knowledge of the energy dependence of the sensitivity of our detector.

VI. RESULTS

Values of σ_{t0}, the total cross section for a neutron energy E_n, are given in Table I. The statistical uncertainty in σ_{t0} was calculated from the extrapolation formula and the reproducibility of the cross section for each scatterer during the run. The probable errors assigned to the cross section include the uncertainty in the background count and the uncertainty in the correction for the low energy group of neutrons.

The data have been analyzed by Schwinger's shape independent method described by Blatt and Jackson[3] and by Bethe.[4] To calculate the triplet scattering cross section, we have used Hughes'[13] value of -3.75×10^{-13} cm as the coherent scattering amplitude, 20.36 barns[14] as the zero energy neutron proton scattering cross section, and 2.23 Mev as the binding energy[15] of the deuteron. In terms of this shape independent approximation the effective range and scattering length are 1.73×10^{-13} cm and 5.39×10^{-13} cm, respectively for the triplet interaction. The value of the singlet scattering length is -23.68×10^{-13} cm. The singlet scattering cross section was obtained by subtracting the triplet contribution from the total cross section. The values of the phase shift function, $K\cot^1\delta$ for the singlet interaction were calculated from the singlet scattering cross section and are shown as a function of neutron energy in Fig. 3. The singlet effective range for the neutron proton interaction appears to be approximately equal to the proton-proton effective range[4] of 2.7×10^{-13} cm. One can then infer, to within the accuracy of the above experiments, the charge independence of nuclear forces in the energy range up to 5 Mev.

We wish to express our thanks to Mr. M. Fulk for assistance in taking the data.

This research was assisted by the University of Minnesota Technical Research Fund subscribed to by General Mills, Inc., Minneapolis Star Journal and Tribune Company, Minnesota Mining and Manufacturing Company, and Northern States Power Company.

---

[13] Hughes, Burgy, and Ringo, Phys. Rev. **77**, 291 (1950).

[14] Melkonian, Rainwater, and Havens, Phys. Rev. **75**, 1295 (1949).

[15] R. E. Bell and L. G. Elliot, Phys. Rev. **74**, 1552 (1948).

PHYSICAL REVIEW VOLUME 80, NUMBER 5 DECEMBER 1, 1950

Pressure Measurement in Superflow

R. Bowers, B. S. Chandrasekhar, and K. Mendelssohn
Clarendon Laboratory, Oxford University, England

(Received July 24, 1950)

The pressure of flowing liquid HeII has been measured in the capillary link as well as on its ends. Flow under gravitational and thermal potential has been studied in narrow slits and tubes packed with fine powder. On the basis of ordinary pressure measurement the results are not consistent and it is concluded that the definition of "pressure" in superflow requires additional parameters.

I. INTRODUCTION

THE experiments on liquid HeII have disclosed the existence of a peculiar type of flow which has not been observed in any other fluid. The main characteristic of this "superflow" seems to be the disappearance of viscous dissipation of energy. Linked with this is often the phenomenon of pressure independent flow which is completely free of friction. However, the first condition is evidently the more general one since, as the present work shows, there exist cases in which superflow is accompanied by a loss in kinetic energy. The situation is unfortunately complicated by the fact that liquid HeII in bulk seems to exhibit superflow and ordinary viscous flow side by side and that it is usually difficult to separate the two components of the transport mechanism. In first approximation the flow phenomena can be interpreted by means of a semi-empirical "two-fluid model"[1] which treats the liquid as an interpenetrating mixture of a normal and an anomalous component, the relative concentration of which varies with temperature. The question as to whether or not there exists any theoretical justification for such a concept must be left open, but it is already clear that in spite of the general success of the two-fluid model it cannot be applied in this simple form to some of the observed facts. In particular, most of the results on capillary flow near the lambda-point[2,3] cannot be represented as a simple superposition of viscous and frictionless flow.

A simplification of the flow phenomena can be achieved experimentally by decreasing the diameter of the flow channel and thereby suppressing the viscous component. Pure frictionless flow is observed in film transfer[4] and these results are closely approximated by

---

[1] L. Tisza, Nature **141**, 913 (1938).

[2] J. F. Allen and A. D. Misener, Proc. Roy. Soc. A **172**, 467 (1939).

[3] R. Bowers and K. Mendelssohn, Proc. Roy. Soc. A (to be published).

[4] J. G. Daunt and K. Mendelssohn, Proc. Roy. Soc. A **170**, 423 (1939).

the flow in very narrow capillaries.[2] In all of these experiments the flow rate was found to be independent of the length of the flow channel and of the pressure difference at its ends, but flow through a tube closely packed with fine powder[2] was clearly pressure dependent. This discrepancy shows that the "pressure" in superflow may not be a straightforward concept and requires further elucidation. Pressure measurements were therefore carried out on these two types of superflow.

II. METHOD

In the conventional type of flow experiment two volumes of liquid A and B are connected by a capillary link C (Fig. 1) and the mass of liquid transported per unit time through C is determined in relation to the pressure difference $p_A - p_B$. The flow experiments on HeII have so far been carried out in the same manner but it is clear that in the case of pressure independent flow determinations of the pressure at the ends of the capillary link are insufficient. We have therefore also measured[5] the intermediate pressure p_I at a point I in the capillary link between A and B. Care was taken in all experiments to make the dimensions of the apparatus so that p_I was not falsified by the Bernoulli force.

The experiment was varied in two different ways. The flow was studied on:

(a) narrow slits or channels; and
(b) channels tightly packed with fine powder.

The pressure under which the flow took place was produced by:

(1) gravitational potential; and
(2) thermal potential.

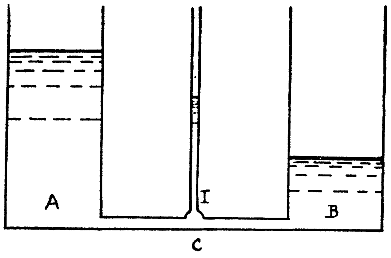

FIG. 1. Scheme of measurements.

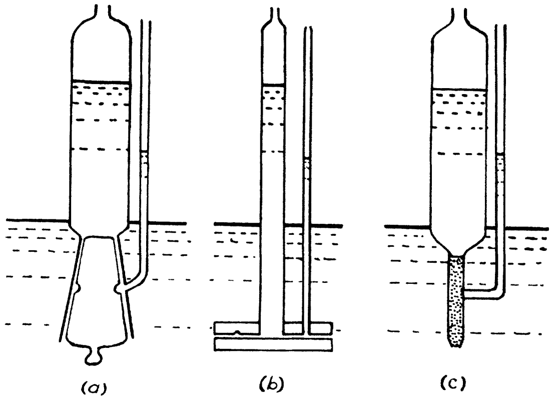

FIG. 2. Experimental arrangements.

Two types of slit were used, one in the form of a conical joint and the other between two optically flat glass plates (Fig. 2a, b). In each case the slit was divided into two lengths by means of a groove to which a narrow side tube was connected. The pressure at the ends of C was then given by the height of the helium levels in the experimental vessel and the bath while the intermediate pressure was indicated by the level in the side tube. The present paper deals only with the narrowest slits produced in this way which were of the order of 1 to 5×10^{-4} cm. The general arrangement for the experiments with packed channels (Fig. 2c) was similar, except that the slits were replaced by glass or metal tubes into which jewellers rouge was pressed tightly. While the results on slits differed from those on packed channels those in each of the two groups were completely consistent in themselves.

The experiments were carried out in a combined liquefier-cryostat[6] in which the helium is free from contamination[7] and the heat influx is reduced to a minimum. The light used in the observations was supplied by a 12-watt bulb at one meter distance from the cryostat and passed through screens of copper sulfate solution in water, liquid air and liquid hydrogen. Under these conditions the total evaporation from the helium cryostat was 2 cc of liquid per hour. All measurements described here were carried out at $\sim 1.3°$K where the rate of superflow is practically independent of temperature.

III. RESULTS

Under equilibrium conditions the levels in the helium vessel and in the bath adjusted themselves to equal height while the position of the side level was ~ 1.5 mm higher owing to surface tension forces. In the experiments under gravitational potential the vessel was partly lifted out of the bath and the change with time of its level and that in the side tube were noted. In the case of slits (Fig. 3, 1a) the drop in the vessel was almost linear, showing that the flow was, in first approximation, pressure independent. The side level fell rapidly immediately on lifting and adjusted itself to the height of the bath level. The side level also took up the position of the bath level when the direction of the flow was reversed. Thus, as was expected[8] in the case of pressure independent flow, the total pressure drop was concentrated at the narrowest part of the slit. In the present case the ratio of resistances to ordinary laminar flow between the two parts of the slit was $\sim 1:4$, but in similar experiments on narrow channels ratios of $\sim 1:2$

---

[5] R. Bowers and K. Mendelssohn, Proc. Phys. Soc. A **63**, 178 (1950).

[6] J. G. Daunt and K. Mendelssohn, J. Sci. Inst. **25**, 318 (1948).

[7] R. Bowers and K. Mendelssohn, Nature **163**, 870 (1949).

[8] K. Mendelssohn, Proc. Phys. Soc. London **57**, 371 (1946).

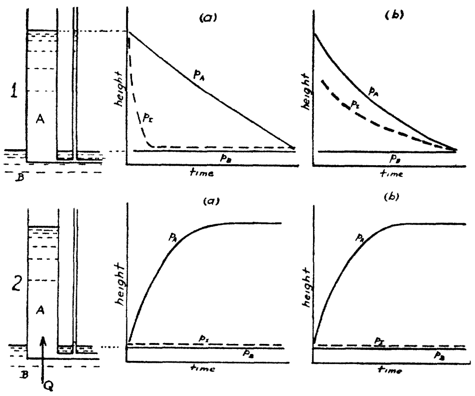

FIG. 3. Pressure distribution: (1) under gravity, (2) under thermal potential, (a) in slits, (b) in tubes packed with powder.

were used and even under these conditions the whole of the pressure drop occurred in the slightly more constricted channel.

The flow through powder (Fig. 3, 1b) is completely different, the rate depending on $(p_A - p_B)^{\frac{1}{2}}$ which indicates turbulence. The dissipation of kinetic energy is clearly not an end effect but takes place over the length of the powder tube as is shown by the position of the side level taking up an intermediate position. Comparison experiments above the lambda-point showed that the flow of HeI is more than 500 times smaller, which means that the dissipation cannot simply be caused by the viscosity of the "normal" component of liquid HeII. Estimates[9] of the viscosity of this component vary at ~1.3°K between 0.8 and 0.1 of that of HeI but a factor 0.002 seems to be quite inadmissable. The type of transport in the packed tubes therefore is evidently pressure dependent but non-viscous and might be described as turbulent superflow. Thus this observation also seems to contradict the two-fluid model in its simple form.

For the observations on flow under thermal potential heating coils were inserted into the helium vessels and the small apertures at the top were sealed off. In this case the results on slits and packed tubes were identical (Fig. 3, 2a and b). On application of a steady heating current the level in the helium vessels was found to rise and ultimately to reach a constant thermo-mechanical pressure.[10] In *all* cases the side level remained constant at the height of the bath.

IV. CONCLUSIONS

The four experiments illustrated in Fig. 3 present a curious inconsistency. They show that in superflow the intermediate pressure p_I is not determined unambiguously by the pressures p_A and p_B at the ends of the tube. In the cases 1a, 2a, and 2b for any given value of $p_A - p_B$; p_I is equal p_B but in 1b, $p_A > p_I > p_B$. The inconsistency remains even when a distinction is made between hydrostatic and thermo-mechanical pressure. On the basis of such a distinction one can argue that the flow in 2a and b takes place under zero hydrostatic pressure. However, then we are faced with the difference between cases 1a and 1b together with the fact (from 2b) that the powder tube can carry flow under zero hydrostatic pressure. Taking the discrepancy between cases 1b and 2b we must conclude that for a given flow velocity a pressure gradient may or may not exist along the tube. It thus appears that, in contradistinction to ordinary flow, the intermediate pressure is not simply a function of the end pressures but requires for its determination one or more additional parameters.

---

[9] F. London, Phys. Soc. Camb. Conf. Rep. 2, 1 (1947).

[10] J. F. Allen and J. Reekie, Proc. Camb. Phil. Soc. 35, 114 (1939); L. Meyer and J. H. Mellink, Physica 13, 180 (1947).