Momentum Distribution of Charged Cosmic-Ray Particles at Sea Level*

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A new measurement of the momentum distribution of high energy charged cosmic-ray particles at sea level has been made with a magnetic spectrograph which uses two counter-controlled cloud chambers and a large electromagnet. The spectrum is extended beyond previous determinations to a momentum of 80 Bev/c. For the 1547 measureable sets of tracks recorded the positive–negative ratio is 1.26±0.06. Up to the highest momenta measured the differential spectrum can be represented by an expression of the form $1/p^s$ with $s=1.9$. The positive excess and positive–negative ratio appear to be functions of momentum with the positives predominating at about 1.3 Bev/c. Except for this the positive and negative spectra are very similar.

Although the effect is barely outside statistical uncertainty there seems to be an anomalous dip in the momentum distribution at about 3 Bev/c. There seems to have been a small diurnal variation in the spectrum during the time occupied by the experiment, 1 Bev/c particles being favored during the day, and 3 Bev/c and 6 Bev/c particles being favored at night. These effects are discussed and the present results are compared with data of other experimenters.

I. INTRODUCTION

SEVERAL determinations of the cosmic-ray momentum spectrum have been made by observing curvatures of tracks in a counter-controlled cloud chamber immersed in a uniform magnetic field.[1–8] All of them agree in the gross features of the distribution; a rise at the low end to a maximum at about 1 Bev/c and then a decrease in intensity which falls off roughly as the inverse square of the momentum out to a momentum of about 10 Bev/c which is the upper limit of previous measurements.

This experiment is an extension of the magnetic deflection method to higher momenta and to higher accuracy in the range already investigated. It was undertaken in order to study the form of the spectrum above 10 Bev/c, and to reexamine the part below 10 Bev/c with higher resolving power in a search for properties of the distribution such as positive-negative ratio that might vary with momentum. Two cloud chambers are used, one placed above and the other below a region of fairly strong and uniform magnetic field in air. Straight tracks are observed in the chambers and the angular deflections of the particles in traversing the field are obtained from stereographic pictures which give the angles between the tracks. A pair of Geiger counters in coincidence triggers the cloud chambers. Figure 1 shows the main parts of the apparatus schematically.

In spite of its increased difficulty as compared with the conventional method, the two-chamber method offers several advantages for this kind of measurement. Since the chambers are not in the gap, the required field can be produced more economically; the chambers are easier to photograph and insulate thermally against distortions due to convection currents in the chambers, and tracks which are not straight because of scattering or distortions can be rejected easily and objectively. A final advantage of the present method is the ease and accuracy with which the deflection angle can be measured as compared with the difficult task of measuring curvatures directly. We have introduced a new source of uncertainty with the present arrangement, however, that is, the scattering in the material separating the sensitive volumes of the two chambers. It turns out that the effect of scattering is not serious as will be shown by a simple calculation which has been checked experimentally.

II. THE APPARATUS AND ITS OPERATION

The experiment described here was performed at Pasadena, California, at an elevation of about 800 feet above sea level. All of the particles observed entered the apparatus from a nearly vertical direction after passing through a roof of duraluminum about 2 mm thick which constituted the only solid material above the top counter. The magnetic field was oriented nearly north and south and was horizontal. Data included in this survey were recorded between May 27 and September 9, 1949, during which time the apparatus was operated continuously day and night except when repairs and adjustments were necessary. Pictures were taken at the rate of six or seven an hour during normal operation. The magnet gives a very nearly uniform field of about 9000 Gauss at 3.3 kilowatts. It will be shown later that this uniformity in the vertical direction is not necessary for this experiment, but that the quantity characterizing the magnetic field which is relevant here is the line integral $\int B dl$, taken along a vertical straight line at

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[5] P. M. S. Blackett, Proc. Roy. Soc. **A159**, 1 (1937).
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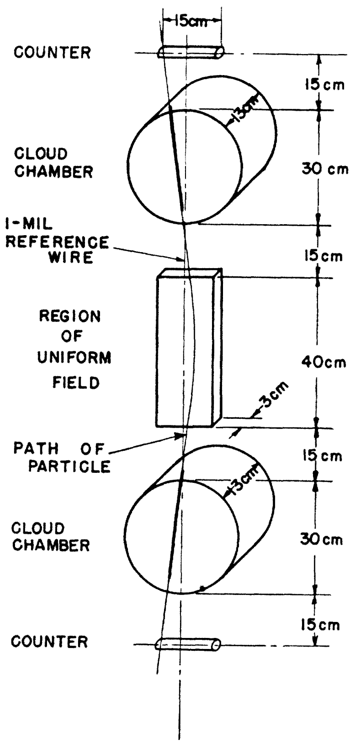

FIG. 1. Arrangement of apparatus showing the path of a typical particle which passes through the two counters, two cloud chambers, and the region of uniform magnetic field. The 1-mil tungsten wire furnishes a reference direction for measuring angles in the photographs.

the center of the gap. Its value is $(3.9\pm0.2)\times10^5$ Gauss-cm for this experiment.

The cloud chambers are of the conventional moving-piston design and have a sensitive volume 30 cm in diameter and 13 cm deep. Each chamber is photographed stereographically by means of two cameras placed about 140 cm in front of the chamber, one of which looks along the chamber axis. The axis of the second camera is parallel to the axis of the first but is displaced 25 cm to the side. A 6-foot length of 1-mil tungsten wire is stretched vertically the whole length of the apparatus just in front of the cloud chambers to provide a reference direction in the pictures with respect to which angles may be measured on the films. Two Geiger counters in a double coincidence circuit are placed one above the top chamber and one below the bottom one in such a way that only a particle passing through the uniform portion of the field and the center portions of the cloud chambers trip the apparatus except, of course, for scattered particles and accidental coincidences.

III. THEORY OF THE MEASUREMENTS

To an approximation sufficient for this experiment, the magnetic field used here can be considered to be uniform except for variations due to fringing at the top and bottom of the magnet. Particles which get near the edge of the pole pieces in the horizontal direction are quickly lost from the field. Those few which do get bent back into the field can be found by examining full scale projections of the pictures.

The equation of motion of a particle carrying a single electronic charge and moving in a plane perpendicular to a magnetic field of flux density B is

$$pc=3\times10^{-7}B(y)\rho(y'',y') \qquad (1)$$

if y is the distance in cm measured in the direction in which B varies (vertical here), ρ is the radius of curvature of the path in cm, B is the magnetic induction in Gauss, y' and y'' are derivatives with respect to distance taken in the plane of motion and perpendicular to the y axis, and p is the momentum in Bev/c. If ds is the arc length along the path and φ is the angle between the path and the y axis

$$\rho=ds/d\varphi=dy/(\cos\varphi d\varphi). \qquad (2)$$

Since p is a constant of the motion we can substitute (2) into (1) and integrate to get

$$pc=\left[3\times10^{-7}\int_{y_1}^{y_2}B(y)dy\right]\Big/(\sin\varphi_2-\sin\varphi_1). \qquad (3)$$

Hence the momentum of the particle is determined by a vertical line integral of the magnetic induction and the angles of entry and exit of the particle. The integral is evaluated from a plot of the magnetic flux density in the region between the two chambers including the region of fringing. Inside the chambers the field is too weak to cause observable deflections. Once the magnetic flux integral is measured, we need only two angles to determine the momentum.

IV. ERRORS AND CORRECTIONS TO THE OBSERVATIONS

Since the particles observed here move in very nearly plane curves, it is sufficient for a calculation of the momentum to determine the projections of the angles of entry and exit on a plane perpendicular to the magnetic field. The maximum error in momentum that can result from neglecting the angle of tilt of the actual motion is less than 0.02 percent. These projected angles can be found from the central cameras alone with sufficient accuracy for the particles of momentum less than 10 Bev/c, but for the higher energy particles it is necessary to measure the side views and make a sterographic correction.

In the absence of the magnetic field the solid angle into which a given point on the top counter will accept particles which pass through both counters depends only the size and position of the bottom counter. For the two counters used here the integrated aperture of the instrument is 0.0822 cm^2-steradian. Due to the magnetic field the particles move in curved trajectories so that the limiting paths are different from those found without the field in such a way that the effective solid angle of the instrument is reduced. A straightforward though somewhat tedious calculation has been made to evaluate the amount of this effect. At 3 Bev/c and above the solid angle is within one percent of the no-field value. Below this momentum one must apply a correction to the observed momentum distribution to obtain the true distribution. At 2.4 Bev/c one must add three percent, at 1.5 Bev/c, 10 percent, at 1.0 Bev/c, 24 percent, and at 0.7 Bev/c, 60 percent. Particles of momentum less than about 0.3 Bev/c cannot trip the apparatus.

In addition to the magnetic deflections on which this experiment is based, the charged particles observed will suffer various amounts of angular deviation due to scattering by the walls of the cloud chambers and the gas in the space between the chambers. To estimate the effect of this scattering on the momentum distribution, we assume all the particles observed to be mu-mesons and consider coulomb scattering only since nearly all of the penetrating component at sea level is thought to consist of high energy mu-mesons and the nuclear cross sections for fast mu-mesons are small. Williams[9] has considered the problem of the multiple and plural scattering of fast charged particles and concludes that, for all but the largest net angles of scattering, a suitable Gaussian distribution represents the distribution of scattering angles quite well. In the present experiment large deflection angles are excluded by the counter geometry. Using Williams' result calculated for mu-mesons it is found that the probability of a given fractional change in apparent momentum due to scattering is independent of the momentum. Most scatterings (35 out of 36) are found to change the momentum by less than 10 percent of itself. One out of 600 scatterings changes it by more than 15 percent and one out of 10,000 by more than 20 percent for the 2 cm of glass scatterer assumed here.

By starting with some hypothetical momentum distribution which can be represented analytically, one can estimate the effect of scattering on the shape of the momentum spectrum. A distribution that rises linearly from zero to a peak at 1 Bev/c and then falls off as $1/p^2$ is a fairly good approximation to the observed curve. When Williams' Gaussian scattering results are applied to this distribution, it is found that the sharp peak is reduced by about nine percent, that the ordinates of the distribution are increased by less than 0.3 percent between 0.1 Bev/c and 0.5 Bev/c and increased by about the same percent between 3 Bev/c and 100 Bev/c. Hence, the general form of the spectrum is not altered by scattering of the particles although any sharp peaks that might be present in the true distribution will be smeared out somewhat. A no-field run of 107 tracks gives good agreement with the theoretically predicted root mean square scattering angle.

In each picture the angle between the track image

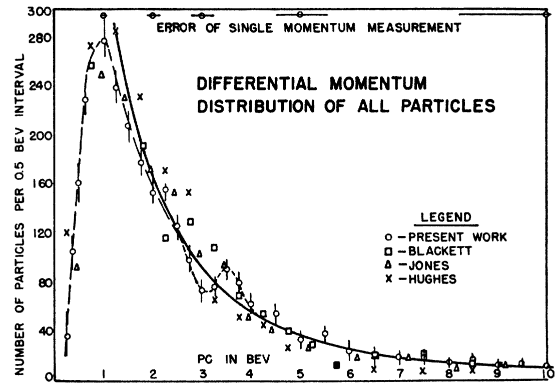

FIG. 2. The dashed curve represents the raw data; the full curve results after correction for the magnetic cut-off effect at low momenta. A possible anomaly at 3 Bev/c is shown dotted.

and the reference wire image was measured by projecting the films full scale onto a protractor of one meter radius. It was possible in this way to measure angles to about 0.001 radian. The errors in momentum assigned on the various curves shown are derived from the root mean square deviations in angle measurements obtained by several remeasurements of some of the tracks. These independent remeasurements permit an estimate of errors due to chamber distortions and track thickness. We have assumed that it is impossible for a whole track 10 inches long to remain straight after being rotated by gas flow in the chamber. Such gas flow results in kinking the tracks into a more or less severe S shape. Photographic distortions were found to be negligible compared with the errors due to the thickness of the tracks and chamber distortions.

V. RESULTS AND CONCLUSIONS; COMPARISON WITH OTHER EXPERIMENTS

1547 measureable pairs of photographs were taken in this experiment, 127 of them representing particles of momentum greater than 10 Bev/c. Figure 2 shows the differential momentum distribution of the particles of momentum less than 10 Bev/c. Each point represents the number of particles found in 0.5 Bev/c interval centered about the momentum plotted as abscissa. Vertical bars indicate the standard deviations of these numbers and the horizontal bars are errors in momentum derived from the mean error of the angle measurements. The dashed curve follows the raw data; the full curve has been corrected for the variation of the aperture of our instrument with momentum. After the correction is applied it is at best difficult to determine the position of the maximum point of the distribution since the instrument is very strongly biased against particles of low momentum when operated at high magnetic field strengths. An unusual feature of the distribution is the apparent irregularity at about 3.5 Bev/c which is shown dotted in Fig. 2. Overlapping intervals have been used to plot the low end of the spectrum in order to represent the data in more detail. As a result only half the points

[9] E. J. Williams, Proc. Roy. Soc. **A169**, 531 (1939).

TABLE I. Summary of momentum distribution surveys.

Reference	Resolving power in Bev/c	Number of particles	Peak of distribution in Bev/c	Exponent of power law	Positive–negative ratio	Means of limiting electrons
Kunze[a]	3	61	—	—	1	single tracks*
Anderson and Neddermeyer[b]	5	78	1.0	−1.7	1.2	single tracks
LePrince-Ringuet and Crussard[c]	20	300	1.0	—	1.7	magnetic cutoff
Blackett and Brode;[d] Blackett[e]	20	829	1.0	−1.9	1.2	magnetic cutoff
Jones[f]	10	923	1.0	−1.9	1.3	10 cm lead
Hughes[g]	16	674	1.0	−2.5	1.2	10 cm lead
Wilson[h]	20	424	0.8	−1.8	—	lead
Present work	80	1547	—	−1.8 below 10 Bev/c; −2.1 above 10 Bev/c	1.26±0.06	magnetic cutoff

* Not counter-controlled.
[a] See reference 1.
[b] See reference 2.
[c] See reference 3.
[d] See reference 4.
[e] See reference 5.
[f] See reference 6.
[g] See reference 7.
[h] See reference 8.

shown at the low end are statistically independent. Blackett[5] found a similar irregularity at somewhat lower momentum. At Manchester where his measurements were made, the minimum momentum a proton requires to arrive from the vertical direction is about 2 or 3 Bev/c; at Pasadena this value is 7 Bev/c. One might reasonably expect some characteristics of the spectrum, therefore, to be different in the two places.

Table I contains a summary of the salient features of all of the surveys of the momentum distribution published to date in which counter-controlled cloud chambers and magnetic fields were used. We have taken the resolving power to be that value of the momentum at which the error in determining the momentum reaches 100 percent. For comparison we have plotted the results of the three of the previous surveys in which the largest numbers of measurements were reported. After normalization to include the same number of particles between about 1 Bev/c and 10 Bev/c we find Jones' data agree excellently with the present work. Blackett's data also lie closely on our curve except for the anomaly at 2.5 Bev/c. Hughes' data seem to lie consistently below our results for momenta above 3 Bev/c and consistently above for lower values of the momentum. Though the statistics are not good enough in any one of these experiments to establish the presence of an irregularity around 3 Bev/c, all of them seem to show some evidence for it. Adding all the data together might be the wrong thing to do, however, if the suspected effect is latitude sensitive. In comparing the various sets of data, it should be observed that the resolution of the present experiment is roughly twice that of Blackett's in the range below 10 Bev/c. The other authors did not give a detailed error estimate. Figures 3 and 4 show the differential momentum distributions of positive and negative particles, respectively.

In Fig. 5 is plotted the distribution of the excess of positive over negative particles, the full curve representing the data after correction for magnetic biasing. There seems to be a concentration of positive particles around 1.3 Bev/c with a gradual tailing off toward high momenta. Again the normalized data of previous experiments are replotted and verify in a general way the present results although the poorer statistics and resolv-

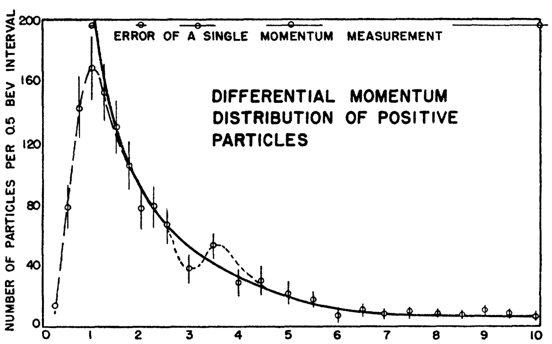

FIG. 3. The dashed curve represents the raw data; the full curve results after correction for the magnetic cut-off effect at low momenta. A possible anomaly at 3 Bev/c is shown dotted.

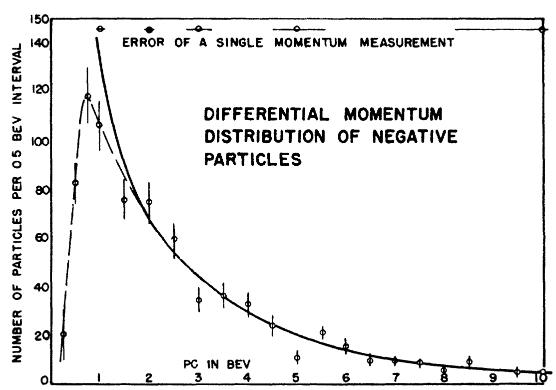

FIG. 4. The dashed curve represents the raw data; the full curve results after correction for the magnetic cut-off effect at low momenta. The low point at 3 Bev/c is similar to the corresponding one in Fig. 3.

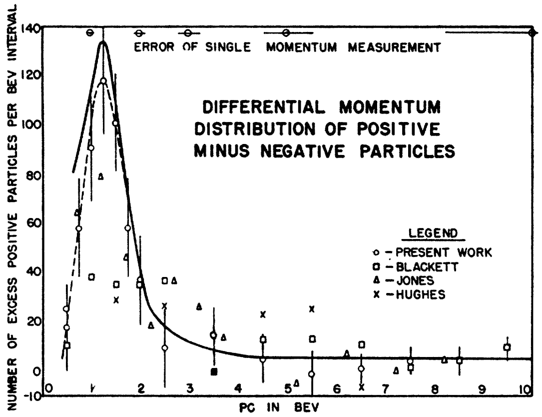

FIG. 5. The dashed curve represents the raw data; the full curve results after correction for the magnetic cut-off effect at low momenta.

ing powers of the earlier results wash out the sharp peak to a large extent.

Corresponding to the variation of the positive–negative excess with momentum, the ratio of positives to negative seems to vary with momentum, the ratio having a peak at about 1.5 Bev/c. In Table II are listed the values of the positive–negative ratio computed for 2 Bev/c intervals centered about the momentum values tabulated.

Actually the apparent variation of the positive–negative ratio is not outside the statistical uncertainty for this experiment alone, but Owen and Wilson[10] have recently found an indication of similar behavior with better statistics. We also agree with Brode[11] who finds the ratio 1.37±0.04 in a range around 1.5 Bev/c.

The over-all ratio of positives to negative is 1.26 ±0.06; the ratio for the 127 particles of momentum greater than 10 Bev/c is 1.2±0.2.

Figure 6 shows the differential momentum distribution for particles between 10 Bev/c and 100 Bev/c. Plots of the positive and negative spectra separately are similar but the statistics do not permit a very critical comparison.

In order to compare these results with the theory and make various calculations, it is desirable to try to find some simple analytical representation for the cosmic-ray momentum distribution. As is shown in Fig. 7 the data are well-fitted by a distribution of the form $1/p^s$ with $s=1.8\pm0.2$ from 2.5 Bev/c to 10 Bev/c and $s=2.1\pm0.6$

TABLE II. Ratio of numbers of positive to negative particles.

Center of 2-Bev/c momentum interval	Positive–negative ratio
1.5 Bev/c	1.36±0.1
2.5	1.24±0.1
3.5	1.15±0.1

[10] B. G. Owen and J. G. Wilson, Proc. Phys. Soc. London **A62**, 601 (1949).

[11] R. B. Brode, Phys. Rev. **76**, 468 (1949).

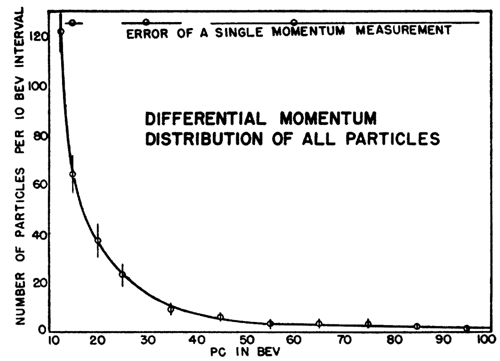

FIG. 6. Distribution for 112 particles of momenta between 10 Bev/c and 100 Bev/c.

from 10 Bev/c to about 70 Bev/c. For simplicity in calculations $s=1.9$ is quite good for the whole range. A fairly good fit can also be obtained with an exponential law, $\exp(-kp)$, but at least three different values of k are needed to cover the whole range of momenta. Similarly the integral spectrum can be represented by $1/p^n$ with $n=1.2$ as shown in Fig. 8 in good agreement with the result for the differential spectrum. The upper end of the integral spectrum is somewhat uncertain because of the uncertainty in the number of particles above 100 Bev/c apparent momentum. Fifteen such particles were found and a $1/p^2$ law for the differential spectrum when extended to infinite momentum predicts twelve particles. No significant differences between the analytical behaviors of the positive and negative distributions were found.

It was possible to divide all the particles observed here into two groups; those that occurred at night and those that came in when the sun was above the horizon in Pasadena. By chance the running times of the apparatus at night and during the day gave nearly equal

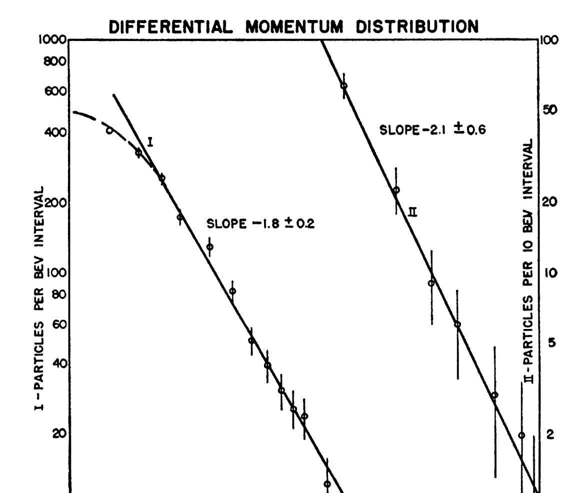

FIG. 7. Logarithmic plot of total differential momentum distribution.

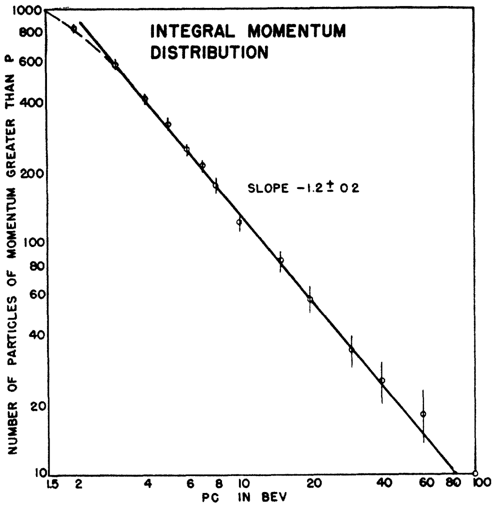

FIG. 8. Logarithmic plot of total integral momentum distribution.

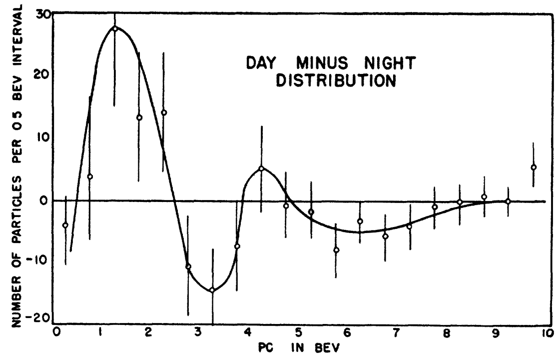

FIG. 9. Plot of the diurnal variation of the differential momentum distribution. Each ordinate is the difference between the number of particles occurring during the day and the number occuring during the night for the 0.5 Bev/c interval placed symmetrically about the plotted abscissa.

numbers of night time and daytime particles. In Fig. 9 is plotted the momentum distribution of the day particles minus the night particles. Although the statistical accuracy is poor, there seems to be some indication of a variation favoring 1 Bev/c particles during the day and 3 Bev/c and 7 Bev/c particles at night. Perhaps it is not an accident that this 3 Bev/c irregularity coincides with that of the whole momentum spectrum. Above 10 Bev/c the same number of particles occurred during the day as at night. In a recent paper, K. Dwight[12] gives a calculation of the diurnal variation in primary intensity at the top of the atmosphere due to the combined magnetic fields of the earth and the sun, assuming that the sun actually has a permanent field. A rough extrapolation of his results to latitude 33° gives the following expectation. At Pasadena no singly charged primary particle of momentum less than 7 Bev/c can hit the earth at the vertical. Above 10 Bev/c little diurnal effect is expected at Pasadena, while between 7 Bev/c and 10 Bev/c one expects an effect, the time of maximum intensity occurring at different times for different momenta. To calculate how rapidly the time of maximum intensity varies with momentum requires considerable numerical work and, in any event, the total variation is not expected to exceed 10 percent. Since the curve shown in Fig. 9 is presumably for meson secondaries, it would be surprising if any diurnal variation should survive at sea level at all. This is a point that might bear further examination experimentally. There was no evidence of a correlation with sidereal time.

Rossi[13] has compared previous measurements of the sea level momentum spectrum with the intensity *versus* depth curve for charged cosmic-ray particles deep underground to see whether ionization loss alone is sufficient to explain the rate of absorption of the cosmic rays. He finds good agreement out to a momentum of 10 Bev/c, one point at 20 Bev/c being in apparent disagreement indicating that the more energetic particles lose energy at a greater rate than can be accounted for by ionization loss alone. When our data are normalized to compare with Rossi's curve, we confirm in a rough way the discrepancy above 10 Bev/c. Calculations are now under way to determine whether radiation and pair production losses are sufficient to account for the discrepancy.

In conclusion the authors wish to thank Professor Carl D. Anderson for his continued interest and assistance and Professor Robert B. Leighton for assistance in the design of the cloud chambers.

[12] K. Dwight, Phys. Rev. **78**, 40 (1950).

[13] B. Rossi, Rev. Mod. Phys. **20**, 537 (1948).