## High Energy Neutron Scattering by Nuclei*

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THE so-called transparent model[1] of a nucleus seems to be a useful model in treating the scattering of 90-Mev neutrons by nuclei, since the nuclear radii fitted from experiment by this model agree well with an $A^{\frac{1}{3}}$ law.[2] However, the calculated differential scattering cross sections deviate somewhat from the experimental observations,[3] the latter being 10 to 20 percent higher at low scattering angles.

The differential scattering calculations for the transparent model were made by assuming the nucleus to be representable by a square well with potential 30.8 Mev, plus absorption, leading in the classical (W.K.B.) approximation to a sphere with a complex constant index of refraction. The angular distribution amplitude was found to be

$$f(\theta)=k\int_0^R\{1-\exp[(-K+2ik_1)(R^2-\rho^2)^{\frac{1}{2}}]\}J_0(k\rho\sin\theta)\rho d\rho,$$

where $k$ is the neutron wave number, $K$ is the absorption coefficient, and $k_1/k$ the real part of the index of refraction. This integral was evaluated by converting it to a sum of approximately $kR$ terms.

The integral for $f(\theta)$ can be evaluated by means of a series representation due to Van de Hulst,[4] viz.,

$$f(\theta)=kR^2\left\{\frac{J_1(z)}{z}-\frac{\rho}{w^2}e^{iw}(1-iw)+\frac{J_0(z)}{\rho^2}-\frac{1\cdot3zJ_1(z)}{\rho^4}+\frac{1\cdot3\cdot5z^2J_2(z)}{\rho^6}-\cdots\right\},$$

where

$$\rho=(2k_1+iK)R;\quad z=kR\sin\theta;\quad w=(\rho^2+z^2)^{\frac{1}{2}}.$$

This series for $f(\theta)$ is particularly useful for large mass nuclei and for high energies, for which $kR$ (the approximate number of terms in the summation method) is large. Van de Hulst derived this formula by a double series expansion of $f(\theta)$ in powers of $z$ and $\rho$. A simpler proof can be obtained by writing

$$f(\theta)=kR^2\left\{[J_1(z)/z]+i(\partial/\partial\rho)\int_0^{\frac{1}{2}\pi}e^{i\rho\cos\gamma}J_0(z\sin\gamma)\sin\gamma d\gamma\right\}.$$

If we define

$$I_n=\int_0^{\frac{1}{2}\pi}J_n(z\sin\gamma)e^{i\rho\cos\gamma}\sin\gamma d\gamma/(z\sin\gamma)^n,$$

then integration by parts yields

$$I_n=(e^{i\rho}/2^n n!i\rho)-[J_n(z)/z^n i\rho]+2z^2\partial I_{n+1}/\partial\rho^2.$$

Repeated integrations by parts yield the series for $f(\theta)$.

Since the complex constant index of refraction is equivalent to the assumption of a square well with a complex constant potential, it was considered desirable to check the validity of the classical approximation by making an exact partial wave analysis for the complex square well, using the corresponding values of the parameters. This was done for aluminum, the results being shown in Fig. 1. The circles represent the experimental points of Bratenahl, *et al.*, the dotted line the values of $\sigma(\theta)$ calculated using the classical approximation, and the solid line the values of $\sigma(\theta)$ calculated by means of the exact partial wave analysis. It is seen that the apparent deviations from experiment are at least partly due to the calculational method rather than to the use of the complex square well model.

The calculated scattering and absorption cross sections differ somewhat from those obtained with the classical approximation. The scattering cross section becomes 0.83 instead of 0.75 barn, and the absorption cross section is 0.45 instead of 0.36 barn. To make a closer comparison of the calculated differential scattering cross section with the experimental results, it would be necessary first to adjust the complex potential parameters to fit the experimentally determined scattering and absorption cross sections.

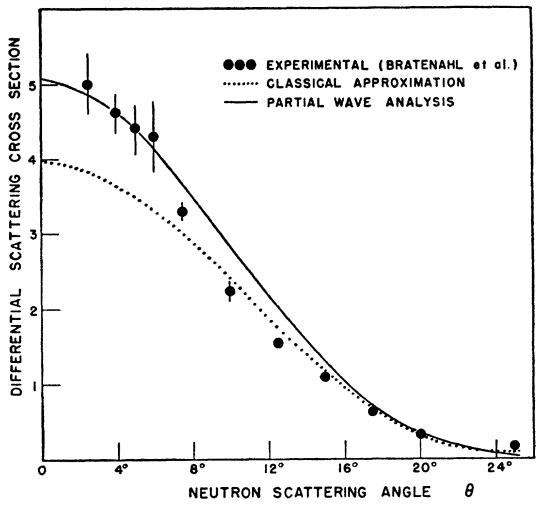


FIG. 1. Comparison of experimental differential scattering cross sections with those calculated from transparent model theory, in units of barns per steradian.

The phase shifts calculated by the exact partial wave analysis deviate considerably from those obtained by the W.K.B. method.[2] The latter gives, for aluminum,

$$\delta_l=(1.35+0.452i)\{1-(l+\tfrac{1}{2})^2/73.1\}^{\frac{1}{2}}\quad l\leqq8$$
$$=0\quad l>8.$$

The former method yields the following phase shifts for aluminum (for $l=0, 1, 2, \cdots$): $1.29+0.38i$, $1.44+0.56i$, $1.20+0.40i$, $1.36+0.40i$, $1.12+0.49i$, $1.01+0.29i$, $1.11+0.32i$, $0.85+0.47i$, $0.27+0.19i$, $0.065+0.025i$, $0.012+0.005i$, $0.002+0.001i$, etc.

We would like to express our appreciation to Richard J. Weiss for some helpful discussion, and to William Donoghue, Theresa Danielson, and Dale Meyer for performing the numerical work.

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[2] Fernbach, Serber, and Taylor, Phys. Rev. **75**, 1352 (1949).
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## Penetrating Showers from Lithium*

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THE analysis of penetrating showers originating in heavy elements is complicated by the possibility that more than one collision may occur inside a single nucleus. Thus, it is difficult to determine whether a number of mesons can be created in a single nucleon-nucleon collision from the data obtained on penetrating showers in lead, gold, aluminum, or even carbon. Hydrogen would be ideal as a generator, but the obvious experimental difficulties have led us to try lithium first.

A block of sodium-free lithium 6 inches thick, 5 inches wide, and 16 inches long was placed over a cloud chamber previously used in penetrating shower investigations.[1] Lead plates in the chamber served to analyze the showers produced in the lithium.

Fourteen showers appeared to have originated in the lithium block and have been analyzed. Origin in the lithium was determined by tracing the paths of the particles observed in the upper section of the chamber to see whether they came from a common point. This was done on a large print of the photograph in question and by re-projecting the stereoscopic pictures through the original camera.

The lithium was encased in a tin-coated steel can of $\frac{1}{32}$-inch

TABLE I. Multiplicity of penetrating showers from lithium.

| Event no. | Charged particles traceable to point of origin | Identified penetrating particles | Electron showers starting from first lead plate |
|---|---|---|---|
| 37250 | 10 | 3 | >3 |
| 37260 | 2 | 2 | 1 |
| 37370 | 2 | 2 | 3 |
| 37737 | 2 | 2 | 0 |
| 38287 | 10 | 5 | >2 |
| 38340 | 13 | 10 | 6 |
| 38459 | 4 | 4 | 2 |
| 39514 | 12 | 7 | 4 |
| 40173 | 2 | 2 | 0 |
| 40514 | 5 | 4 | 0 |
| 40971 | 6 | 6 | 0 |
| 41596 | 4 | 3 | 2 |
| 41804 | 7 | 5 | 3 |
| 42586 | 7 | 5 | >2 |

thickness and the top of the cloud chamber was made of $\frac{1}{4}$-inch Dural. About 15 events originating above the chamber were rejected because they appeared to come from the aluminum, the steel, or the counters, or were too close to the edge of the lithium. Any event that appeared to originate within one centimeter of the edge of the lithium was rejected.

The results of the analysis are given in Table I. In the large events the total number of charged particles traceable to the point of origin may be underestimated by one or two, but probably not more than this. Penetrating particles were identified by their passage through two $\frac{1}{4}$-inch lead plates without interaction. In several cases the number of penetrating particles given is probably much lower than the true value because of the difficulty in analyzing the complicated events after the electron showers started in the first lead plate.

The size of the showers observed is certainly influenced by the counter and cloud-chamber arrangement. The chamber was tripped when one and only one of the five one-inch counters above the lithium discharged in coincidence with two 2-inch counters below the chamber covered by $\frac{1}{4}$ inch of lead and separated by one inch of lead. There was a total of $5\frac{1}{2}$ inches of lead in the chamber. The apparatus was located at Berkeley and the counting rate for lithium showers was about three per month. The largest shower observed (event No. 38340) was not an exceptionally high energy event, judging from the lack of high energy electronic radiation that usually orginates in high energy nuclear events.

The maximum number of nucleon-nucleon interactions that can occur within a single nucleus of lithium is seven. If a single charged meson is produced in each interaction,[2] one can imagine on the most extreme assumptions of the plural production theory that 4 negative mesons, 3 positive mesons, and 5 protons could emerge, a total of 12 charged particles. No provision is made in this for neutral mesons. In event No. 38340 the above extremely unlikely event could not explain the number of charged particles observed, even omitting the electron showers presumably caused by the decay of neutral mesons into gamma-rays.

Several of the other showers have multiplicities of charged particles which are too large to be explained by a single interaction under the plural theory. If multiple interactions occur fairly frequently in an element as light as lithium, they must be extremely common in lead; and one might expect that in lead an incoming nucleon would nearly always dissipate all of its energy in one nucleus. Observations of successive events in cloud chambers[1] and with counters[3] contradict this idea.

If mesons are produced multiply, even with very small multiplicities of perhaps 2 to 5 per collision, the large lithium showers would be easily explained. Successive events within the same nucleus very likely occur, but it seems necessary to allow at least a small multiplicity to explain the lithium showers.

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## Two Comments on the Limits of Validity of the P. R. Weiss Theory of Ferromagnetism

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I SHOULD like to make two comments on the P. R. Weiss theory of ferromagnetism.[1] I should emphasize that these comments are literally indications of the limits of validity of this theory, and do not detract much from the main (and very considerable) achievements of the Weiss method, which has recently found gratifying confirmation in the work of Zehler.[2]

The first comment is a very simple one. In spite of the fact that until now the criteria of the Bloch spin-wave[3] and Weiss theories have agreed in giving ferromagnetism in precisely the same lattice structures and no others, a cursory glance tells one that these criteria are in principle very different. The Bloch criterion is merely that the lattice be three-dimensional; the Weiss criterion, however, is that the nearest neighbors of a given atom have certain topological—not spatial—relations to each other and to the central atom. That is, it involves counting the number of nearest neighbors, finding the nearest neighbor relationships among these, and similar purely topological considerations, which without explicit proof or disproof do not seem to have anything to do with the dimensionality of the lattice. For instance, if no nearest neighbors are nearest neighbors of each other, all that is required in the Weiss theory is that there be six or more nearest neighbors. A counter-example which shows the non-equivalence of the two criteria is the diamond lattice with nearest-neighbor spin interaction. In this lattice each atom has only four nearest neighbors, so it would not be ferromagnetic according to Weiss, although it would be ferromagnetic according to Bloch, since it is three-dimensional. Since the Bloch theory is rigorous in its low-temperature domain of applicability, this seems to indicate that the Weiss theory criterion is not always correct.

The second comment shall be stated primarily in the form of results. It is found that at sufficiently low temperatures, for all lattices, ferromagnetism is no longer present in the Weiss theory; and thus at some low temperature there must be an "anti-Curie point" below which ferromagnetism vanishes. This is true in the Weiss theory in spite of the fact that it is not true for the non-quantum-mechanical Ising model of Peierls and Bethe[4] upon which it is based, and indeed the effect can be traced to a specifically quantum-mechanical cause.

In either case at very low temperatures, only the very lowest state of the Bethe "cluster" of an atom and its neighbors is appreciably populated. However, this state is aligned completely, and thus the moment $m_0$ of the central atom is rigorously equal to that ($m_1$) of a boundary atom; and to find the result of the consistency condition $m_0=m_1$, we must appeal to higher order effects.[5] In the Ising case[4] the only such effect is thermal excitation of the next higher level, which leads easily to $m_0>m_1$ at very low ordering fields $H_1$, but to $m_1>m_0$ at high fields. Thus, $m_1=m_0$ is certainly satisfied for a finite intermediate field $H_1$, at low temperatures.

In the quantum-mechanical problem, however, there is a new effect, of exponentially $[\exp(J/kT)]$ greater magnitude than thermal excitation at low enough temperatures: the second-order perturbation of the cluster levels due to the ordering field $H_1$. It is easily shown that this effect has the wrong sign at low fields $H_1$, giving $m_1>m_0$; and thus, since $m_1>m_0$ always at high fields, we have no crossing point $m_1(H_1)=m_0(H_1)$ and no ordering field or ferromagnetism. Actual computations have been carried out using the formulas (30, 36, and 37) of reference 1, and it is found that an anti-Curie point (for the simple cubic lattice) can be located at $kT=0.269J$. ($J$ is the exchange integral.) This is at a temperature one-seventh of that of Weiss' computed "true" Curie point $T_c$; however, it is clear from other computations made that Weiss' method fails completely at temperatures lower than $\frac{1}{2}T_c$.