Comments on Truesdell's Paper on Bernoulli's Theorem for Viscous Compressible Fluids

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1. The necessity[1] of the $\partial\mathbf{V}/\partial t=\mathbf{grad}U$ restriction is questionable since a Bernoulli theorem of the type defined by the author can be written even when the "local acceleration potential," U, does not exist. In that case Eq. (3) of reference 1 can be written as

$$\mathbf{w}\times\mathbf{v}+\mathbf{grad}\left(\frac{v^2}{2}+\frac{\lambda+2\mu}{\rho}\theta\right)=\bar{\mathbf{f}},$$

where

$$\bar{\mathbf{f}}=\mathbf{f}-\frac{1}{\rho}\,\mathbf{grad}p+(\lambda+2\mu)\frac{\theta}{\rho}\,\mathbf{grad}\,\log\rho-\frac{\mu}{\rho}\,\mathbf{curl}\,\mathbf{w}-\frac{\partial\mathbf{v}}{\partial t}.$$

Of course the additional generality obtained in this way increases the complexity of the definition of the curves $\mathcal{C}$ through the additional term $-\partial\mathbf{V}/\partial t$ in $\bar{\mathbf{f}}$.

2. In Eq. (2), and consequently in (3), there has been used the identity

$$\frac{\lambda+2\mu}{\rho}\,\mathbf{grad}\theta\equiv\mathbf{grad}\left[(\lambda+2\mu)\frac{\theta}{\rho}\right]+(\lambda+2\mu)\frac{\theta}{\rho}\,\mathbf{grad}\,\log\rho,$$

presumably to introduce viscosity terms into the expression for the Bernoulli constant. Viscosity terms also appear, however, in the expression for $\bar{\mathbf{f}}$. It would seem preferable to put (3) in the form $\mathbf{w}\times\mathbf{v}+\mathbf{grad}(U+\frac{1}{2}v^2)=\bar{\mathbf{f}}$, where

$$\bar{\mathbf{f}}=\mathbf{f}-\frac{1}{\rho}\,\mathbf{grad}p+\frac{\lambda+2\mu}{\rho}\,\mathbf{grad}\theta-\frac{\mu}{\rho}\,\mathbf{curl}\,\mathbf{w}.$$

In this form both the Bernoulli constant and $\bar{\mathbf{f}}$ are simplified.

3. The utility of the classical Bernoulli theorems rests in the fact that they connect pressure with velocity. The author's grouping of the variables discards this feature except for incompressible flow. Thus the "generalized" Bernoulli theorem given in the paper does not reduce directly to the classical form when the appropriate simplifying assumptions are made.

A theorem which does contain the pressure, and which does reduce as desired is readily obtained. Assuming a conservative body force and a barotropic fluid, as well as steady vorticity, we have

$$\partial\mathbf{v}/\partial t=\mathbf{grad}U,\quad \mathbf{f}=-\mathbf{grad}\psi,\quad p=p(\rho).$$

Equation (3) of reference 1 can then be written as

$$\mathbf{w}+\mathbf{v}+\mathbf{grad}B=\bar{\mathbf{f}},$$

where

$B\equiv U+\psi+\frac{1}{2}v^2+\int dp/\rho$ is the "Bernoulli constant,"

and

$$\bar{\mathbf{f}}\equiv\frac{\lambda+2\mu}{\rho}\,\mathbf{grad}\theta-\frac{\mu}{\rho}\,\mathbf{curl}\,\mathbf{w}.$$

Here B resembles closely the classical form, and the definition of $\bar{\mathbf{f}}$ is simpler than the author's in that it does not contain either $\mathbf{f}$ or p. This is a matter of no small importance if the curves $\mathcal{C}$ are to be located in an actual flow problem.

4. In the special case $\mathbf{w}\times\mathbf{v}=0$, it follows at once from Eq. (3) that

$$\bar{\mathbf{f}}\cdot\mathbf{curl}\,\bar{\mathbf{f}}\equiv 0$$

since

$$\mathbf{curl}\,\bar{\mathbf{f}}=\mathbf{curl}\,\mathbf{grad}B\equiv 0.$$

Thus the condition on the existence of the surfaces is in this case automatically satisfied.

5. The application of the theorem to the solution of flow problems is difficult because of the complex definition of the curves $\mathcal{C}$. The writers regret that the author did not include some example of its use.

6. Finally, there are three typographical errors in Eq. (3), which should read:

$$\bar{\mathbf{f}}=\mathbf{f}-\frac{1}{\rho}\,\mathbf{grad}p+\frac{\lambda+2\mu}{\rho}\theta\,\mathbf{grad}\,\log\rho-\frac{\mu}{\rho}\,\mathbf{curl}\,\mathbf{w}.$$

[1] C. E. Truesdell, Phys. Rev. **77**, 535 (1950).

On Bailey's Theory of Growing Circularly Polarized Waves in a Sunspot

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IN a recent paper Bailey[1] developed a theory to show that circularly polarized waves in an ionized medium would be amplified, in certain frequency bands, if a d.c. axial magnetic field $\mathbf{B}_0$ was present and if the medium possessed a drift velocity $\mathbf{u}_0$ with a component parallel to the axial magnetic field. It was argued that the ordinary waves[2] would be amplified if

$$\mathbf{u}_0\cdot\mathbf{B}_0>0$$

and that the extraordinary waves would be amplified if

$$\mathbf{u}_0\cdot\mathbf{B}_0<0$$

the frequency bands in which amplification can take place being, for small u_0 just those bands that are attenuating when the ionosphere has zero drift velocity. Furthermore, the rate of growth of those amplified, or growing, waves is approximately independent of the magnitude of the drift velocity, being approximately equal to the rate of attenuation in a stationary ionosphere.

This result is so surprising that a detailed discussion both of the physical mechanism by which the energy amplification takes place and of the mechanism by which the growing wave is excited seems necessary before it can be accepted. Professor Bailey's theory is essentially mathematical in nature and his criterion for the existence of amplification is simply that the phase velocity of the growing wave

$$\exp(\alpha z)\cdot\exp[i(\beta z-wt)]$$

be positive. That is that

$$\alpha>0,\quad \beta>0.$$

If the phase velocity be positive it follows that the flow of energy associated with this wave is also positive, that is, out of the medium. Bailey concludes that, in this case, the growing wave is not a wave reflected from the far end of the medium but can be directly excited by an electromagnetic wave, of suitable polarization, incident in the initial surface of the medium. However, this conclusion is unjustified. If the transient response of the medium to a signal incident on the initial phase $z=0$ is analyzed, it can be shown that this growing wave will never be excited if the medium extends, without discontinuity, to infinity. If there is a surface of discontinuity at $z=d$, then the growing wave at z cannot be excited until a time greater than $(2d-z)/c$ has elapsed, where c is the velocity of light in vacuum.

Thus, despite the fact that it has a positive phase velocity, this growing wave must be regarded as a reflected wave, whose amplitude, at the surface of discontinuity, bears a relation to the amplitude at this point of the incident wave which is determined solely by the impedances of the true media on either side of the discontinuity.

Ideally, by suitable choice of this impedance, it should be possible to obtain power amplification in a laboratory device, but in the sun, where the change of space-charge density and magnetic field with distance is presumably slow, it does not seem that Bailey's theory can account for the observed phenomenon.

It is hoped to publish elsewhere a detailed criticism of Bailey's theory, with particular attention to the physical aspects, together with an alternative explanation for the excess noise from sunspots.

[1] V. A. Bailey, Phys. Rev. **78**, 428 (1950).
[2] In the magneto-ionic theory ordinary waves have a sense of rotation of polarization opposite to that of a free electron under the action of the axial magnetic field while the reverse is true for the extraordinary waves.

Search for Be[7] States Using the Li[6] Technique*

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IN Be[7], the mirrored counterpart of the single 480-kev excited state in Li[7] is now well established at 429±15 kev by two independent methods.[1] However, two additional levels in Be[7] have been reported[2] in spectrum studies on neutrons from LiCl and LiF bombarded with 5.1-Mev (cyclotron) protons. Subsequently others[3] using similar techniques at lower proton energies have not found such levels. We have used the improved neutron energy resolution of the Li[6] technique[4] to measure Li[7](p,n)Be[7] neutron spectra in an extended search for any additional levels in Be[7]. The possibility[2] of a proton energy threshold for excitation of these additional levels led us to extend E_p nearly 0.5 Mev above the maximum Van de Graaff beam energy previously employed.

The proton beam from the Berkeley electrostatic generator was deflected through a stationary 90°-analyzing magnet into a double slit system defining E_p to ±15 kev. A spectroscopically pure lithium target was used to eliminate extraneous (p,n) reactions from the negative radical of any Li compound. Metallic Li was evaporated onto a thin copper backing disk and transferred in an argon atmosphere to the target chamber. Target thickness measured less than 30 kev for all E_p. An oil-vapor trap was used to prevent carbon deposits on the target. Li[6] plates in Cd containers were mounted tangentially to neutron direction at radial angles $\eta=0$, 30, and 90° to the proton beam. Exposures were monitored by proton collection on the insulated target with a conventional current integrator to measure the charge. Exposed plates were faded, processed[5] and examined under 90× oil-immersion objectives and 8× oculars fitted with AO-1407 reticules. The sum of alpha- and triton ranges was measured to the nearest *RU* (Fig. 1); an eyepiece goniometer gave laboratory angles, ϕ, between neutron and triton, to the nearest degree. Details of data selection and analysis using the Li[6] technique for collimated and/or isotropic neutrons are available.[4,5]

Two separate runs were made at each $E_p=3.00$, 3.60, and 4.35 Mev. Approximately 10,000 disintegrations have been measured. Figure 1 shows measured histograms for $\eta=0°$. From mean energies of the resolved neutron groups in nine different spectra an average value of 433±26 kev for the level excitation energy in Be[7] is obtained. It is noteworthy that this level was definitely excited even for the lowest $E_p=3.00$ Mev. Though some of the data suggest a level intermediate between 433 kev and the ground state, no single group corresponding to such a level was considered sufficiently resolved or sufficiently intense to be so identified with certainty. Spectra were corrected for the energy dependence of the geometrical selection criteria[5] but not for σ_c(Li[6]), since this is not established for $E_n>0.6$ Mev. Preliminary evidence[5] indicates a broad resonance near 2 Mev and probable $1/E$ flatness elsewhere. Applying this tentative σ-correction, no additional group is observed with more than 15 percent of ground-state intensity. In regions of large $d\sigma/dE_n$ (e.g., near 2 Mev) clearly there has been slight shifting and considerable spreading of the observed peaks. Most accurate comparison of relative group intensities and widths must await precison measurements, now underway, of σ_c(Li[6]) for $E_n>0.6$ Mev.

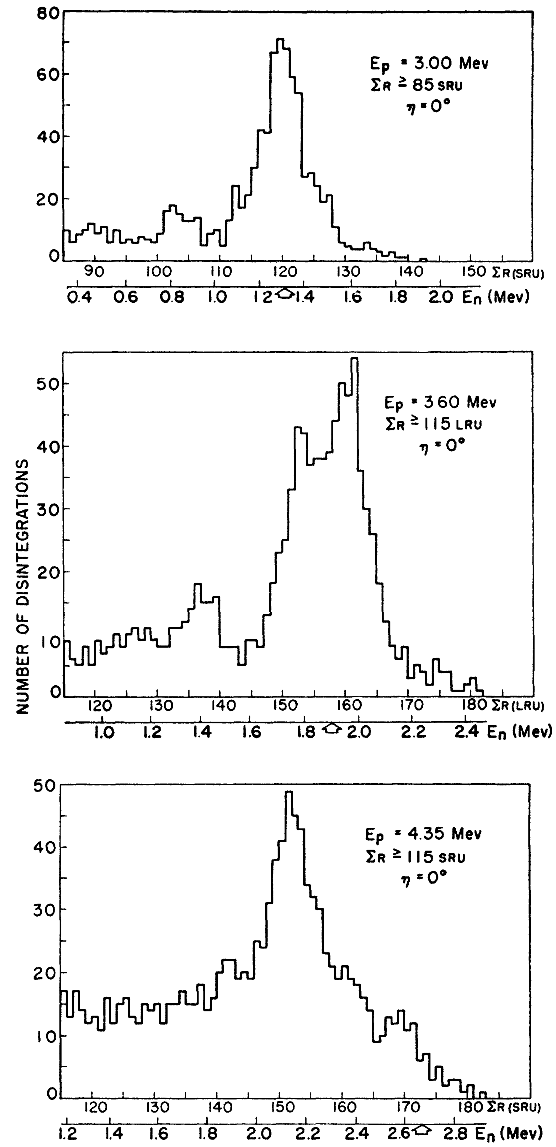

FIG. 1. Li[7](p,n)Be[7] neutron spectra (uncorrected for σ_{Li^6}) observed at 0° to the proton beam. Arrow indicates ground-state energy as calculated from E_p, η, and $Q=-1.63$ Mev. One *LRU* $=0.60\mu$; one *SRU* $=0.70\mu$.

Possible sources of error are considered in detail in the complete report of this work, UCRL-924; extraneous (p,n) reactions in the target assembly are shown to be negligible at these values of E_p, while any (n,p) reactions in the emulsion are readily distinguished from Li[6] disintegrations by track length or grain density variation. Li[6] monitor plates exposed near the target assembly indicated a less than one percent neutron background for all E_n.

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[1] Brown, Chao, Fowler, and Lauritsen, Phys. Rev. **78**, 88 (1950). T. Lauritsen and R. G. Thomas, Phys. Rev. **78**, 88 (1950).
[2] J. Grosskreutz and K. Mather, Phys. Rev. **77**, 580 (1950).
[3] Johnson, Laubenstein, and Richards, Phys. Rev. **77**, 413 (1950); Freier, Rosen, and Stratton, Phys. Rev. **79**, 721 (1950); B. Hammermesh and V. Hummel, Phys. Rev. **78**, 73 (1950).
[4] G. Keepin, Jr. and J. Roberts, Rev. Sci. Inst. **21**, 163 (1950); A. J. F. Siegert and G. Keepin, Jr., LAMS 937 (1950), unpublished.
[5] G. Keepin, Jr., UCRL 790 (1950), unpublished.