and recoil electrons produced at the mouth of the collimator or in the gas. The contributions of these different sources were determined and are given in Table I.

While it was not essential to the experiment to separate the background into its components, this was done with the purpose of deciding whether any further reduction in the background rate were possible. Attempts to reduce the contribution due to scattering from the collimator by reshaping the end of the hole and lining it with aluminum were unsuccessful.

Angular Distribution of Coincidences

The first experiment consisted in measuring the coincidence rate with the foil at various distances from the end of the collimator. For each foil position the coincidence rate was measured over a total period of 24 hr. or more to obtain between 1000 and 3000 coincidences. The constancy of the rates in the separate

TABLE I. Sources of background coincidences.

Source	Coincidences/hr.
Accidentals	0.5
Cosmic rays and room background	3.8
Scattering in helium	2.5
Scattering from collimator	7.5
Total	14.3

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[7] W. H. Jordan and P. R. Bell, Rev. Sci. Inst. **18**, 703 (1947).

[8] W. G. Cross, Phys. Rev. **78**, 185 (1950).

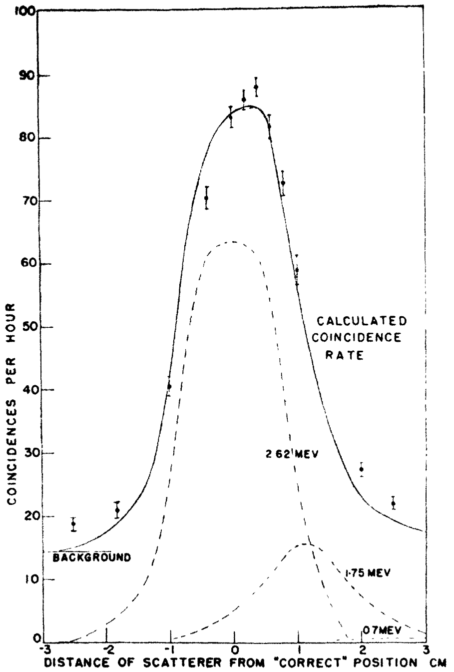

Fig. 2. Angular distribution of coincidences in horizontal plane.

counters served as a check on their continued correct operation, while both the single channels and the coincidence circuit were tested periodically by measuring rates and accidental coincidences due to a Co^{60} source.

Interspersed among these observations were measurements of the background without the foil. The resulting coincidence rates with their statistical errors are shown in Fig. 2. The full curve is the rate calculated as described later.

Coplanarity

To verify that the incident beam and scattered quantum and recoil electron directions lie in a plane, the foil was left in its correct position and the coincidence rate compared for the electron counter in its usual position and with its center raised 2.5 cm above the horizontal plane. Because about half of the background coincidences are due to scattering at the mouth of the collimator, the electrons so produced, and which correspond to quanta passing through the gamma-counter, would be expected to be concentrated in the horizontal plane. Raising the counter was consequently found to reduce the background by a factor of two. The rates obtained are given in Table II.

More Accurate Determination of the Recoil Angle

A third experiment was carried out to locate more accurately the peak of the coincidence distribution curve. For this purpose the method of leaving the counters fixed and moving the foil is not so suitable, since uncertainties in corrections which have to be made when this method is used might shift the peak of the curve by a small amount. Further, the effect of lower energy components in shifting the peak, is less when one of the counters is moved.

Accordingly the foil was left fixed and the electron counter rotated through about 2° on each side of the correct position. As expected, no appreciable variation in the background rate was found over this range. The coincidence rates (including background) with the foil in position and their statistical errors are given in Fig. 3. The individual angular settings may be in error by 0.4°.

The curve drawn in Fig. 3 is the calculated distribution with its maximum at 31.3°. Within the experimental error the measured maximum is in agreement with this value, and almost certainly cannot differ from it by more than 1°.

Simultaneity

In the previous experiments and in the discussion which follows it is shown that the expected number of electrons appear in the direction predicted by the conservation laws, simultaneous (within 0.3 μsec.) with the scattered photons. A fourth experiment was carried out to reduce the uncertainty in the simultaneity. This experiment was similar in principle to those of Bothe and Geiger and others[5] in that simultaneity alone was determined, and not the relative angles of the recoil electron and scattered photon. While simultaneity alone discriminates against the theory of Bohr, Kramers, and Slater, it does not by itself prove that energy and momentum are conserved.

For this experiment, Co^{60} gamma-quanta, collimated in a narrow beam were scattered from one anthracene counter crystal, which detected the recoil electrons, into a second anthracene counter whose output was set in coincidence with that of the first. The second crystal was covered with a plastic shield to make it insensitive to scattered electrons. The 1*P*21 photo-multiplier voltages were increased to 190 volts per stage so that no external amplification was necessary. Output pulses were limited in amplitude, shortened to 8×10^{-9} sec. with a delay-line clipper and fed into a crystal diode coincidence circuit. A delay of 1.5×10^{-8} sec. or more introduced into either channel (by adding lengths of coaxial cable) reduced the coincidence rate by a factor of fifty, the remaining coincidences being attributable to accidentals. Adding this delay in both channels decreased the coincidence rate by less than ten percent. The accidental rate was determined both by adding

Table II. Variation of coincidence rate with counter position.

Position of electron counter	Coincidences/hr.		
	No foil	Foil in position	Rate due to foil
In plane	14.3±0.6	83.0±1.4	68.7
2.5 cm above plane	7.3±0.5	23.7±1.0	16.4

long delays in one channel and by shielding the second counter from gamma-rays scattered by the first and restoring the original rate in the second counter by means of an additional small source. The experiment was carried out with the second counter at mean scattering angles of 45° and 70°.

The result, in agreement with the recent measurement of Hofstadter and McIntyre,[5] indicates that the recoil electrons produced in two separate, successive Compton scattering events are simultaneous within 1.5×10^{-8} sec. A consideration of the various possibilities shows that this requires that there be no delay (greater than 0.015 μsec.) between the arrival of the incident quantum and the emission of the scattered quantum. Further, except for the extremely unlikely possibility that the recoil electron is delayed by a fixed amount in *all* such events, the electrons must have been emitted simultaneous with the arrival of the incident quanta.

A similar but more accurate measurement of simultaneity has recently been made by Bell and Graham[9] of the Chalk River Laboratory, National Research Council of Canada. Using coincidence methods described previously[10] they have verified that for scattering of Co^{60} gamma-rays at 90°, the successive Compton recoil electrons are simultaneous to within less than 5×10^{-10} sec.

Determination of the Strength of the Beam

To calculate the coincidence rates to be expected in the first experiment it was necessary to know the number of 2.62 Mev quanta which struck the scattering foil per second. The strength of the gamma-beam was determined both by direct measurement and by calculation from the known strength of the source and the geometry of the collimator.

Direct measurement was made with a scintillation counter at the position of the foil. For a measured rate of 8420 counts/sec. corrections were necessary for counting losses in the scaler and for a pile-up effect which tended to increase the rate. The net magnitude of these effects was determined by comparing the rate due to two sources together with the sum of their separate rates. It was necessary to repeat this a number of times, the source strengths being progressively increased (so that at each stage the effect in the rate for a single source was known) until the total rate equaled that from the collimator. A further one percent of the measured rate may have been due to electrons from the collimator.

Of the remaining rate, the part due to 2.62-Mev quanta must be separated from that due to lower energy quanta in the beam. The gamma-ray spectrum[10a] of RaTh has been determined[11] both by direct measurements and by indirect information from alpha- and beta-spectra. While there is no general agreement on the relative intensities of the lines, and even some doubt as to whether some of the lines reported belong to the spectrum of Th*C* and its decay products, the intensities are probably known with sufficient accuracy for our purposes. From the available data we have selected the intensities given in Table III. Lines of energies lower than 0.510 Mev will be removed by the filtering. In addition, there may be a weak 3.2-Mev line, whose intensity is probably much less than one percent of the 2.62-Mev line.[12]

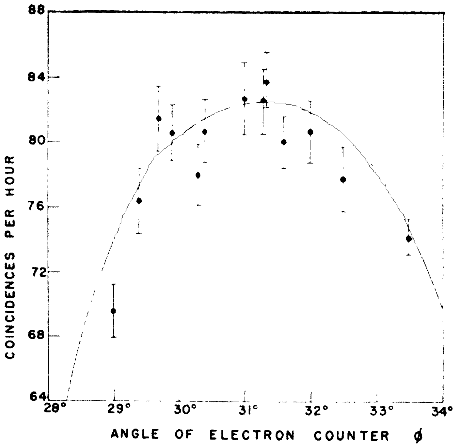

FIG. 3. More accurate determination of recoil angle.

From the intensities given and the known efficiencies of the gamma-counter for the different energies, the fraction of the measured rate which was due to 2.62-Mev quanta was calculated to be 60 percent, and the beam strength to be 78,000 quanta per second of 2.62 Mev.

The strength of the source was determined (in April, 1949) by the Radiochemical Centre, Amersham, Bucks., to be 195 mg Ra equivalent when measured through 5 mm Pb. From the measurements of Shenstone and Schlund[13] and of Gurney[14] and using the branching ratio

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[9] Private communication.

[10] R. E. Bell and H. E. Petch, Phys. Rev. **76**, 1409 (1949); R. E. Bell and R. L. Graham, Phys. Rev. **78**, 490 (1950); T. D. Newton, Phys. Rev. **78**, 490 (1950).

[10a] It is actually the spectrum of the active deposit from thoron which was measured in the experiments quoted. The transitions from RaTh to Th*B* are not believed to give rise to any gamma-rays with energies above 0.5 Mev.

[11] C. D. Ellis, Proc. Roy. Soc. **A138**, 318 (1932), **A143**, 350 (1933); D. V. Skobelzyn, Comptes Rendus **194**, 1486 (1932); F. Oppenheimer, Proc. Camb. Phil. Soc. **32**, 328 (1936); R. Arnoult, Ann. de physique **12**, 241 (1939); A. Flammersfeld, Zeits. f. Physik **114**, 227 (1939); Curran, Dee, and Strothers, Proc. Roy. Soc. **A174**, 546 (1940); J. Itoh and Y. Watase, Proc. Phys. Math. Soc. Japan **23**, 142 (1941); G. D. Latyshev and L. A. Kulchitsky, J. Phys. USSR **4**, 515 (1941); A. I. Alichanov and V. P. Dzelepov, Doklady **20**, 113 (1938); G. D. Latyshev, Rev. Mod. Phys. **19**, 132 (1947); A. Johansson, Arkiv. f. Mat. Ast. o. Fys. **34A**, No. 9 (1947); D. G. E. Martin and H. O. W. Richardson, Proc. Roy. Soc. **195A**, 287 (1948); Martin, Richardson, and Hsü, Proc. Phys. Soc. London **60**, 466 (1948); H. O. W. Richardson, Nature **161**, 516 (1948); D. G. E. Martin and H. O. W. Richardson, Proc. Phys. Soc. London **63**, 223 (1950).

[12] R. E. Bell and L. G. Elliott, Can. J. Research **26A**, 379 (1948).

[13] A. G. Shenstone and H. Schlund, Phil. Mag. **43**, 1038 (1922).

[14] R. W. Gurney, Proc. Roy. Soc. **A112**, 380 (1926).

TABLE III. Intensities in incident gamma-beam.

Energy	Assumed initial intensity	Relative intensity after filtration through 2 cm Pb
2.62	100	100
2.20	5	4.8
1.80	6	5.3
1.62	10	8.0
1.50	3.5	2.6
1.35	4	2.8
0.859	18	5.2
0.726	16	3.6
0.582	100	12.8
0.510	20	1.7

66:34 for[15] ThC′:ThC″, and 3.60×10^7 as the number of disintegrations per second[16] of 1 mg of Ra, 1 mg Ra equivalent of RaTh so measured emits per second 1.38×10^7 quanta of 2.62 Mev. This value is in close agreement with the calculations of Bouchez[17] based on measurements of the heating effect[18] and total ionization[19] of the alpha-particles from the active deposit of thoron.

The strength of the source was hence estimated to be (in September, 1949) 2.32×10^9 quanta per second of 2.62 Mev, and the strength of the beam 65,000 quanta per second of 2.62 Mev. Scattering in the collimator probably increased this value. In view of the uncertainties involved in the calculations of both these methods, we do not consider these to be in disagreement, and believe that the value which will be used—73,000 quanta per second of 2.62 Mev—can be relied upon only to within ten percent.

V. DISCUSSION

In these experiments it has been shown clearly that there are many more coincidences in the direction predicted by the conservation laws than for other directions. There remains to be explained quantitatively the observed width of the angular distribution of coincidences, and the absolute coincidence rate for the correct position. Contributing factors to this width will be scattering of the electrons in the foil and gas, coincidences produced by other energy components in the gamma-spectrum of RaTh, and to a small extent, the finite width of the initial gamma-beam.

For each position of the foil the absolute rate in the gamma-counter due to 2.62-Mev quanta scattered from the foil is

$$R_\gamma = N\sigma(\theta)t\Omega\epsilon_\theta, \qquad (2)$$

where N is the strength of the incident gamma-beam in 2.62-Mev quanta per second, $\sigma(\theta)$ the Compton scattering coefficient per unit solid angle (at angle θ) per cm of Be, t the thickness of the Be foil in cm, Ω the solid angle subtended by the counter at the foil, and ϵ_θ the absolute efficiency of the gamma-counter for the gamma-energy corresponding to scattering of 2.62-Mev quanta through an angle θ. A similar expression gives the contribution of the Pliofilm support.

To these quanta corresponds a cone of electrons. For an "ideal" experiment the electron counter in the correct direction would just intercept all of this cone and the coincidence rate would equal R_γ. When the electron counter is at other angles the coincidence rate would be multiplied by the fraction of the cone intercepted by the counter. Because of electron scattering, lower energy gamma-rays and geometrical factors the rate will be altered.

In the Appendix is calculated an expression for the fraction f_hf_v of this electron cone which, due to scattering of the electrons, actually enters the electron counter when the latter is at an angle η from the "correct" angle. $R_\gamma f_hf_v$ then gives the theoretical rate for a particular foil position and for the 2.62-Mev quanta. For each foil position used in the experiment, this calculation should in principle be repeated for each energy component present in the initial gamma-beam, the result multiplied by the component's intensity, and summed over all components. Because of the length of these calculations we have replaced the numerous lower energy components (given in Table III) with two fictitious "equivalent" lines at 1.75 and 0.70 Mev and with respective intensities 24 and 23 percent of the main line, and for each foil position have summed the rates due to these three components. The resulting curve is given in Fig. 2 where are also plotted the separate contributions of the three components.

In these calculations we have neglected the effect of the finite size and divergence of the incident gamma-beam. The resultant calculated distribution is therefore slightly too narrow. However, for the correct position of the foil the effect of beam size has been included. [The value of f_h is now given by putting $\eta=0$ in Eq. (I-4) of the Appendix.] For our conditions these equations lead to a value of f_hf_v which is four percent lower than was obtained when the size of the incident beam was neglected. For this position about 38 percent of the true coincidences are lost due to electron scattering.

Finally, we must consider the loss of coincidences due to the fact that the cone of electrons which corresponds to quanta passing through the gamma-counter does not have a rectangular cross section (as the electron counter does) but is distorted. For the correct position of the foil this distortion was calculated to result in a four percent loss. In Fig. 4 is shown the shape of this distorted area. To show the deformation more clearly a gamma-area almost twice the width of our gamma-counter has been drawn. The distribution in the experiment will differ from a rectangle by a smaller amount than is shown. For other positions of the foil this

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[15] L. Meitner and K. Freitag, Zeits. f. Physik **37**, 481 (1926); R. Gregoire, Ann. de physique **2**, 161 (1934); A. F. Kovarick and N. I. Adams, Jr., Phys. Rev. **54**, 413 (1938).

[16] T. P. Kohman *et al.*, MDDC 852 (unpublished).

[17] R. Bouchez, J. de phys. et rad. **10**, 415 (1949).

[18] L. Winand, J. de phys. et rad. **10**, 361 (1939).

[19] A. Ricoux, J. de phys. et rad. **8**, 388 (1937).

distortion will be somewhat increased, resulting in the loss of more coincidences and making the actual coincidence distribution slightly narrower than the calculated one.

Comparison between Calculated and Measured Coincidence Rates

The calculated value for the absolute coincidence rate, for the electron counter in the correct position, was 74.5 coincidences per hour: the observed rate was 68.7±1.5. The most important uncertainties in the calculated absolute rate lie in the determination of the beam strength (ten percent), in the efficiency of the gamma-counter (five percent), and in the correction for scattering of the electrons (three to four percent). While it is difficult to assign a probable error to some of these calculations we believe that the estimated value would not be in error by more than 15 percent. Within this error, therefore, the expected number of recoil electrons was emitted at the angle predicted by the conservation laws. The uncertainty provides an upper limit on the fraction of the electrons which could be emitted in other directions.

Turning now to the angular distribution of coincident electrons, in Fig. 2 we have not shown the absolute calculated values but have multiplied these by 68.7/74.5 in order to make the curve agree with the experimental value for the correct position of the foil. This was done so that the shape of the experimental and theoretical distributions could be compared, irrespective of the errors in the absolute value of the latter. In the calculation of electron scattering in the foil, the neglect of the finite width of the gamma-beam and the use of a mean foil thickness rather than averaging the expressions obtained over the thickness of the foil (see Appendix) both make the calculated distribution too narrow at larger angles of scattering. A wider distribution would also have resulted had we used Molière's theory of multiple scattering[20] (which for angles greater than the mean scattering angle is probably more correct than that of Williams) rather than Williams' theory[20] as a basis for the calculations of electron scattering. On the other hand, neglecting the distortion of the electron cone from a rectangular cross section will tend to make our calculated curve wider than it should be. In all the calculations the gamma-counter crystal has been treated as being concentrated all at the same distance from the foil. Actually some quanta whose directions do not pass through this cross section of the counter can be counted in the front half of the crystal. This will further broaden the lower part of the experimental distribution.

Considering these errors, as well as the uncertainties in the theory of scattering of electrons and in the energy spectrum of RaTh gamma-rays, the agreement between the calculated and experimental distributions is better than we probably have a right to expect, and is certainly well within the errors of calculation. By choosing intensities for the lower energy lines of RaTh and its decay products other than are given in Table III but still consistent with values given in the literature, the agreement can be made slightly better or worse than is shown in Fig. 2.

When in the second experiment the electron counter was moved out of the plane, the coincidence rate dropped to less than one quarter of what it had been for the counter in the plane. This rate, however, is still twice as large as was calculated. We do not know the cause of this discrepancy: unfortunately it was not discovered until after it was impossible to repeat the experiment. Since close to the calculated number of coincidences was observed in the correct position, this indicates an inconsistency between the second experiment and the others rather than a definite disagreement with the conservation laws.

VI. CONCLUSIONS

The experiments described have shown that in Compton scattering: (1) the peak of the angular distribution of electrons coincident with quanta scattered in a given direction is in the direction required by the conservation laws within ±1°; (2) the observed width of the angular distribution about the predicted direction (about 14° total at half-maximum) can be accounted for quantitatively by the scattering of the electrons in the foil, by the components of other energies in the incident gamma-beam, and by small geometrical factors; (3) the scattered quantum and recoil electron are both emitted within 1.5×10^{-8} sec. of the time the incident quantum arrives at the scattering center.

The results therefore indicate that certainly in most Compton encounters simultaneity and the conservation laws are satisfied within the limits indicated above.

We are grateful to Drs. R. W. Birge, D. Bodansky and L. S. Lavatelli for suggestions and assistance.

APPENDIX: SCATTERING OF ELECTRONS IN THE GAS AND SCATTERING FOIL

Expressions for the angular distribution of a collimated beam of monoenergetic electrons after multiple scattering have been

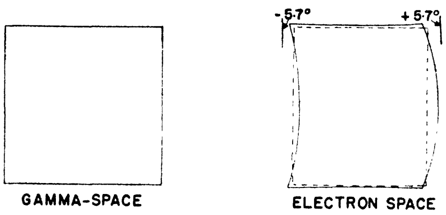

FIG. 4. Corresponding areas in gamma- and electron space.

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[20] W. Bothe, *Handbuch der Physik* (1933), Bd. XXII/2, p. 17; Bethe, Rose, and Smith, Proc. Am. Phil. Soc. **78**, 573 (1938); E. J. Williams, Proc. Roy. Soc. **169A**, 531 (1939), Phys. Rev. **58**, 292 (1940); S. Goudsmidt and J. L. Saunderson, Phys. Rev. **57**, 24 (1940), **58**, 36 (1940); B. Rossi and K. Greisen, Rev. Mod. Phys. **13**, 240 (1941); A. F. Kompaneetz, J. Phys. USSR. **9**, 17 (1944); G. Molière, Zeits. f. Naturforschung **3a**, 78 (1948); W. T. Scott, Phys. Rev. **76**, 212 (1949); H. S. Snyder and W. T. Scott, Phys. Rev. **76**, 220 (1949).

given by a number of authors.[20] For angles less than the mean scattering angle the distribution is essentially Gaussian and its width can be conveniently characterized by the "half-width" Δ, that angle at which the intensity is down to $1/e$ of its maximum. For the conditions of our experiment—electrons of about 1 Mev and a Be scatterer of effective thickness about 17 mg/cm^2, the values of Δ given by nearly all these theories agree to within about ten percent and are also within ten percent of the measured values.[21] We shall use a Gaussian distribution with Δ as given by Williams,[20] according to which the probability of the projection of the scattering angle in the horizontal plane lying between ϕ and $\phi+d\phi$ after multiple scattering is

$$P(\phi)d\phi=(2/\Delta\pi^{\frac{1}{2}})\exp[-(\phi/\Delta)^2]d\phi, \qquad \text{(I-1)}$$

where Δ depends on the atomic number and thickness of the foil and on the energy of the electrons. It can be shown that for our conditions the effect of scattering of the electrons in the helium is equivalent to increasing the half-width Δ of the scattering foil by about one percent.

Consider first only the horizontal components of the directions of electrons. Neglecting, temporarily, the finite width of the initial gamma-beam, the number of the recoil electrons which correspond to quanta which pass through the gamma-counter, and which are scattered by the foil into $d\alpha$ at angle α, is

$$F(\alpha)d\alpha=[\mathit{erf}((\alpha+\omega)/\Delta)-\mathit{erf}((\alpha-\omega)/\Delta)]d\alpha/4\omega, \qquad \text{(I-2)}$$

where

$$\mathit{erf}x\equiv 2/\pi^{\frac{1}{2}}\int_0^x \exp(-t^2)dt,$$

and ω is the horizontal semi-angle in electron space which corresponds [according to Eq. (1)] to the boundaries of the gamma-counter. This gives the angular distribution of the electron beam about its mean direction. If now the center line of the electron counter is not on the center line of the beam but off by a distance which at the foil subtends an angle η (the "error angle"), the fraction f_h of the electron directions whose horizontal components lie within the boundaries $\eta\pm\omega_\beta$ of the electron counter is given by integrating $F(\alpha)$;

$$f_h=\frac{\Delta}{8\omega}\left\{\mathit{ierf}\left(\frac{\eta+\omega+\omega_\beta}{\Delta}\right)+\mathit{ierf}\left(\frac{\eta-\omega-\omega_\beta}{\Delta}\right)\right.$$
$$\left.-\mathit{ierf}\left(\frac{\eta+\omega-\omega_\beta}{\Delta}\right)-\mathit{ierf}\left(\frac{\eta-\omega+\omega_\beta}{\Delta}\right)\right\}, \qquad \text{(I-3)}$$

where we define

$$\mathit{ierf}(x)=\int_0^x \mathit{erf}(t)dt=x\,\mathit{erf}(x)+\frac{\exp(-x^2)-1}{\pi^{\frac{1}{2}}}.$$

The fraction of the electrons whose vertical direction components lie within the boundaries of the electron counter is given by a similar expression, but with η, ω, and ω_β referring to vertical angles. The relation between vertical angles in electron and gamma-space is not given by Eq. (1), as is the relation for horizontal angles. For θ and ϕ almost equal (as they are) the approximation that a 1° vertical angle in gamma space corresponds to a 1° vertical angle in electron space is not greatly in error and has been used.

To take account of the fact that electrons are produced at different depths in the foil, Eq. (I-3) must be averaged over the thickness of the foil. Taking Δ_x, the half-width for electrons produced at depth x to be proportional to $x^{\frac{1}{2}}$, leads to an expression in the second integral of the error function. We have evaluated this expression and find that the result differs by only a few percent from that obtained by using in Eq. (I-3) a half-width corresponding to a mean foil thickness $4d/9$, where d is the actual thickness.

In now taking into account the finite size of the scattering foil and the angular divergence of the incident beam we have assumed, to simplify the calculations, that the beam has a square, rather than circular, cross section. The intensity distribution of the electron beam in the horizontal plane (before scattering) corresponding to quanta entering the gamma-counter can then be shown to be approximately

$$\begin{array}{ll} d\phi/2\omega & \text{for } \phi<y \\ (d\phi/2\omega)(z-\phi)/(z-y) & \text{for } y<\phi<z \\ 0 & \text{for } \phi>z, \end{array}$$

where y and z are constant angles, determined by the width and divergence of the incident beam, by the angles subtended by the gamma-counter at the foil and by the cross section of the beam at the gamma-counter, and by the mean angles of the electron and gamma-counter.

The fraction f_h of the electron directions whose horizontal components lie within the boundaries of the electron counter then becomes

$$f_h=\frac{\Delta}{y^2-z^2}\left\{\mathit{iierf}\left(\frac{z+\eta+\omega_\beta}{\Delta}\right)-\mathit{iierf}\left(\frac{z+\eta-\omega_\beta}{\Delta}\right)\right.$$
$$-\mathit{iierf}\left(\frac{z-\eta-\omega_\beta}{\Delta}\right)+\mathit{iierf}\left(\frac{z-\eta+\omega_\beta}{\Delta}\right)$$
$$-\mathit{iierf}\left(\frac{y+\eta+\omega_\beta}{\Delta}\right)+\mathit{iierf}\left(\frac{y+\eta-\omega_\beta}{\Delta}\right)$$
$$\left.+\mathit{iierf}\left(\frac{y-\eta-\omega_\beta}{\Delta}\right)-\mathit{iierf}\left(\frac{y-\eta+\omega_\beta}{\Delta}\right)\right\}, \qquad \text{(I-4)}$$

where

$$\mathit{iierf}x\equiv\int_0^x \mathit{ierf}(t)dt,$$

which for our geometry gives a result two percent lower than does Eq. (I-3). Because of the length of the calculation and the fact that the difference is small, these expressions were not averaged over the depth of the foil, but a mean thickness $4d/9$ was used in calculating Δ.

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[21] C. W. Sheppard and W. A. Fowler, Phys. Rev. **57**, 273 (1940); M. M. Slawsky and H. R. Crane, Phys. Rev. **56**, 1203 (1939); Oleson, Chao, and Crane, Phys. Rev. **60**, 378 (1941); L. A. Kulchitsky and G. D. Latyshev, J. Phys. USSR **5**, 249 (1941); Andrievsky, Kulchitsky, and Latyshev, J. Phys. USSR **6**, 278 (1942).