High Energy Neutron Scattering by Nuclei*

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THE so-called transparent model[1] of a nucleus seems to be a useful model in treating the scattering of 90-Mev neutrons by nuclei, since the nuclear radii fitted from experiment by this model agree well with an $A^{\frac{1}{3}}$ law.[2] However, the calculated differential scattering cross sections deviate somewhat from the experimental observations,[3] the latter being 10 to 20 percent higher at low scattering angles.

The differential scattering calculations for the transparent model were made by assuming the nucleus to be representable by a square well with potential 30.8 Mev, plus absorption, leading in the classical (W.K.B.) approximation to a sphere with a complex constant index of refraction. The angular distribution amplitude was found to be

$$f(\theta)=k\int_0^R\{1-\exp[(-K+2ik_1)(R^2-\rho^2)^{\frac{1}{2}}]\}J_0(k\rho\sin\theta)\rho d\rho,$$

where k is the neutron wave number, K is the absorption coefficient, and k_1/k the real part of the index of refraction. This integral was evaluated by converting it to a sum of approximately kR terms.

The integral for $f(\theta)$ can be evaluated by means of a series representation due to Van de Hulst,[4] viz.,

$$f(\theta)=kR^2\left\{\frac{J_1(z)}{z}-\frac{\rho}{w^2}e^{iw}(1-iw)+\frac{J_0(z)}{\rho^2}-\frac{1\cdot3zJ_1(z)}{\rho^4}+\frac{1\cdot3\cdot5z^2J_2(z)}{\rho^6}-\cdots\right\},$$

where

$$\rho=(2k_1+iK)R;\quad z=kR\sin\theta;\quad w=(\rho^2+z^2)^{\frac{1}{2}}.$$

This series for $f(\theta)$ is particularly useful for large mass nuclei and for high energies, for which kR (the approximate number of terms in the summation method) is large. Van de Hulst derived this formula by a double series expansion of $f(\theta)$ in powers of z and ρ. A simpler proof can be obtained by writing

$$f(\theta)=kR^2\left\{[J_1(z)/z]+i(\partial/\partial\rho)\int_0^{\frac{1}{2}\pi}e^{i\rho\cos\gamma}J_0(z\sin\gamma)\sin\gamma d\gamma\right\}.$$

If we define

$$I_n=\int_0^{\frac{1}{2}\pi}J_n(z\sin\gamma)e^{i\rho\cos\gamma}\sin\gamma d\gamma/(z\sin\gamma)^n,$$

then integration by parts yields

$$I_n=(e^{i\rho}/2^n n!i\rho)-[J_n(z)/z^n i\rho]+2z^2\partial I_{n+1}/\partial\rho^2.$$

Repeated integrations by parts yield the series for $f(\theta)$.

Since the complex constant index of refraction is equivalent to the assumption of a square well with a complex constant potential, it was considered desirable to check the validity of the classical approximation by making an exact partial wave analysis for the complex square well, using the corresponding values of the parameters. This was done for aluminum, the results being shown in Fig. 1. The circles represent the experimental points of Bratenahl, *et al.*, the dotted line the values of $\sigma(\theta)$ calculated using the classical approximation, and the solid line the values of $\sigma(\theta)$ calculated by means of the exact partial wave analysis. It is seen that the apparent deviations from experiment are at least partly due to the calculational method rather than to the use of the complex square well model.

The calculated scattering and absorption cross sections differ somewhat from those obtained with the classical approximation. The scattering cross section becomes 0.83 instead of 0.75 barn, and the absorption cross section is 0.45 instead of 0.36 barn. To make a closer comparison of the calculated differential scattering cross section with the experimental results, it would be necessary first to adjust the complex potential parameters to fit the experimentally determined scattering and absorption cross sections.

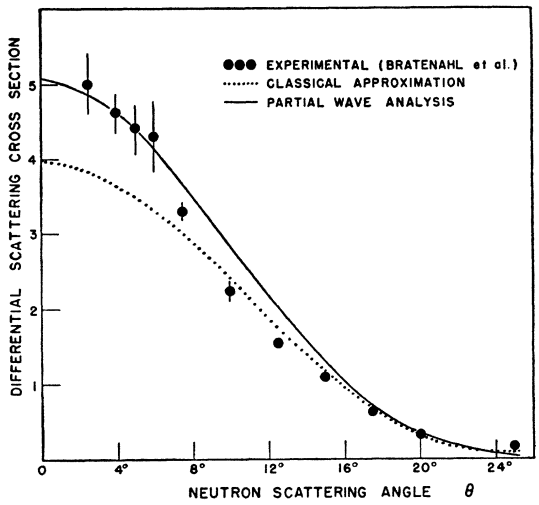

FIG. 1. Comparison of experimental differential scattering cross sections with those calculated from transparent model theory, in units of barns per steradian.

The phase shifts calculated by the exact partial wave analysis deviate considerably from those obtained by the W.K.B. method.[2] The latter gives, for aluminum,

$$\delta_l=(1.35+0.452i)\{1-(l+\tfrac{1}{2})^2/73.1\}^{\frac{1}{2}}\quad l\leqq8$$
$$=0\quad l>8.$$

The former method yields the following phase shifts for aluminum (for $l=0, 1, 2, \cdots$): $1.29+0.38i$, $1.44+0.56i$, $1.20+0.40i$, $1.36+0.40i$, $1.12+0.49i$, $1.01+0.29i$, $1.11+0.32i$, $0.85+0.47i$, $0.27+0.19i$, $0.065+0.025i$, $0.012+0.005i$, $0.002+0.001i$, etc.

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[2] Fernbach, Serber, and Taylor, Phys. Rev. **75**, 1352 (1949).
[3] Bratenahl, Fernbach, Hildebrand, Leith, and Moyer, Phys. Rev. **77**, 597 (1950).
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Penetrating Showers from Lithium*

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THE analysis of penetrating showers originating in heavy elements is complicated by the possibility that more than one collision may occur inside a single nucleus. Thus, it is difficult to determine whether a number of mesons can be created in a single nucleon-nucleon collision from the data obtained on penetrating showers in lead, gold, aluminum, or even carbon. Hydrogen would be ideal as a generator, but the obvious experimental difficulties have led us to try lithium first.

A block of sodium-free lithium 6 inches thick, 5 inches wide, and 16 inches long was placed over a cloud chamber previously used in penetrating shower investigations.[1] Lead plates in the chamber served to analyze the showers produced in the lithium.

Fourteen showers appeared to have originated in the lithium block and have been analyzed. Origin in the lithium was determined by tracing the paths of the particles observed in the upper section of the chamber to see whether they came from a common point. This was done on a large print of the photograph in question and by re-projecting the stereoscopic pictures through the original camera.

The lithium was encased in a tin-coated steel can of $\frac{1}{32}$-inch