TABLE I. Multiplicity of penetrating showers from lithium.

Event no.	Charged particles traceable to point of origin	Identified penetrating particles	Electron showers starting from first lead plate
37250	10	3	>3
37260	2	2	1
37370	2	2	3
37737	2	2	0
38287	10	5	>2
38340	13	10	6
38459	4	4	2
39514	12	7	4
40173	2	2	0
40514	5	4	0
40971	6	6	0
41596	4	3	2
41804	7	5	3
42586	7	5	>2

thickness and the top of the cloud chamber was made of ¼-inch Dural. About 15 events originating above the chamber were rejected because they appeared to come from the aluminum, the steel, or the counters, or were too close to the edge of the lithium. Any event that appeared to originate within one centimeter of the edge of the lithium was rejected.

The results of the analysis are given in Table I. In the large events the total number of charged particles traceable to the point of origin may be underestimated by one or two, but probably not more than this. Penetrating particles were identified by their passage through two ¼-inch lead plates without interaction. In several cases the number of penetrating particles given is probably much lower than the true value because of the difficulty in analyzing the complicated events after the electron showers started in the first lead plate.

The size of the showers observed is certainly influenced by the counter and cloud-chamber arrangement. The chamber was tripped when one and only one of the five one-inch counters above the lithium discharged in coincidence with two 2-inch counters below the chamber covered by ¼ inch of lead and separated by one inch of lead. There was a total of 5½ inches of lead in the chamber. The apparatus was located at Berkeley and the counting rate for lithium showers was about three per month. The largest shower observed (event No. 38340) was not an exceptionally high energy event, judging from the lack of high energy electronic radiation that usually orginates in high energy nuclear events.

The maximum number of nucleon-nucleon interactions that can occur within a single nucleus of lithium is seven. If a single charged meson is produced in each interaction,[2] one can imagine on the most extreme assumptions of the plural production theory that 4 negative mesons, 3 positive mesons, and 5 protons could emerge, a total of 12 charged particles. No provision is made in this for neutral mesons. In event No. 38340 the above extremely unlikely event could not explain the number of charged particles observed, even omitting the electron showers presumably caused by the decay of neutral mesons into gamma-rays.

Several of the other showers have multiplicities of charged particles which are too large to be explained by a single interaction under the plural theory. If multiple interactions occur fairly frequently in an element as light as lithium, they must be extremely common in lead; and one might expect that in lead an incoming nucleon would nearly always dissipate all of its energy in one nucleus. Observations of successive events in cloud chambers[1] and with counters[3] contradict this idea.

If mesons are produced multiply, even with very small multiplicities of perhaps 2 to 5 per collision, the large lithium showers would be easily explained. Successive events within the same nucleus very likely occur, but it seems necessary to allow at least a small multiplicity to explain the lithium showers.

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† Reported at the Mexico City Meeting of the American Physical Society, June 22, 1950.
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Two Comments on the Limits of Validity of the P. R. Weiss Theory of Ferromagnetism

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I SHOULD like to make two comments on the P. R. Weiss theory of ferromagnetism.[1] I should emphasize that these comments are literally indications of the limits of validity of this theory, and do not detract much from the main (and very considerable) achievements of the Weiss method, which has recently found gratifying confirmation in the work of Zehler.[2]

The first comment is a very simple one. In spite of the fact that until now the criteria of the Bloch spin-wave[3] and Weiss theories have agreed in giving ferromagnetism in precisely the same lattice structures and no others, a cursory glance tells one that these criteria are in principle very different. The Bloch criterion is merely that the lattice be three-dimensional; the Weiss criterion, however, is that the nearest neighbors of a given atom have certain topological—not spatial—relations to each other and to the central atom. That is, it involves counting the number of nearest neighbors, finding the nearest neighbor relationships among these, and similar purely topological considerations, which without explicit proof or disproof do not seem to have anything to do with the dimensionality of the lattice. For instance, if no nearest neighbors are nearest neighbors of each other, all that is required in the Weiss theory is that there be six or more nearest neighbors. A counter-example which shows the non-equivalence of the two criteria is the diamond lattice with nearest-neighbor spin interaction. In this lattice each atom has only four nearest neighbors, so it would not be ferromagnetic according to Weiss, although it would be ferromagnetic according to Bloch, since it is three-dimensional. Since the Bloch theory is rigorous in its low-temperature domain of applicability, this seems to indicate that the Weiss theory criterion is not always correct.

The second comment shall be stated primarily in the form of results. It is found that at sufficiently low temperatures, for all lattices, ferromagnetism is no longer present in the Weiss theory; and thus at some low temperature there must be an "anti-Curie point" below which ferromagnetism vanishes. This is true in the Weiss theory in spite of the fact that it is not true for the non-quantum-mechanical Ising model of Peierls and Bethe[4] upon which it is based, and indeed the effect can be traced to a specifically quantum-mechanical cause.

In either case at very low temperatures, only the very lowest state of the Bethe "cluster" of an atom and its neighbors is appreciably populated. However, this state is aligned completely, and thus the moment m_0 of the central atom is rigorously equal to that (m_1) of a boundary atom; and to find the result of the consistency condition $m_0=m_1$, we must appeal to higher order effects.[5] In the Ising case[4] the only such effect is thermal excitation of the next higher level, which leads easily to $m_0>m_1$ at very low ordering fields H_1, but to $m_1>m_0$ at high fields. Thus, $m_1=m_0$ is certainly satisfied for a finite intermediate field H_1, at low temperatures.

In the quantum-mechanical problem, however, there is a new effect, of exponentially $[\exp(J/kT)]$ greater magnitude than thermal excitation at low enough temperatures: the second-order perturbation of the cluster levels due to the ordering field H_1. It is easily shown that this effect has the wrong sign at low fields H_1, giving $m_1>m_0$; and thus, since $m_1>m_0$ always at high fields, we have no crossing point $m_1(H_1)=m_0(H_1)$ and no ordering field or ferromagnetism. Actual computations have been carried out using the formulas (30, 36, and 37) of reference 1, and it is found that an anti-Curie point (for the simple cubic lattice) can be located at $kT=0.269J$. (J is the exchange integral.) This is at a temperature one-seventh of that of Weiss' computed "true" Curie point T_c; however, it is clear from other computations made that Weiss' method fails completely at temperatures lower than $\frac{1}{2}T_c$.

The reason for this failure is clear. The small dimensions of the cluster exclude the low energy spin-wave states, which are all-important in ferromagnetic behavior at low temperatures. If it were possible to increase the cluster size, these states would come in in increasing numbers and the "anti-Curie point" would move to lower and lower temperatures.

It is not possible to draw many conclusions from this work as to the validity of Weiss' method at or near the Curie point. It does seem that the differences between quantum and Ising ferromagnetism are not entirely summed up in the concept of spin waves. We may also conclude that the Weiss method is probably a somewhat worse approximation to the true statistical problem than the Bethe method is to its corresponding Ising model problem.

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Two-Fluid Theory of Liquid Helium II below 1°K

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Recently, Usui[1] has developed the general formalism of the two-fluid theory of liquid He II, avoiding ingeniously the use of the Tisza's relation, $\rho_n/\rho = s/s_\lambda$, the role of which has been critically discussed by us[2] in connection with the second-sound velocity[3] below 1°K. We are now applying the general formalism to the construction of the two-fluid theory below 1°K. New results obtained are as follows.

Usui has assumed that the two fluid components are always in local equilibrium, the normal fluid concentration, $x=\rho_n/\rho$, being a function of the pressure and temperature determined by the condition of minimum Gibbs free energy. Thus, we may use the total density, ρ, and the concentration, x, as the independent thermodynamic variables. The entropy per unit mass, s, is regarded as the function of ρ and x. Usui has shown that the entropy flow is given by

$$\rho s \mathbf{V}_1 + (\partial s/\partial x)_\rho \rho_n (\mathbf{V}_n - \mathbf{V}_1), \tag{1}$$

where $\mathbf{V}_1$ is the velocity of the center of gravity. He has obtained the generalized expressions for the fountain pressure and the second-sound velocity, respectively, as follows:

$$x(\partial s/\partial x)_\rho \operatorname{grad} T \tag{2}$$

and

$$C_2^2 = x(1-x)(\partial s/\partial x)_\rho (\partial T/\partial x)_\rho. \tag{3}$$

The latter expression is not identical with that used by Landau,[4] as we predicted in our previous report.[2] C_2 calculated on the basis of Landau's model by means of Eq. (3) is shown by curve II in Fig. 1.

Now, following Tisza's idea,[5] we extend the two fluid theory to include the phonon entropy:

$$s = s_\lambda x + s_{\mathrm{ph}}. \tag{4}$$

The entropy flow, Eq. (1), then takes the form

$$(s_\lambda + (\partial s_{\mathrm{ph}}/\partial x)_\rho)\rho_n \mathbf{V}_n + (s_{\mathrm{ph}} - x(\partial s_{\mathrm{ph}}/\partial x)_\rho)\rho \mathbf{V}_1. \tag{5}$$

Even in the limiting case of the extremely narrow capillary ($\mathbf{V}_n \rightarrow 0$), the phonon entropy is transferred by the superfluid flow. Accordingly the experimental evidence proved by Kapitza[6] and, therefore, H. London's expression for the fountain pressure should be invalid below 1°K, as we have suggested previously.[2]

In order to calculate C_2 by means of Eqs. (3) and (4), one needs the expressions for x, s_λ, and s_{ph}. x would be determined by the analysis of the viscosity measurement and the proportionality relation between the fountain pressure and heat current.[7] The available data, however, are lacking at present. If we assume that

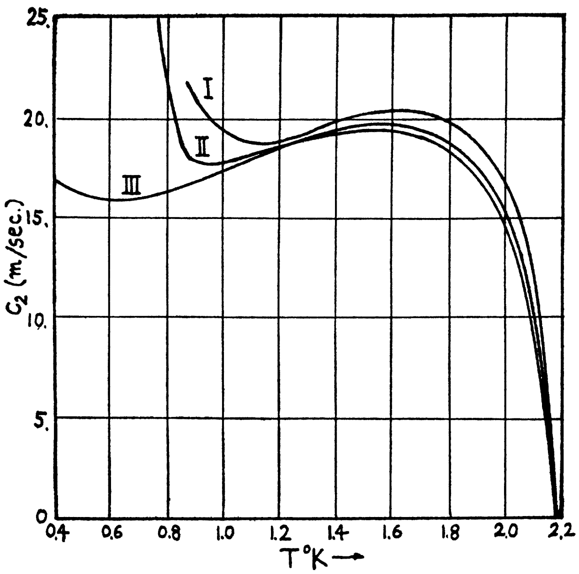

FIG. 1. Second-sound velocity *vs.* temperature. I: observed curve (M.H.); II: Landau's model; III: generalized model of Tisza.

$x=(T/T_\lambda)^r$, the second-sound velocity has a non-zero or infinite value as $T\rightarrow 0$, only when $r \geqq 4$. The preliminary result of the calculation is shown by curve III in Fig. 1, where we have used the following values: $s_\lambda = 0.397$ cal./g-deg., $r=6$, and $s_{\mathrm{ph}} = 0.75 \times 10^{-3} T^3$ cal./g-deg. The calculated C_2 has a minimum and increases as the temperature decreases. C_2 increases too rapidly in Landau's model and too slowly in Tisza's model in comparison with the experimental results.

At any rate, we may say that one should not decide on the superiority of Landau's theory from the analysis of the second-sound velocity alone. In our opinion, the generalized model of Tisza, Eq. (4), has the advantage that it can be connected easily with Bose-Einstein statistics by the idea of B-E liquid. The detailed discussion will appear in the forthcoming paper.

One of us has enjoyed stimulating discussions with Associate Professor T. Usui at the University of Tokyo.

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A 1.7×10^{-9}-Sec. Isomeric State in $_{68}Er^{166}$

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The half-life of the 80-kev transition in Er^{166} has been measured as $(1.7 \pm 0.2) \times 10^{-9}$ sec. with a delayed coincidence scintillation spectrometer.

The β-spectrum of Ho^{166} (27 hr.) is known to consist of two components of 1.84- and 0.55-Mev maximum energy and intensities 89 and 11 percent, respectively.[1] Siegbahn and Slätis showed that the 80-kev transition follows the higher energy component of the β-spectrum and that the L shell conversion coefficient is $\sim$0.4.

The delayed coincidence scintillation spectrometer is similar to that described previously.[2] In addition, a second differential