Magnetoresistance of Bismuth at 3000 Megacycles

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The magnetoresistance of bismuth for frequencies of 3.5 Mc has been found to be of the same magnitude as for steady direct currents. On the other hand, for long infra-red waves (8μ) the magnetoresistance is zero. Using a resonating bismuth cavity and a slotted wave-guide measurements of the standing wave ratio have indicated that magnetoresistance at 3000 Mc is not more than half as big as for direct currents. This small value of magnetoresistance is possibly due to the fact that the skin depth is of the same order of magnitude as the mean free path of electrons in the metal.

I. INTRODUCTION

THE effect of a magnetic field on the resistance of bismuth for electric currents of high frequency has been investigated at 3.5 Mc by Blunt[1] and found to be of the same magnitude as for direct currents. For frequencies in the infra-red no magnetoresistance is to be observed[2] for wave-lengths of 8μ. It appears, therefore, that somewhere in the region between wave-lengths of 85 meters and 8μ the magnetoresistance of bismuth drops to zero. In order to investigate this phenomenon more precisely some experiments have been performed at wave-lengths of 9.2 cm using a klystron oscillator and a resonating cavity made of bismuth.

II. APPARATUS

Power was fed from the klystron through a flexible coaxial line and a slotted coaxial wave guide into a bismuth cavity which was loop coupled to the end of the slotted section. This cavity was of the cylindrical re-entrant type, of diameter 2.1 cm and length 2.3 cm. A central coaxial rod of length 1.8 cm and diameter 0.57 cm extended from the top of the cavity to within 0.5 cm of the bottom. Tuning for resonance was effected by a screw through the bottom. All parts were of cast and machined bismuth.

In operation much of the incident energy was reflected from the cavity and the standing wave in the slotted section was investigated in the usual way with a probe, crystal rectifier, and sensitive galvanometer. Large attenuation was inserted between the klystron and the slotted section in order to cut down the power to the microwatt region where the crystal rectifier gives readings accurately proportional to the power. This attenuation also served to insulate the klystron from the effects of the reflected energy.

The bismuth cavity was placed between the poles of an electromagnet with pole faces 5.1 cm in diameter and 4.1 cm apart. A field of 6900 oersteds was used in most of the experiments. The cavity was excited in the fundamental TEM mode and thus the predominant current direction was transverse to the magnetic field.

The experimental procedure consisted in adjusting the probe carefully to a point of minimum or maximum power in the slotted section, observing the deflection of the galvanometer, then throwing on the magnetic field and observing the change of galvanometer deflection. These observations sufficed for a calculation of the fractional change of electrical resistance produced in the bismuth by the magnetic field. This fractional change was found to decrease in a regular way with the magnetic field.

The wave-length, measured with a TFS-5 coaxial wave meter, was 9.2 cm.

III. THEORY

Let P_i be the power incident upon the bismuth cavity, P_a the power absorbed in the cavity, and P_r the total reflected power. Then $P_a = P_i - P_r$. Some of the power is reflected at the cavity window. This reflection will not be affected by the magnetic field. When the field is applied the absorbed power will be increased dP_a if the resistance of the bismuth is increased, so that the reflected energy will be changed by dP_r. The incident power is unchanged and thus

$$dP_a = -dP_r.$$

The power loss in a metal cavity[3] is proportional to ρ/δ, where ρ is the resistivity of the wall material, and the skin depth $\delta = [\rho/(\pi f \mu)]^{\frac{1}{2}}$. Here f = frequency of the current, μ = magnetic permeability of the metal. Hence P_a is proportional to $\rho^{\frac{1}{2}}$ and we get $dP_a/P_a = d\rho/(2\rho)$. Substitution of the equivalent values of dP_a and P_a gives

$$d\rho/\rho = -2dP_r/(P_i - P_r).$$

Let A and B be, respectively, the amplitudes of the incident and reflected waves, so that $P_i = KA^2$, $P_r = KB^2$, where K is a constant. We then have

$$d\rho/\rho = -4BdB/(A^2 - B^2).$$

Let the power at a node of the standing wave be P_n,

[1] R. F. Blunt, Phys. Rev. **73**, 654 (1948).
[2] C. W. Heaps, Phys. Rev. **27**, 764 (1926); McLennan, Allin, and Burton, Phil. Mag. **14**, 508 (1932); E. Englert and K. Schuster, Zeits. f. Physik **79**, 194 (1932).
[3] M. I. T. Radar School, *Principles of Radar* (McGraw-Hill Book Company, Inc., New York, 1946), p. 10–57.

and at a loop or antinode, P_a. Then

$$P_n = K(A-B)^2, \qquad P_a = K(A+B)^2,$$
$$dP_n/P_n = -2dB/(A-B), \quad dP_a/P_a = 2dB/(A+B).$$

The probe current at any point in the slotted section is proportional to the power at that point, so the quantities $\Delta_1 = dP_n/P_n$ and $\Delta_2 = dP_a/P_a$ are determined by observing galvanometer deflections. When Δ_1 and Δ_2 are expressed in terms of the amplitudes and added together we get

$$d\rho/\rho = \Delta_1 + \Delta_2.$$

The reflected amplitude B is not greatly different from A so that Δ_2 is much smaller than Δ_1. dB is always a negative quantity so that Δ_1 is positive and Δ_2 negative.

IV. RESULTS

Table I gives representative results, including the largest and smallest values obtained.

The machined surface for the set of data beginning December 3 was several weeks old. Each of the remaining sets for different surfaces began within about an hour after the surface was prepared.

The values of $d\rho/\rho$ from Table I are distributed over a considerable range. It is probable that lack of stabilization of the oscillator is responsible for some of this fluctuation. However, it seems to be possible to draw certain general conclusions. First, the magnetoresistance observed at this frequency is smaller than that found with steady, direct currents. A small bar of this polycrystalline bismuth when tested in this same transverse field was found to have a direct current value of $d\rho/\rho = 0.25$. Second, the character of the surface appears to have a significant effect on the magnitude of $d\rho/\rho$. A freshly machined surface gave the largest value, a surface freshly etched with nitric acid and then carefully washed, the smallest.

It is to be expected that roughness of the surface[4] would affect the magnitude of $d\rho/\rho$ because the direction of current flow would be altered by scratches or projections on the surface. The average direction of the current with respect to the magnetic field would thus be altered and so the magnitude of the magnetoresistance would be changed.

Another important factor would be the chemical purity of the surface. If a film of oxide or other bismuth compound formed on the surface and a substantial fraction of the current flowed in this film the magnetoresistance would not be characteristic of pure bismuth.

TABLE I. Fractional resistance change of bismuth under different conditions for a magnetic field of 6900 oersteds.

Date	Surface	Δ_1	Δ_2	$d\rho/\rho$
Dec. 3	Machined	0.084	−0.013	0.071
Dec. 5	Machined	0.088	−0.013	0.075
Dec. 6	Machined	0.093	−0.028	0.065
Dec. 6	Etched	0.036	−0.018	0.018
Dec. 6	Etched	0.057	−0.011	0.046
Dec. 8	Machined	0.13	−0.006	0.12
Dec. 9	Machined	0.11	−0.000	0.11
Dec. 27	Machined	0.038	−0.013	0.025
Dec. 28	Machined	0.064	−0.018	0.046
Dec. 28	Polished	0.067	−0.000	0.067

In case the skin depth approaches a magnitude comparable to the mean free path of the conduction electrons an effect on magnetoresistance may be expected. Statistical theories[5] explain magnetoresistance as being due to the fact that the conducting electrons possess different values of L/v, where L is the free path and v the velocity of agitation. When the skin thickness is less than the mean free path those electrons moving normal to the surface make a smaller contribution to the current during a free path than those moving parallel to the surface. Their effective mean free path is changed. Hence the fluctuation of L/v among the electrons is different when the skin effect is important, so the magnetoresistance is different.

The skin depth in bismuth for the frequency used in these experiments is about 1×10^{-3} cm. It has been suggested that the mean free path of electrons in bismuth is abnormally large. Eucken and Förster[6] have deduced from experiments on very fine wires of bismuth a value $L=1.08\times10^{-3}$ cm at 273°K. If this estimate is accepted the skin depth and the mean free path are of the same order of magnitude and it is therefore not surprising that the magnetoresistance is considerably smaller than for direct currents.

Mott and Jones[7] have stated that it is not necessary to assume an abnormally large mean free path for bismuth. They believe that the experiments of Eucken and Förster should have some other explanation.

The results of this experiment, if we assume the high frequency resistance of a freshly machined bismuth surface to behave like that of bulk material, seem to favor the idea of an abnormally long free path for electrons in bismuth.

[4] It is interesting to note in this connection that A. B. Pippard, Proc. Roy. Soc. **A191**, 385 (1947), has found the skin resistivity of a drawn copper wire to be nearly equal to that of the bulk material. Polishing the wire resulted in deterioration of the surface and an increase of skin resistivity.

[5] A. Sommerfeld and N. H. Frank, Rev. Mod. Phys. **3**, 1 (1931).

[6] A. Eucken and F. Förster, Göttingen Nachrichten **1**, 43 (1934).

[7] N. F. Mott and H. Jones, *Properties of Metals and Alloys* (Oxford University Press, London, 1936), p. 303.