On the Magnetic Moments of the Proton and Neutron

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September 14, 1950

ONE of the principal difficulties encountered by the theory of elementary particles is the explanation of the "anomalous" magnetic moments of the proton and neutron, for the moments are markedly different from the values of one nuclear magneton ($\mu_N = e\hbar/2Mc$) and zero, respectively, which would be expected from the Dirac equation. Current theories ascribe this difference to the effect of the circulation of mesons in the vicinity of a bare nucleon. All attempts at obtaining quantitative agreement between values calculated from this model and experiment seem to have been uniformly unsuccessful.[1] As a result, it may be of value to approach the problem from a somewhat different level of sophistication. In fact, as we shall see below, the most rudimentary semiclassical and dimensional considerations lead directly to extremely accurate formulas for the moments; in particular, the proton moment obtained in this way agrees *exactly* with the best available experimental value.

We begin by remembering that a characteristic length a which can be associated with the proton is the corresponding classical radius of a charged particle,[2] *viz.*, $a = 2e^2/3Mc^2$. In addition, we recall that the existence of an isolated magnetic pole has been shown by Dirac to be compatible with the requirements of quantum mechanics.[3] The magnitude of this pole is $g = \hbar c/2e$. Since the dimensions of magnetization are those of field, i.e., pole/(length)^2, it is natural to take $I_0 = g/a^2$ as the simplest formula for any magnetization which may be characteristic for this case. A sphere of radius a which is uniformly magnetized with magnetization I_0 has a magnetic moment

$$\mu_p = 4\pi a^3 I_0/3 = 4\pi a g/3 = (8\pi/9)\mu_N.$$

Identifying this moment with that of a proton, we see that its value in units of μ_N is $8\pi/9 = 2.79253$. Within the experimental uncertainty, this agrees exactly with the value 2.79255 ± 0.00010 given by Mack.[4]

Magnetization can also be expressed as pole/area, so one may wonder what the consequences may be of considering a circle of radius a to be the fundamental area; this leads to a characteristic magnetization $I_1 = g/\pi a^2$. Of more interest for us, however, is the difference, $\delta I = I_0 - I_1 = (\pi - 1)I_0/\pi$. For a sphere of radius a, the similar use of δI leads to a moment

$$|\mu_N| = 4(\pi - 1)a^3 I_0/3 = (8/9)(\pi - 1)\mu_N.$$

In this case $(8/9)(\pi - 1) = 1.90364$, while the neutron moment is 1.91280.[4] The discrepancy in the two values is less than $\frac{1}{2}$ percent of the experimental value; this discrepancy may result from the necessity of using a radius for the neutron slightly different from that which is useful for the proton. From above, we immediately obtain the convenient formula: $|\mu_N|/\mu_p = (\pi - 1)/\pi$. This has the value 0.6817 as compared to the experimental value of 0.6850.

Although these models are suggestive and lead to the correct values for the moments, they are probably more of heuristic value than as a picture which can be taken literally. For example, the radius used is much smaller than that usually associated with nucleons, the latter being more of the order of the Compton wavelength, $h/Mc \approx e^2/mc^2 \approx$ range of nuclear forces. One would naturally prefer to obtain these numerical values directly from some general equation involving only e, $\hbar$, c, and m, say, rather than by invoking models of such specificity, but, in any case, the results obtained above are accurate and covenient formulas.

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[1] For example: K. M. Case, Phys. Rev. **74**, 1884 (1948); J. M. Luttinger, Phys. Rev. **75**, 309, 1277 (1949); S. D. Drell, Phys. Rev. **76**, 427 (1949); S. Borowitz and W. Kohn, Phys. Rev. **76**, 818 (1949).

[2] M. Abraham and R. Becker, *Theorie der Elektrizität*, II, (Teubner, Leipzig, 1933), p. 43.

[3] P. A. M. Dirac, Phys. Rev. **74**, 817 (1948).

[4] J. E. Mack, Rev. Mod. Phys. **22**, 64 (1950).

Excited State of C^{14} from the $C^{13}(d, p)C^{14*}$ Reaction*

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October 2, 1950

AT present there is very little information concerning the excited states of C^{14}, and that which does exist has been confined to the $C^{13}(d,p)C^{14*}$ reaction. Bennett *et al.*,[1] using 1.0-Mev deuterons, found only the ground-state group with a Q-value of 6.09 ± 0.2 Mev. They concluded that there were probably no levels in C^{14} below 2.8 Mev. Humphreys and Watson,[2] using 3.8-Mev deuterons, reported two proton groups with Q-values of 5.82 ± 0.2 and 0.59 ± 0.3 Mev, which they assigned to the ground state of C^{14} and an excited level at 5.2 Mev. The increase in yield of the latter group with increased amount of C^{13} is not obvious from their published results. Recently, Curling and Newton,[3] using 0.93-Mev deuterons, measured a Q-value of 5.91 ± 0.03 Mev for the ground-state $C^{13}(d,p)C^{14}$ group. In addition, they observed a proton group, which, if assigned to the $C^{13}(d,p)C^{14*}$ reaction, would have a Q-value of 0.32 ± 0.03 Mev, corresponding to an excited state of C^{14} at 5.59 ± 0.04 Mev. This group occurred at the same range as a $N^{14}(d,p)N^{15*}$ group; however, they considered the group to be five times more intense than expected for the $N^{14}(d,p)N^{15*}$ group.

The gamma-radiations from $C^{13}+d$ have been measured by Thomas and Lauritsen.[4] They found a gamma-ray of 6.115 ± 0.030 Mev at 0.6-Mev bombarding energy which could only be assigned to the $C^{13}(d,p)C^{14*}$ reaction on the basis of energy considerations. In addition, they assigned a gamma-ray of 5.69 ± 0.05 Mev to the $C^{13}(d,p)C^{14*}$ reaction on the basis of the results of Curling and Newton.[3]

In view of the paucity of reported levels in C^{14} and also because C^{13} occurs as a contaminant on all targets (1.1 percent of natural carbon), it was decided to investigate the $C^{13}(d,p)$ reaction using the M.I.T. magnetic spectrometer. The targets used for this investigaton were prepared by allowing a few drops of a suspension of $BaCO_3$ in water to evaporate onto a thin film of Formvar stiffened by a thin layer of evaporated gold. By this method, targets were prepared of both normal $BaCO_3$ and $BaCO_3$ in which the C^{13} was enriched to 52 percent of the carbon content.[5] Direct evaporation of the $BaCO_3$ was not possible, inasmuch as it decomposes at high temperatures. The target thickness was estimated to be about 90 kev for the $C^{13}(d,p)C^{14}$ ground-state proton group. It was found that these targets survived long exposures to the bombarding deuterons, and their considerable thickness was not a disadvantage in the present work.

The lower curve in Fig. 1 indicates the results of a partial survey made at 1.507-Mev bombarding energy using an enriched $BaC^{13}O_3$

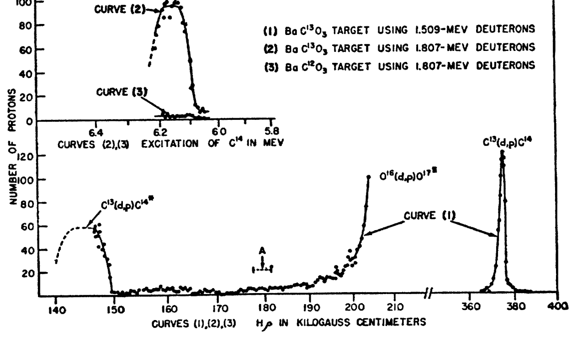

FIG. 1. Proton groups observed from targets of normal $BaCO_3$ and enriched $BaC^{13}O_3$ at bombarding energies of 1.509 and 1.807 Mev.

target. The observed $C^{13}(d,p)C^{14}$ ground-state group had a measured Q-value of 5.948±0.014 Mev, in good agreement with Curling and Newton's value[3] of 5.91±0.03 Mev. In addition, proton groups were investigated with energies between 2.0 and, 1.03 Mev, corresponding to a region of excitation of C^{14} from 5.1 to 6.14 Mev. One proton group was observed which, if assigned to the $C^{13}(d,p)C^{14*}$ reaction, had a Q-value of -0.148 ± 0.005 Mev, corresponding to an excited state of C^{14} at 6.096±0.015 Mev. This group was estimated to be about three times more intense than the ground-state $C^{13}(d,p)C^{14}$ group. No additional proton groups were observed in this partial survey, except the low energy side of the $O^{16}(d,p)O^{17*}$ group with $Q=1.049$ Mev. The remainder of the $O^{16}(d,p)O^{17*}$ group was not observed because of the presence of a large background of deuterons elastically scattered from the target material.

The assignment of the previously unreported $C^{13}(d,p)C^{14*}$ group was verified in two ways. The yield of this group was observed at 1.807-Mev bombarding energy from both enriched $VaC^{13}O_3$ and normal $BaCO_3$ targets. The results shown in curves 2 and 3 indicated that this proton group should be attributed to the C^{13} isotope. In addition, a precise measurement was made of the change in energy of this proton group for a change in bombarding energy of 298±2 kev. The expected shift in energy of a $C^{13}(d,p)$ group would be 238±2 kev, while the shift for a $C^{12}(d,p)$ or $N^{14}(d,p)$ group would be 234±2 kev or 242±2 kev, respectively. The observed energy shift was 240±2 kev, indicating that the target mass could not be different by more than one unit from 13. Since there are no known $C^{12}(d,p)$ or $N^{14}(d,p)$ groups in this region, the assignment of this group to the $C^{13}(d,p)C^{14*}$ reaction appears to be correct.

It is concluded that a level has been found in C^{14} at 6.095±0.015 Mev, which is in excellent agreement with Thomas and Lauritsen's assignment[4] of a 6.115±0.030-Mev gamma-ray to an excited state of C^{14}. No additional $C^{13}(d,p)C^{14*}$ groups were found in a region of excitation of C^{14} from 5.2 to 6.1 Mev with an intensity greater than 0.2 the intensity of the proton group corresponding to the 6.096-Mev level. Hence, there was no evidence for the proton group reported by Curling and Newton which would correspond to a level of C^{14} at 5.59±0.04 Mev (indicated by region A in Fig. 1). Thomas and Lauritsen have assigned to this same level a gamma-ray of 5.69±0.05 Mev which was 0.3 as intense as the 6.115-Mev gamma at 1.58-Mev bombarding energy. The present results indicate no corresponding proton group with this relative intensity at a bombarding energy of 1.51 Mev.

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* This work has been assisted by the joint program of the ONR and AEC.

[1] Bennett, Bonner, Hudspeth, Richards, and Watt, Phys. Rev. **59**, 781 (1941).

[2] R. F. Humphreys and W. W. Watson, Phys. Rev. **60**, 542 (1941).

[3] C. D. Curling and J. O. Newton, Nature **165**, 609 (1950).

[4] R. G. Thomas and T. Lauritsen, Phys. Rev. **78**, 884 (1950).

[5] We are indebted to Professor J. D. Roberts of the M.I.T. Chemistry Department for the enriched $BaC^{13}O_3$.

Erratum: Scattering of Positrons and Electrons by Nuclei

[Phys. Rev. **79**, 892 (1950)]

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October 6, 1950

THROUGH an editorial slip the errors in the figures given in Table I were indicated incorrectly. The correct form of Table I is as follows:

Table I. Ratio of e^-/e^+ scattering by platinum.

Kinetic energy (Mev)	e^-/e^+ (experimental)	e^-/e^+ (unscreened theory)
0.7	3.15±0.15	2.74
1.0	3.13±0.12	2.90
1.3	3.60±0.36	2.98