On the Directional Correlation of Successive Nuclear Radiations

J. A. SPIERS
The Clarendon Laboratory, Oxford, England
September 18, 1950

IN a recent paper with the above title, Falkoff and Uhlenbeck[1] quote a general expression for the angular correlation function $W(\theta)$ between successive nuclear emissions:

$$W(\theta)=\sum_{M_0M_1M_2} P_{M_0M_1}(0)P_{M_1M_2}(\theta), \tag{1}$$

where M_0, M_1, M_2 are the z-components of total angular momentum of the possible spin orientations ("sub-levels") of the initial, intermediate, and final nuclear states, respectively; $PM_0M_1(0)$ $PM_0M_1(0)$ is the relative probability of emission of the first particle along the direction $\theta=0$ (axis of quantization) and $PM_1M_2(\theta)$ the relative probability of emission of the second particle along a direction making an angle θ with this axis.

This expression has hitherto only been verified for particular applications. A general proof, involving only properties of the angular momentum, can be given as follows. The transitions in which the first and second particles, respectively, are emitted can be written

$$\psi J_0M_0\rightarrow\sum_{j_1a_1} A_{j_1a_1}\sum_{m_1+M_1=M_0} c^{J_0J_1j_1}_{M_1m_1}\psi J_1M_1\phi^{(1)}_{j_1m_1a_1}, \tag{2}$$

$$\psi J_1M_1\rightarrow\sum_{j_2a_2} A_{j_2a_2}\sum_{m_2+M_2=M_1} c^{J_1J_2j_2}_{M_2m_2}\psi J_2M_2\phi^{(2)}_{j_2m_2a_2}. \tag{3}$$

Here the C's are the usual Wigner coefficients, ψ's are nuclear wave functions, ϕ's are wave functions of the emitted particles with quantized total angular momenta (intrinsic spin plus orbital) j_1m_1, j_2m_2; a_1, a_2 denote any other quantities, e.g., polarization, required to specify the ϕ-states completely; $A_{j_1a_1}$ and $A_{j_2a_2}$ are complex amplitudes related to the transition probabilities for the various possible values of j_1a_1, j_2a_2, respectively.

Define

$$\Phi^{(1)}_{M_0M_1}=\sum_{j_1a_1} A_{j_1a_1}c^{J_0J_1j_1}_{M_1,\,M_0-M_1}\phi^{(1)}_{j_1,\,M_0-M_1,\,a_1}, \tag{4}$$

$$\Phi^{(2)}_{M_1M_2}=\sum_{j_2a_2} A_{j_2a_2}c^{J_1J_2j_2}_{M_2,\,M_1-M_2}\phi^{(2)}_{j_2,\,M_1-M_2,\,a_2}. \tag{5}$$

The final state $\psi_f{}^{M_0}$ of the system as a whole (after emission of both particles) can then be written, using (2)–(5), as

$$\psi_f{}^{M_0}=\sum_{M_1M_2}\Phi^{(1)}_{M_0M_1}\Phi^{(2)}_{M_1M_2}\psi J_2M_2. \tag{6}$$

Hence, by integrating the square modulus of $\psi_f{}^{M_0}$ over all nuclear variables and summing over spin orientations, we obtain the probability

$$W(\theta_1\phi_1,\theta_2\phi_2)=\sum_{M_0}\sum_{\sigma_1\sigma_2}\sum_{M_2}\Big|\sum_{M_1}\Phi^{(1)}_{M_0M_1}(r_1\sigma_1)\cdot\Phi^{(2)}_{M_1M_2}(r_2\sigma_2)\Big|^2. \tag{7}$$

($r_1\equiv r_1\theta_1\phi_1$ are the coordinates of the first particle, σ_1 the z-component of its intrinsic (spin) angular momentum, and similarly for the second particle. r_1 and r_2 are assumed to be given some large but constant value in expression (7).)

The crux of the proof is to show that the summation over M_1 can also be brought outside the square modulus when we put $\theta_1=\phi_1=0$. Now the wave function of an emitted particle of intrinsic spin S has the form

$$\phi_{jma}(r\theta\phi\sigma)=\sum_{l=|j-s|}^{l=j+s} a_l c^{jlS}_{m-\sigma,\,\sigma} f_l(r)Y_l{}^{m-\sigma}(\theta,\phi). \tag{8}$$

The quantities a_l are constants depending on the state of polarization of the particle.

From (8) it can be seen that $\phi_{jma}(r_100\sigma_1)=0$ unless $\sigma_1=m_1$, hence that $\Phi^{(1)}_{M_0M_1}(r_100\sigma_1)=0$ unless $M_0-M_1=\sigma_1$. Thus there is only one value of M_1 which gives a non-vanishing term in expression (7) for given values of M_0, σ_1, and we may write

$$W(00,\theta_2\phi_2)=\sum_{M_0M_1M_2}\Big\{\sum_{\sigma_1}|\Phi^{(1)}_{M_0M_1}(r_100\sigma_1)|^2\cdot\sum_{\sigma_2}|\Phi^{(2)}_{M_1M_2}(r_2\theta_2\phi_2\sigma_2)|^2\Big\}, \tag{9}$$

whence the expression (1) follows immediately.

The argument given above applies to photons as well as to nuclear particle emissions; in this case the wave functions ϕ_{jma} are replaced by the Heitler spherical wave expansions for the electromagnetic vector potential[2] which can be put in the form (8) with $S=1$; also with a slight increase in complexity to β-emissions, (occurrence of suitable combinations of electron and neutrino "large" and "small" components, each of the form (8) with $S=\frac{1}{2}$, instead of single particle functions).

It should be pointed out incidentally that the absence of interference between transitions via different intermediate levels implied by (1) enables this expression to be put in the more symmetrical form[3]

$$W(\theta)=\sum_{M_0M_1M_1'M_2} P_{M_0M_1}(0)\cdot[D^{J_1}_{M_1M_2'}(\theta)]^2\cdot P_{M_1'M_2}(0), \tag{10}$$

where $D^{J_1}_{M_1M_1'}(\theta)$ is the irreducible representation of the spatial rotation group of order J_1; a form which is useful when the emitted particles (or photons) have high angular momentum (or multipole order), since the relative probabilities have only to be evaluated for emission along the axis of quantization.[4]

[1] D. L. Falkoff and G. E. Uhlenbeck, Phys. Rev. **79**, 323 (1950) (Eq. (2a), with a slight change in notation).
[2] G. Goertzel, Phys. Rev. **70**, 897 (1946); D. S. Ling and D. L. Falkoff, Phys. Rev. **76**, 1639 (1949).
[3] J. A. Spiers, *Directional Effects in Radioactivity*, Nat. Res. Council of Canada, Chalk River, Ontario, April, 1949, p. 13. Goertzel, reference 2, verifies this expression for the particular case of $\gamma-\gamma$ correlation.
[4] This expression was used to calculate the correlation functions given in J. A. Spiers, Phys. Rev. **78**, 75 (1950).

A Note on Neutron-Deficient Europium Activities*

GEOFFREY WILKINSON† AND HARRY G. HICKS‡
Radiation Laboratory and Department of Chemistry, University of California, Berkeley, California
September 13, 1950

A STUDY has been made of europium activities produced by bombardment of samarium oxide with 10-Mev protons from the 60-in. Crocker Laboratory cyclotron. The techniques of bombardment, chemical separation by ion exchange resin-column procedures, and determination of radiation characteristics by measurements with beryllium, aluminum, and lead absorbers have been described previously.[1]

In addition to the very long-lived activities Eu^{154}, Eu^{152}, and possibly lighter europium isotopes which have not been individually resolved, proton bombardment of samarium produces activities of half-lives 15.0 ± 0.5 hours, 14 ± 1 days, and 54 ± 1 days.

The 15-hr. activity was determined by study on a simple magnetic spectrometer and decayed with emission of positrons of maximum energy 1.8 Mev. The negative sweep showed beta-particles of maximum energy approximately 1.8 Mev decaying with the half-life of the well-known 9.2-hr. Eu^{152} activity. The decay and lead absorption curves of the electromagnetic radiations were complex and difficult to resolve but no evidence of gamma-radiation harder than about 0.5 Mev associated with the 15-hr. activity was found. From the ratios of negative to positive electrons, of the total electron to electromagnetic radiations, and the known[2] K/β^--ratio of 0.22 for the 9.2-hr. Eu^{152}, and assuming 0.5 percent counting efficiency for the soft gamma-radiation in the standard argon-alcohol tubes used, the ratio γ/β^+ for the 15-hr. activity was found to be 2 to 2.3, showing that orbital electron capture accounts for less than about one-third of the disintegrations. On these assumptions the yield of the 15-hr. activity was about one-third that of the 9.2-hr. activity in the proton bombardment of samarium suggesting allocation to mass 150.

A europium activity of half-life 27 hours, decaying by positron-emission, was reported to be formed by $(n,2n)$ reactions in europium[3] while recently a 15-hr. activity was produced by bombardment of europium with 23-Mev gamma-rays, presumably by (γ,n) reaction;[4] no radiation characteristics were given for these activities. Allocation of the 15-hr. activity to mass 150 seems therefore to be fairly certain.

In addition to very long-lived activities (5.3-yr. Eu^{154} and 5.2-yr. Eu^{152}), two additional activities of half-lives 14 ± 1 days and 54 ± 1 days were observed in the chemically separated europium fraction. The radiation characteristics were obtained by resolution of decay and aluminum, beryllium, and lead absorption curves after subtraction of the experimentally determined contributions of the long-lived activities. The 14-day activity decays with emission of soft electrons of about 100-kev energy, L and K x-rays, and a hard gamma-ray of around 1-Mev energy; the curves were rather difficult to resolve. The 54-day activity showed electrons of approximately 0.4-Mev energy, L and K x-radiations, and gamma-rays of energies about 0.4 Mev and 1.0 Mev. The ratios of various radiations of the 54-day activity corrected for counting efficiencies of x- and gamma-radiation, absorption in counter windows, etc., were $\sim$0.4 Mev e^-:L x-ray:K x-ray:$\sim$0.4-Mev gamma-ray:1.0-Mev gamma-ray$=\sim0.05:\sim0.7:1:\sim0.3:\sim0.3$.

No positrons were observed in the radiations of either the 14-day or the 54-day activity and both presumably decay by orbital electron capture with gamma-rays arising from subsequent transitions. Marinsky and Glendenin have independently reported[5] similar activities in the europium fraction from deuteron bombarded samarium and have allocated the 14-day and 54-day activities to masses 149 and 147, respectively. The present data agree closely with this earlier work.

Of the europium isotopes which could be formed by (p,n) reactions in samarium, only Eu^{144} and Eu^{148} have not yet been observed; both might be expected to have short half-lives. In the present work half-lives less than a few hours would not have been detected.

* This work was done under the auspices of the AEC.
† Present address: Chemistry Department, Massachusetts Institute of Technology, Cambridge, Massachusetts.
‡ Present address: Chemistry Division, General Electric Company, Hanford Works, Richland, Washington.
1 G. Wilkinson and H. G. Hicks, Phys. Rev. **75**, 1370 (1949).
2 J. H. Reynolds and M. G. Inghram, Phys. Rev. **75**, 1500 (1949).
3 M. L. Pool and L. L. Quill, Phys. Rev. **53**, 437 (1938).
4 F. D. S. Butement, Nature **165**, 149 (1950).
5 J. A. Marinsky and L. E. Glendenin, *Radiochemical Studies: The Fission Products* (McGraw-Hill Book Company, Inc., New York) (to be published), Paper No. 336, National Nuclear Energy Series, Plutonium Project Record, Vol. 9.

The Stars Initiated by Gamma-Rays

SEISHI KIKUCHI
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York
September 14, 1950

WHILE the stars initiated by π-mesons, alpha-particles,[1] deuterons,[2] protons,[3] and neutrons[4] have been investigated by the photographic emulsion method, little is known about the stars initiated by gamma-rays. The following is a brief report of an experiment on the stars initiated by 300-Mev synchrotron gamma-rays.

Ilford C2 plates, 100-μ thick and insensitive to electrons, were exposed directly in the beam of gamma-rays about 3.3 meters from the synchrotron target. The intensity of the beam at the axis of the beam measured by a Victoreen Condenser r-meter behind a one-eighth-inch thick lead converter was about 600 r per hour. An exposure of a few minutes was most suitable to get a clear picture of the stars. An area of about 7.5 cm² was scanned and 750 stars were observed. That at least most of them are not due to π-mesons emitted by gamma-rays is clear from the difference in features of the stars in this case from that of π-meson stars. The fact that the most of the stars are obviously not associated with π-meson tracks also provides strong evidence that they are not due to π-mesons. A probability consideration shows that it is highly improbable that they are due to neutrons emitted by gamma-rays from the glass plate on which the emulsion is fixed or from the emulsion itself.

TABLE I. Distribution of prongs.

		2	3	4	5	6	7
Number of prongs		2	3	4	5	6	7
Number of events		103[a]	64	57	24	4	1
Relative number of events relative to 3-prong star	gamma-rays	160	100	89	38	6.4	1.6.
	π-mesons	180	100	51	12	—	—

[a] This figure is subjected to a larger error compared with other figures since two-prong stars are easily confused with a single track suffering a large angle scattering. Only such cases were counted where either the grain density is obviously different in both prongs or it was obvious from the change in grain density that the both prongs started from the point where they meet.

Two hundred and fifty-two stars obtained from one-third of the total area scanned were examined rather carefully. About 4 or 5 of them were suspected to be due to π-mesons. The distribution of prongs in the stars observed is shown in Table I. Prongs shorter than 5μ are not counted. The stars with four or more prongs are more abundant in this case than in the meson-produced stars.

To show the energy distribution of prongs more or less quantitatively the prongs were classified in five groups A, B, C, D, and E according to their energies. The energy of each prong was estimated by its range or by the visual comparison of the grain density with that of tracks of known residual ranges. In the latter case the accuracy is very low. The results are shown in Table II.

The exact comparison with the case of π-meson is difficult because of the large error involved in these figures. But it seems[5] that the prongs are on the average more energetic in this case than in the case of π-mesons.

TABLE II. Energy distribution of prongs.

A	B	C	D	E
>4500μ	4500–600μ	600–50μ	50–5μ	<5μ
60–30 Mev	30–10 Mev	10–2 Mev	2–0.2 Mev	<0.2 Mev
39	109	427	181	67

The cross section of the process averaged over all kind of atoms except hydrogen is estimated to be 4×10^{-28} cm². In this estimation the result of Blocker, Kenny, and Panofsky[6] was used, assuming that the r-value of the intensity at the beam axis 1 meter from the synchrotron target determined by us would give the same value as measured by the Berkeley method. The small difference in energy is also ignored. The error arising from these circumstances together with those arising from the inaccuracy involved in this experiment was estimated to be a factor of 5. The cross section is close to that for the π-meson production.

Many thanks are due to Professor R. R. Wilson for valuable suggestions and discussions, to other members of the laboratory for discussions as well as for the operation of the synchrotron and to Mrs. M. R. Keck for her valuable help in examining the plates.

1 E. Gardner, Phys. Rev. **75**, 379 (1949).
2 E. Gardner and V. Peterson, Phys. Rev. **75**, 364 (1949).
3 Camerini, Fowler, Lock, and Muirhead, Phil. Mag. **41**, 413 (1950).
4 Smith, Gardner, and Bradner, UCRL 527, November 23, 1949.
5 See for instance W. B. Cleston and L. J. B. Goldfarb, Phys. Rev. **78**, 683 (1950).
6 Blocker, Kenny, and Panofsky, Phys. Rev. **79**, 419 (1950).