A Geometrical Model for the Unified Theory of Physical Fields*

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IT is possible to give a unified field theory based on the generalized geometry introduced by Finsler.[1] Finsler's geometry, as is well known,[2] is a generalization of Riemann's and can be regarded as a manifold of line elements in the space of which the metric is defined by

$$ds=L(x^i, \dot{x}^i)dt \tag{1}$$

where x^i represents a point of the n-dimensional space and $\dot{x}^i$ will be in our case the derivative of x^i with respect to the arc length of the curve $x^i=x^i(t)$. The space has a general covariant formalism, which also is a direct generalization of Riemann's. As a consequence of (1), the metrical ground tensor, being a symmetric covariant tensor of the second order is

$$g_{ik}=g_{ik}(x^i, \dot{x}^i)=\tfrac{1}{2}\partial^2(L^2)/\partial\dot{x}^i\partial\dot{x}^k \tag{2}$$

and the parallel displacement of a covariant vector ξ_i is defined by

$$d\xi_i=C_i{}^r{}_s\,\xi_r d\dot{x}^s+\Gamma_i{}^r{}_s\,\xi_r dx^s \tag{3}$$

where the coefficients of the affine connection $C_i{}^r{}_s$ and $\Gamma_i{}^r{}_s$ are deducible from $L(x^i, \dot{x}^i)$ as follows

$$\begin{aligned}
&C_i{}^k{}_j=g^{kr}C_{irj},\\
&C_{ikj}=\frac{1}{2}\frac{\partial g_{ik}}{\partial\dot{x}^j}=\frac{1}{4}\frac{\partial^3(L^2)}{\partial\dot{x}^i\partial\dot{x}^k\partial\dot{x}^j},\\
&\Gamma_i{}^k{}_j=g^{kr}\Gamma_{irj},\\
&\Gamma_{ijk}=\frac{1}{2}\left\{\frac{\partial g_{ij}}{\partial x^k}+\frac{\partial g_{jk}}{\partial x^i}-\frac{\partial g_{ik}}{\partial x^j}\right\}+C_{ikr}\frac{\partial G^r}{\partial\dot{x}^j}-C_{jkr}\frac{\partial G^r}{\partial\dot{x}^i},\\
&G^r=G_i g^{ir},\\
&G_i=\frac{1}{4}\left\{\frac{\partial^2(L^2)}{\partial\dot{x}^i\partial x^r}\dot{x}^r-\frac{\partial(L^2)}{\partial x^i}\right\},\\
&\Gamma^*{}_i{}^r{}_k=\Gamma_i{}^r{}_k-C_i{}^r{}_s\,\Gamma_k{}^s{}_k\,\dot{x}^h.
\end{aligned} \tag{4}$$

The space is characterized by its curvature and torsion. It is usual to introduce the totally symmetric covariant tensor of the third order

$$A_{ikh}=L(x^i, \dot{x}^i)\cdot C_{ikh} \tag{5}$$

which determines the deviation of Finsler's space from Riemann's, which arises from the original space by discarding the dependence of the g_{ik} on the $\dot{x}^i$ (this is equivalent to $A_{ikh}\equiv 0$). The curvature and torsion are given by the tensors

$$\begin{aligned}
R^k{}_{ihj}=&\frac{\partial\Gamma^*{}_i{}^k{}_h}{\partial x^j}-\frac{\partial\Gamma^*{}_i{}^k{}_h}{\partial\dot{x}^s}\frac{\partial G^s}{\partial\dot{x}^j}-\left\{\frac{\partial\Gamma^*{}_i{}^k{}_j}{\partial x^h}-\frac{\partial\Gamma^*{}_i{}^k{}_j}{\partial\dot{x}^s}\frac{\partial G^s}{\partial\dot{x}^h}\right\}\\
&+\Gamma^*{}_i{}^s{}_h\cdot\Gamma^*{}_s{}^k{}_j-\Gamma^*{}_i{}^s{}_j\cdot\Gamma^*{}_s{}^k{}_h,\\
P_{ijkh}=&\,\Gamma^*{}_j{}^r{}_k\cdot A_{ihr}-\Gamma^*{}_i{}^r{}_k\cdot A_{jkr}-L\cdot\frac{\partial C_{ijh}}{\partial x^k}+L\cdot\frac{\partial\Gamma_{ijk}}{\partial\dot{x}^h},\\
S_{ijkh}=&\,A_j{}^r{}_k\cdot A_{ihr}-A_j{}^r{}_h\cdot A_{ikr}.
\end{aligned} \tag{6}$$

We now suppose that, in accordance with Einstein's original idea, the curvature of the space is determined by the distribution of the matter localized in the space and the deviation from Riemann's geometry; as an example, the torsion of the space originates from the electromagnetic, or generally from the meson field.

To realize this assumption we give the connection between the structure of the Finsler space and the electromagnetic (meson) field. We characterize the field by its symmetrized stress-energy tensory S_{ik}, and from this we deduce the following total symmetric covariant tensor of the third order

$$A_{ikh}=-\tfrac{1}{2}\cdot\{S_{ik|h}+S_{kh|i}+S_{hi|k}\} \tag{7}$$

(where $S_{ik|h}$ is the covariant derivative of S_{ik}) which we shall regard as the torsion tensor of Finsler's geometry.

Assumption (7) has also a physical meaning ,and it is easy to see that it can be considered as a generalization of the Lorentz force. To justify this remark we deduce from (7), by contraction of the indices k and h, the following covariant vector

$$A_i=A_i{}^k{}_k=g^{kr}\cdot A_{irk}=-\partial S_i{}^k/\partial x^k, \tag{8}$$

which is just the density of the Lorentz force of the field.

Mathematically (7) gives in the case of the four-dimensional space-time continuum 20 independent partial differential equations for the determination of the dependence of L (or g_{ik}) on the $\dot{x}^i$.

For the determination of the dependence on the x^i we have also Einstein's original equations

$$R_{ik}-\tfrac{1}{2}g_{ik}R=\kappa T_{ik} \tag{9}$$

but now R_{ik} and R are deduced from (6) by the well-known method. T_{ik} is as usual the energy-momentum tensor of the matter. Equation (9) gives also the further 10 partial differential equations for the determination of the dependence of the g_{ik} on the x^i.

Finally, we must pay attention to the fact that the proposed connection between the field theories and the geometrical basis is not only a formal one. We have mentioned that Finsler's space can be regarded as a manifold of line elements defined along curves of the space. If we regard such a curve as the world-line of a particle (electron, nucleon, etc.) by which the field is originated, the line elements have also a direct physical meaning since the unit vector

$$l^i=\dot{x}^i/L \tag{10}$$

deduced from $\dot{x}^i$, can be regarded as the velocity of the particle along the world-line. It is well known that the field of such a moving particle has a strong inhomogeneity. Now this inhomogeneity is faithfully expressed by Finsler's geometry. In cases in which the inhomogeneity would be dissolved (e.g., the static case) every component of A_{ikh} vanishes, as can be verified by elementary calculations, taking account of some of the methods of Finsler's geometry and the usual relations of homogeneous functions. Thus we are not led by our method away from the Riemann space. This theory is also interesting especially in the non-static case, which will be discussed in detail in the near future.

* Extract of the author's habilitation lecture.

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The Diamagnetic Correction for Protons in Water and Mineral Oil

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THE proton nuclear resonance shifts between H_2 gas and water, mineral oil, and the standard sample used in the determination of the proton gyromagnetic ratio[1] have been measured as previously suggested.[1,2] The shifts were measured in a large electromagnet having a 4½-inch gap with pole faces 24 inches across and regulated[3] by means of nuclear resonance. The spatial field variation over a region somewhat larger than that occupied by the sample was of the order of 10 p.p.m. and produced a line width for the distilled water sample of 0.043 gauss.

The three samples were contained in identical spherical Pyrex glass vessels, 1.4-cm i.d. and 2.0-cm o.d. The hydrogen sample was tank hydrogen at a pressure of 40 atmospheres. The distilled water sample was free of dissolved oxygen and the oil sample was Petrolatum U.S.P.-Light.

The resonance shifts between samples were measured relative to a fixed sample and resonance detector probe by means of calibrated Helmholtz coils. Because it is known that slight unpredictable changes in field distribution with time might occur, a

means was provided for rapidly interchanging and accurately positioning the samples in another resonance detector located close to the reference resonance probe.

The tuned r-f coil of the measuring probe was coupled through a high impedance to a low noise 24-Mc crystal oscillator also supplying the reference probe. The r-f voltage across this coil was detected by a high vacuum diode and the resulting audio output fed into a 30 cycle-sec. lock-in amplifier. The diode method of detection was used to eliminate the effect of the dispersion component of the signal, so that the true center of resonance could be obtained. Even with the H_2 gas, which gives a weak signal, the signal-to-noise ratio was sufficient to obtain a consistency of measurement that resulted in an average deviation of 1 part in four million over 10 determinations of the H_2 gas$\rightarrow H_2O$ shift. This average deviation was somewhat greater than in the determination of the other shifts measured.

It is assumed that the time average of the field seen by a particular molecule in the sample is that which would be computed if it were in a spherical cavity surrounded by a diamagnetic medium of volume susceptibility χ. Hence the time average value of the local field is decreased by the fraction $4\pi\chi/3$. But, since the sample itself is spherical and is located in a medium of zero susceptibility, an increase in the field of $4\pi\chi/3$ is produced which causes the net correction to the local field to be zero, at least in the first order of χ.

The actual shifts from H_2 gas for oil and water are respectively: $\Delta H/H=+1.6\times10^{-6}$; and $\Delta H/H=-0.6\times10^{-6}$. Combining these shifts with the diamagnetic correction for H_2 gas calculated[2] by Ramsey, the diamagnetic corrections for mineral oil and water are, respectively, $\delta_{oil}=28.4\times10^{-6}$, and $\delta H_2O=26.2\times10^{-6}$.

The comparison of the standard sample used in the determination of the proton gyromagnetic ratio with the H_2 gas indicates that a diamagnetic correction of 28.1×10^{-6} should be made on the uncorrected value of γ reported[1] earlier. This makes the present corrected value $\gamma=(2.67530_5\pm0.00006)\times10^4$ sec.$^{-1}$ gauss^{-1} and the previously reported[1] value $e/m=(1.75891_2\pm0.00005)\times10^7$ e.m.u. gram^{-1}.

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Spectrum of the Boron Monosulfide (BS) Molecule

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THE spectrum of the boron monoxide (BO) molecule has been analyzed by Mulliken[1] and Jenkins and McKellar,[2] but the analogous spectrum of boron monosulfide has not yet been described. Recently, however, the writer has observed three extensive band systems in an electric discharge through boron trisulfide and has attributed these bands to the BS molecule.

The apparatus consisted of a vacuum system of Pyrex glass, and a quartz discharge tube of the end-on type. Through this tube helium or argon was circulated at low pressure by means of a mercury diffusion pump. A quartz tube, having a glass stopper at one end, contained amorphous boron in a silica boat, and projected horizontally from the one end of the capillary of the discharge tube. A reservoir containing vacuum distilled sulfur protruded vertically downwards from the center of this tube. By means of a cylindrical electrical heater, the amorphous boron could be heated to the desired temperature, while the amount of sulfur vapor entering the system could be controlled separately. The sulfur vapor was carried over the heated boron by the circulating helium or argon gas.

At about 700°C, boron trisulfide (B_2S_3) was formed, and this substance was carried into the capillary of the discharge tube. When this reaction started, the brilliant S_2 spectrum, excited in the capillary at the early stages of the heating process, disappeared and was superseded by a very intense spectrum resembling a "daylight" fluorescent lamp.

The following three strong band systems were observed in the discharge:

(a) An extensive system of open bands in the visible, red, and infra-red regions of the spectrum, degraded to the red. This system will be referred to as the α-system of BS.

(b) A strong system, consisting mainly of two band groups, each with several heads. This system which is degraded to the violet, has a very close fine structure.

(c) A system of double double-headed bands in the ultraviolet region, degraded to the red. This system also has a close fine structure and will be referred to as the γ-system of BS.

The bands of the α- and γ-systems have been arranged in Deslandres Tables. The band heads were measured on plates taken on a large Hilger spectrograph and in the first order of a 21-foot concave grating in a Paschen mounting.

The bands of the α-system originate from a $^2\Pi\rightarrow{}^2\Sigma$ transition, and the rotational analysis showed that the Π-state is inverted. This is analogous to the α-system of BO. These band heads are given by the following equations:

$$^2\Pi_{3/2}\rightarrow{}^2\Sigma:\ \nu_{\text{head}}=\left.\begin{matrix}15663.0\\15668.4\end{matrix}\right\}+[749.37v'-4.77v'^2]-[1173.93v''-6.37v''^2],$$

$$^2\Pi_{1/2}\rightarrow{}^2\Sigma:\ \nu_{\text{head}}=\left.\begin{matrix}15996.7\\16002.1\end{matrix}\right\}+[749.37v'-4.77v'^2]-[1173.93v''-6.37v''^2].$$

The ultraviolet bands originate from a $^2\Pi\rightarrow{}^2\Sigma$ transition, with a normal splitting of the Π-state. These band heads are given by the following equations:

$$^2\Pi_{1/2}\rightarrow{}^2\Sigma:\ \nu_{\text{head}}=\left.\begin{matrix}38785\\38796\end{matrix}\right\}+[884.7v'-6.60v'^2]-[1173.3v''-6.40v''^2],$$

$$^2\Pi_{3/2}\rightarrow{}^2\Sigma:\ \nu_{\text{head}}=\left.\begin{matrix}38899\\38912\end{matrix}\right\}+[884.7v'-6.60v'^2]-[1173.3v''-6.40v''^2].$$

The $^2\Pi_{3/2}\rightarrow{}^2\Sigma$ components of the (2,1), (3,0), (4,0), and (5,0) bands of the α-system have been analyzed from plates taken in the third order of the grating. The following constants were obtained:

$$B_e''=0.7949_0\ \text{cm}^{-1},\quad \alpha_e''=0.0060_5\ \text{cm}^{-1}.$$
$$B'_{e,\text{eff}}{}^{(1)}=0.6213_1,\quad \alpha'_{e,\text{eff}}{}^{(1)}=0.0058_8\ \text{cm}^{-1}.$$

Preliminary analyses of the (0,0) and (0,1) bands of the γ-system have been made from plates taken in the sixth order of the grating. The following tentative values were obtained:

$$B_e''=0.793\ \text{cm}^{-1},\quad \alpha_e''=0.006\ \text{cm}^{-1}.$$
$$B_0'=0.702\ \text{cm}^{-1}.$$

The vibrational and rotational analyses prove that the α- and γ-systems have the same lower state ($^2\Sigma$).

The bands of the blue-green system have not yet been interpreted.

A paper giving further details of these band systems is being prepared at present and will be published in the course of time.

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