The Relation between Phase Stability and First-Order Focusing in Linear Accelerators

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IT is well known that phase stability and first-order focusing are incompatible in a simple linear accelerator without foils or grids in the path of the particles.[1] However, various attempts have been made to design more complicated field shapes and time variations that will circumvent this limitation. A general proof that the limitation exists in all cases was made by the author in 1945 but was not published at that time. This proof is given below.

A particle of charge e moves parallel to the z-axis with velocity v and is acted on by fields $\mathbf{E}$ and $\mathbf{H}$, which are periodic in time and nearly periodic along the z-axis. The time period is T and the corresponding repeat length L is equal to vT. Changes in velocity and direction during one repeat length will be neglected; this is what we mean by the term "first order." The time corresponding to the position z is then given by $t_0+(z/v)$, where t_0 is an arbitrary starting time.

The focusing effect depends on the x and y force components F_x and F_y, whose mean values are given by

$$\bar{F}_x=(e/vT)\int_0^L[E_x-(v/c)H_y]dz,\quad \bar{F}_y=(e/vT)\int_0^L[E_y+(v/c)H_x]dz, \tag{1}$$

the field components being evaluated at the position and corresponding time of the moving particle. If the line of motion is chosen so that both these forces vanish, the restoring force constants toward this line are

$$k_x=-\partial\bar{F}_x/\partial x,\quad k_y=-\partial\bar{F}_y/\partial y. \tag{2}$$

From Eqs. (1) and (2) we get

$$k_x+k_y=(e/vT)\int_0^L\{-[(\partial E_x/\partial x)+(\partial E_y/\partial y)] +(v/c)[(\partial H_y/\partial x)-(\partial H_x/\partial y)]\}dz. \tag{3}$$

With the aid of the Maxwell equations $\mathrm{div}\mathbf{E}=0$,

$$\mathrm{curl}\mathbf{H}=(1/c)\partial\mathbf{E}/\partial t,$$

this becomes

$$k_x+k_y=(e/vT)\int_0^L[(\partial E_z/\partial z)+(v/c^2)\partial E_z/\partial t]dz. \tag{4}$$

Finally, since $\partial/\partial z=(d/dz)-(1/v)\partial/\partial t$, we can write

$$k_x+k_y=(e/vT)[E_z]_0^L-(e/v^2T)(1-v^2/c^2)\int_0^L(\partial E_z/\partial t)dz. \tag{5}$$

Next consider the energy gain ΔW during one period, given by

$$\Delta W=e\int_0^L E_z dz. \tag{6}$$

This depends on the starting time t_0, and its rate of change with t_0 is

$$\partial(\Delta W)/\partial t_0=e\int_0^L(\partial E_z/\partial t)dz. \tag{7}$$

Combining Eqs. (5) and (7), we obtain the relation

$$k_x+k_y=(e/vT)[E_z]_0^L-(1/v^2T)(1-v^2/c^2)\partial(\Delta W)/\partial t_0. \tag{8}$$

Now, in order to have phase stability, $\partial(\Delta W)/\partial t_0$ must be positive, since an increasing t_0 means that the particle is too slow and therefore has an energy defficiency. In order to have focusing, both k_x and k_y must be positive; this is clearly incompatible with the above requirement unless some help is obtained from the first term on the right of Eq. (8). This term depends on the difference in E_z experienced by the particle on leaving and entering the repeat length, and is obviously zero in the absence of foils or grids. With a foil placed so that the field strength on one side is zero, it is determined by the field strength on the other side at the instant the particle enters the foil. (The repeat length must be taken as ending at the foil, since the field equations used are not valid inside a conductor.) In the case of a grid, the effect is essentially the same, even though there is no field discontinuity through a grid opening; the focusing force can then be considered as arising from the charge lying between the equilibrium path and the displaced path.

[1] V. K. Zworykin, Morton, Ramberg, Hillier, and Vance, *Electron Optics and the Electron Microscope* (John Wiley & Sons, Inc., 1945), pp. 660–664; Ginzton, Hansen, and Kennedy, Rev. Sci. Inst. **19**, 89 (1948).

Production of π^+-Mesons by X-Rays as a Function of Atomic Number*

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THE electronic method of meson detection developed by Steinberger and Bishop[1] has been used to study the relative cross sections of various elements for production of π^+-mesons by x-rays.

This study has been made at an angle of 90±8 degrees to the x-ray beam, at approximately 317-Mev synchrotron energy, and at meson energies of 42±7 Mev and 76±6 Mev. The targets used were of identical shape and approximately 1.5 g/cm² for both x-ray beam and mesons. With two exceptions the targets were of approximately equivalent meson ranges. The tin and lead targets were of about 1/10 the range, and corrections of about 10 percent were required for this. The other corrections required for meson range, attenuation of the x-ray beam, inpurities,[2] and decay of mesons in flight were of the order of 2 percent or less in most cases. No correction has been made for nuclear scattering or nuclear absorption of the mesons in the absorbers. The hydrogen cross section was obtained by a polyethylene, carbon subtraction.

The values of the relative cross sections per proton in the nucleus are given in Table I in arbitrary units.

TABLE I. Relative cross sections per proton for the production of π^+-mesons by x-rays from the synchrotron.

Element	42 Mev	Standard deviation[a] (percent)	76 Mev	Standard deviation[a] (percent)
H	6.6	17	8.07	11
Li	3.32	10	2.80	11
Be	2.82	11	2.13	10
B	3.02	11	2.28	15
C	2.60	6	1.93	5
Al	2.50	11	1.68	9
Cu	1.92	19	1.17	15
Sn	1.66	25	0.51	55
Pb	0.51	91	0.80	65

[a] Additional non-statistical errors may be as large as 10 percent for hydrogen, tin, and lead, and 5 percent for the others.

Steinberger, Panofsky, and Steller have made similar measurements as yet unpublished[3] on the yields of 75 Mev *neutral* mesons, using the method described in their recent paper.[4] Relative cross sections were obtained for hydrogen, lithium, beryllium, carbon, aluminum, copper, and lead. When these are tabulated in terms of the cross section per nucleon, σ_{π^0}/A, and arbitrarily normalized to the σ_{π^+}/Z data at beryllium, all except hydrogen agree within the statistical error. Their hydrogen value is approximately the same as the lithium cross section per nucleon.

It is apparent from the above that at least two factors contribute to the decrease of σ/Z with atomic number, one dependent on and the other independent of the meson charge. Chew and Lewis[5] have suggested that for π^+-mesons σ/Z may be higher for hydrogen than for other elements, since the exclusion principle limits the phase space available for the residual neutron. For other elements lower neutron states are occupied. Such an effect would be less with neutral mesons, since the residual nucleon does not have a changed sign.

The further decrease of σ_{π^+}/Z and σ_{π^0}/A with atomic number is not incompatible with the possibility that only the nucleons on the surface of the nucleus take part in the production of mesons. The decrease cannot readily be explained by screening of the internal nucleons from the x-rays and hence possibly may be caused by an interaction of the outgoing π-mesons with the nucleons.

If it is assumed that such an interaction is the entire cause of the effect, a calculation can be made of the mean free path of mesons in nuclear matter. Although the calculated decrease is relatively insensitive to the mean free path assumed, the data are incompatible with a mean free path greater than approximately five times the proton radius.

It is a pleasure to acknowledge the assistance of Dr. J. Steinberger in all parts of this experiment. I wish also to express my gratitude to Professors E. McMillan and W. K. H. Panofsky for their interest and advice. My sincere thanks also go to Mr. Walter Gibbins and the entire synchrotron crew.

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[1] J. L. Steinberger and A. S. Bishop, Phys. Rev. **78**, 493 (1950).
[2] Analysis of purity performed by Conway and Moore.
[3] Steinberger, Panofsky, and Steller have kindly allowed me to quote their as yet unpublished work.
[4] Steinberger, Panofsky, and Steller, Phys. Rev. **78**, 802 (1950).
[5] G. Chew, private communication.

Isotope Shift in the Atomic Spectrum of Carbon

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The isotope shift between C^{12} and C^{13} in the λ2837 doublet in C II has been resolved and previous results[1] on λ2478 and C I have been verified. The observed shifts are given in Table I. In each case the C^{13} was found to be shifted to lower frequency. The normal mass effect can be calculated readily from the change in the Rydberg constant with a result of $+0.142$ cm^{-1} for λ2478 and $+0.124$ cm^{-1} for λ2837, leaving the algebraic differences from the observed values to be accounted for by the "specific mass effect."[2]

Table I. Isotope shifts for carbon.

		λ	Observed shift (cm^{-1})
C II	$2s2p^2\ ^2S_{1/2}-2s^23p\ ^2P_{3/2}$	2836.7	-0.612 ± 0.002
	$2s2p^2\ ^2S_{1/2}-2s^23p\ ^2P_{1/2}$	2837.6	-0.623 ± 0.003
C I	$2p^2\ ^1S_0-2p3s\ ^1P_1$	2478.5	-0.156 ± 0.003

The hyperfine structure due to the magnetic moment in C^{13} was too small to be observed. Resolution of the isotope shifts was obtained by use of a Fabry-Perot interferometer with aluminized mirrors of approximately 75 percent reflectivity, crossed with a quartz Littrow spectrograph.

The carbon spectrum was obtained from a Schüler liquid-air cooled hollow cathode source of the type developed by Arroe and Mack,[3] charged with KCN enriched with 63 percent C^{13}. A static helium supply[3] provided the carrier gas, and the hollow cathodes were made from both aluminum and magnesium.

Measurements were made with 4-, 4.5-, and 12-mm spacers for the C II doublet and with a 15-mm spacer for the C I line. Positive identification of the components was made by two procedures. In one of these, some $KC^{12}N$ was added to the cathode and the resultant relative change in intensities noted. Also exposures were taken on adjacent sections of the spectrograph slit with no change in the positions of the interferometer and photographic plate, first with a cathode containing the enriched sample and then with another cathode containing ordinary KCN.

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[1] J. R. Holmes, Phys. Rev. **77**, 745 (1950).
[2] D. S. Hughes and C. Eckart, Phys. Rev. **36**, 694 (1930).
[3] O. H. Arroe and J. E. Mack, J. Opt. Soc. Am. **40**, 386 (1950).