We illustrate by calculating the non-diagonal matrix element Q_{ij} between $2p^2 4fP$ and $2p^2 3pD$. After summing over spins this is equal to: $-(1/\sqrt{5})\int Q\bar{\Phi}_{2p^2 4f}(P^1)\Phi_{2p^2 3p}(D^1)$. On inserting the expansions of Table I, Q_{ij} further reduces to:

$$-(1/3\sqrt{5})\int Q\bar{\psi}_{P^1}(F_\pi D_\nu)\psi_{D^1}(P_\pi{}^* D_\nu).$$

The decomposition of $\psi_{P^1}(F_\pi D_\nu)$ in terms of product functions was given in Appendix I; the corresponding development for $\psi_{D^1}(P_\pi{}^* D_\nu)$ is: $6^{-\frac{1}{2}}\{-3^{\frac{1}{2}}P_\pi{}^{1*}D_\nu{}^0+P_\pi{}^{0*}D_\nu{}^1+2^{\frac{1}{2}}P_\pi{}^{-1*}D_\nu{}^2\}$. Integration over the neutron coordinates gives

$$\begin{aligned}\int Q\bar{\psi}_{P^1}(F_\pi D_\nu)\psi_{D^1}(P_\pi{}^* D_\nu)\\ &=210^{-\frac{1}{2}}\{-3\sqrt{2}\int Q\bar{F}_\pi{}^1P_\pi{}^{1*}-\sqrt{3}\int Q\bar{F}_\pi{}^0P_\pi{}^{0*}\\ &\qquad+\sqrt{2}\int Q\bar{F}_\pi{}^{-1}P_\pi{}^{-1*}\}\\ &=2\cdot 210^{-\frac{1}{2}}(r^2)_{3p,4f}\{-3\sqrt{2}I_{3,1;1,1}-\sqrt{3}I_{3,0;1,0}+\sqrt{2}I_{3,1;1,1}\}\end{aligned}$$

where $I_{3,0;1,0}=3\sqrt{21}/35$ and $I_{3,1;1,1}=3\sqrt{14}/35$. Since $(r^2)_{3p,4f}=(2\sqrt{14}/5)\langle r^2\rangle_{2p}$, the resulting value of Q_{ij} is $(4\sqrt{21}/15)(\langle r^2\rangle_{2p}/25)$.

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The Primary Specific Ionization and Intensity of the Cosmic Radiation above the Atmosphere at the Geomagnetic Equator*

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Directional intensities and the primary specific ionization of the charged cosmic-ray flux above the atmosphere were measured by means of a G-M counter telescope in an Aerobee sounding rocket launched at the geomagnetic equator. The intensity at a zenith angle of 45° averaged over all azimuths, was found to be 0.04 particle sec.$^{-1}$ cm^{-2} steradian^{-1}, of which not more than 65 percent can be attributed to primaries, the remainder being due to albedo. The low value ($\sim$40 percent) of the observed east-west asymmetry is most directly explainable in terms of positive proton primaries and a large albedo flux at large zenith angles, although a small contribution of negative primaries cannot be excluded.

The primary specific ionization of the radiation above the atmosphere is found to be essentially the same as that of the sea-level radiation, indicating a predominance of singly charged particles of near minimum ionization. This result strongly suggests that the albedo radiation at the equator does not consist of low energy ($<$100 Mev) protons.

Most of the properties of bursts produced in a small lead block can be accounted for reasonably in terms of known initiating particles and interactions.

I. EXPERIMENTAL ARRANGEMENT

AN Aerobee sounding rocket (Round A-11) was fired at the geomagnetic equator about 600 miles off the coast of Peru from the USS Norton Sound on March 22, 1949. The rocket reached an altitude of over 100 km and spent 217 sec. above the appreciable atmosphere. Its trajectory and general flight history were very similar to those[1,2] of Aerobee A-10 which was fired a few days earlier.

This note is mainly concerned with results obtained with an unshielded telescope (Fig. 1) similar to that used in earlier work.[3] In addition to measuring the directional flux of charged radiation, it determined the average primary specific ionization of the radiation, essentially by measuring the efficiency of a low pressure hydrogen-filled counter. This method is not refined enough to determine the ionization of individual particles which traverse the telescope, but does have the advantage of experimental simplicity. All counters[4] were made of relatively thin-walled (0.020 inch) brass tubing with an inside diameter of 2.44 cm. Their effective length was 14.5 cm. As before,[3] the low efficiency counter B' was filled with pure hydrogen, this time to a pressure of only 2.5 cm Hg. Coincidences ABC, $AB'C$, and ACG were telemetered[5] to ground from the

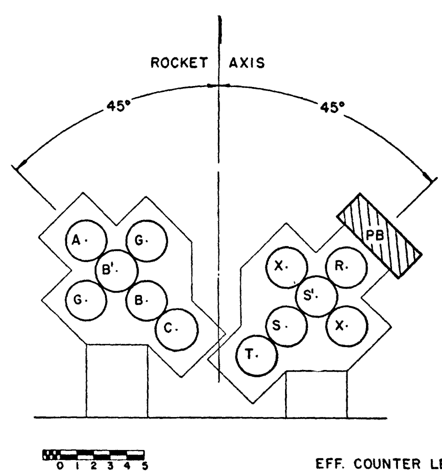

FIG. 1. Disposition in the rocket of telescopes ABC and RST, and lead block.

* Supported by the U. S. Navy, Bureau of Ordnance.

[1] J. A. Van Allen and A. V. Gangnes, Phys. Rev. **78**, 50 (1950).

[2] J. A. Van Allen and A. V. Gangnes, Phys. Rev. **79**, 51 (1950).

[3] S. F. Singer, Phys. Rev. **76**, 701 (1949).

[4] The counters were made up to our specifications by the Nuclear Development Laboratory, Kansas City, Missouri.

[5] G. H. Melton, Electronics **21**, 106 (1948).

TABLE I. High altitude plateau data of telescope ABC.

1 Time interval after launching	2 Time (sec.)	3 Number of coincidences (mutually exclusive) ABC	4 $ABB'C$	5 $ABCG$	6 $ABB'CG$
65–241[a]	167	61	39	43	29
245–295[b]	50	18	9	8	5

[a] Nine seconds of telemetering record were not read because of noise.
[b] The missile started to turn over at 241 sec. and was pointing nose down after 245 sec.

TABLE II. Summary of results (telescope ABC).[a]

	1 Sea level ($\lambda=0°$)	2 High altitude plateau
(Net) counting rate (sec.$^{-1}$) (at 45° zenith angle)	0.0521±0.0028	0.585±0.052
Geometric factor (cm^2-steradian)	11.9	14.5
(Net) directional intensity (at 45° average zenith angle) (sec.$^{-1}$ cm^{-2} steradian^{-1})	0.00437±0.00023	0.0403±0.0035 (av.)
Efficiency of counter B'	0.356±0.015[b]	0.378±0.043 (net) 0.387±0.034 (gross)

[a] All errors quoted are standard errors based only on the number of counts.
[b] This measurement made at Silver Spring, Maryland, ($\lambda=50°$N) with telescope in vertical position.

moving rocket. The experimental efficiency of counter B' was taken as the fraction of coincidences (ABC) accompanied by coincidences ($AB'C$); in terms of mutually exclusive coincidences,

$$\textit{efficiency} = ABB'C/(ABC+ABB'C).$$

The telescope was mounted with its axis at 45° to the missile axis. The solid angle of the telescope was covered only by the 0.040-inch aluminum skin (0.25 g/cm^2) of the rocket in the upward direction and by part of a light electronic chassis ($\sim$1 g/cm^2) in the downward direction. The only major burst producer in the vicinity was a block of lead located off to the side of the telescope ABC (Fig. 1). Most multiple-particle events are believed to have been eliminated by disregarding those cases in which a guard coincidence (ACG) occurred.

II. AVERAGE INTENSITY

The data obtained by telescope ABC are summarized in Table I and compared with sea level results in Table II. Corrections for telescope inefficiency, dead time, and accidental coincidences were negligible. The counting rates given are the net rates; i.e., with guard coincidences subtracted out. The directional intensity (corresponding to an average telescope zenith angle of 45°) was obtained from the net counting rate under the assumption of uniform intensity over the aperture of the telescope of flux incident from above, zero flux incident from the lower hemisphere:[6]

$$j_{\mathrm{av}}(45°) = 0.040\pm0.003 \text{ particle } (\text{sec.}^{-1}\text{-cm}^{-2}\text{-steradian}^{-1}).$$

III. AZIMUTHAL DEPENDENCE OF INTENSITY

A set of 16 photo-cells, together with a magnetometer, were used to determine the aspect and roll of the missile. The roll period during the vacuum trajectory was 1.7 sec.; the angle of the missile axis to the vertical varied from 0° to 15° during the major portion of flight. A histogram of telescope counts *vs.* azimuth angle of the telescope was constructed for the time interval 65–241 sec. and a smooth curve of the form $y=a_0+R\cos(\phi+\delta)$ fitted to it by least squares technique (Fig. 2). This procedure was employed to obtain the maximum and minimum intensities and azimuthal asymmetry of column 2 in Table III. A detailed account of the analysis is given in reference 2. Our results are evidently in very good agreement with data obtained by a telescope of different geometry in Aerobee rocket A-10 (column 3 in Table III).[2] *As anticipated, the maximum of the flux of the radiation above the atmosphere occurs in a westerly direction; the absolute value of the azimuthal asymmetry, however, is markedly less than expected.*

IV. PRIMARY SPECIFIC IONIZATION

The measured sea-level efficiency[7] of counter B' corresponds to particles of about minimum specific ionization $J_{p\,\mathrm{min}}$. The efficiency from the data on the cosmic-ray plateau above the atmosphere (Table II)[8] can be compared with the sea-level efficiency after applying the necessary corrections.

As before,[3] a geometrical correction has to be made to the efficiency, since the average path length through counter B' increases in going above the atmosphere as the zenith angle distribution of the radiation changes from a $\cos^2\theta$ dependence to a more nearly isotropic distribution. This additive correction for our particular case is about -0.02. No correction has been made for knock-on electrons, since they are also present in the sea-level case, and the difference can be considered negligible. No correction is made for nuclear bursts, since it can be assumed that most bursts tripped a guard counter and have, therefore, been already subtracted.

The average primary specific ionization of the radiation above the atmosphere is then found to be $1.0\pm0.2J_{p\,\mathrm{min}}$. Essentially the same result is obtained by comparing the counting rate of a low efficiency single

[6] In a preliminary account of this experiment given at the Echo Lake Cosmic-Ray Symposium, June 1949, the telescope geometric factor for isotropic radiation was given as 13.5. Subsequently, this factor was corrected to 14.5.

[7] The efficiency of the counter can be computed from the formula eff. $=(1-e^{-J_pLP})$, where $P=2.5/76$ atmos., and the efficiency (Fig. 3) is averaged according to telescope geometry and zenith angle dependence of radiation over all path lengths L; it equals the experimental efficiency for a value of J_p of 6 ion pairs/cm/atmos. H_2. This calculation serves as a check on the reasonableness of our experimental efficiency.

[8] It may be noted that the "gross" efficiency (i.e., multiple particle events not subtracted out) is experimentally equal to the "net" efficiency.

counter[9] (column 1 of Table IV) to the full efficiency counter used[1] in Aerobee A-10 (column 3 of Table IV). The higher efficiency of the single counter, compared to the counter as part of a telescope, can be explained qualitatively as due to the longer path lengths possible in a single counter.

The gross conclusion then is that the *bulk of the ionizing radiation above the atmosphere at the geomagnetic equator* carries a *single charge* and has a *primary specific ionization very close to minimum.* This conclusion may be compared with the result obtained at White Sands, New Mexico ($\lambda=41°$N)[3] where the primary specific ionization of the radiation above the atmosphere was found to be $1.30\pm0.15J_{p\text{ min}}$.

V. CHARACTER OF THE PRIMARY RADIATION

The experimental results on the flux, azimuthal asymmetry, and specific ionization can be used for drawing some simple conclusions concerning the character of the charged radiation above the atmosphere at the equator.

It is of interest to find what fraction of particles counted by our telescope is due to albedo; i.e., secondary particles originating in the earth's atmosphere whose directional dependence bears no simple relationship to that of the primaries which produced them. If we take the flux value of 0.028 in the vertical direction[1] as the primary intensity (it is really an upper limit to the primary intensity), the primary intensity at 45° zenith angle, averaged over all azimuths, is about 10 percent less,[10] and is the same for a primary radiation of either sign, or for any mixture of positive and negative particles. Comparing then 0.025 with the experimental intensity of 0.040, we conclude that at least 35 percent of the intensity measured is caused by albedo.

From rocket measurements of the vertical intensity at various latitudes, it may be inferred[11] that a power law spectrum of the form $N(>pc/\text{Ze})=0.48(pc/\text{Ze})^{-1.1}$ relates intensity to magnetic rigidity of the primary radiation for rigidities ranging from 2 to 15 Bv. Extrapolating this spectrum to much higher rigidities, the east-west asymmetry at a zenith angle of 45° at $\lambda=0°$ would be 0.84, much larger than is admissible by our experiment. It is probably more realistic to assume a greater spectrum exponent at these higher energies, which would raise the expected east-west asymmetry still further. To explain our low experimental value of asymmetry it may not be necessary to invoke the existence of negatively charged primaries. Any mixture of negative and positive primaries should still lead to a slight *decrease* of intensity with zenith angle.[10] The increase of intensity with zenith angle, and the low value of azimuthal asymmetry may both be reasonably ascribed to albedo.[12] *Our results are, therefore, consistent with the existence of a primary radiation composed mainly of positive protons.*

VI. CHARACTER OF THE ALBEDO RADIATION

The data, on the primary specific ionization, which was found to be $1.0\pm0.2J_{p\text{ min}}$ in Section IV, can now be used to draw more detailed, but rather qualitative conclusions about the ionizing radiation above the atmosphere. Purely on the basis of the data of this experiment an upper limit of about 20 percent can be given for the percentage of particles with charge greater than unity; however, it seems most reasonable to assume that they constitute a much smaller percentage of the *total* radiation measured, which includes a large fraction of albedo. We can assume safely that the primary protons have a specific ionization near minimum since their momenta are known from geomagnetic considerations. This is not the case for the albedo. However, for each type of albedo particle, provided this type is assumed to constitute the bulk of the albedo radiation, we can give an approximate range in which the energy lies. We make use of the fact that the primary specific ionization has its minimum value, $J_{p\text{ min}}$, near a momentum about four times the particle rest mass (Fig. 4). For much higher momenta J_p shows a slow relativistic increase. For lower momenta, however, J_p rises very rapidly. To satisfy our experimental result, which gives for J_p of the radiation above the

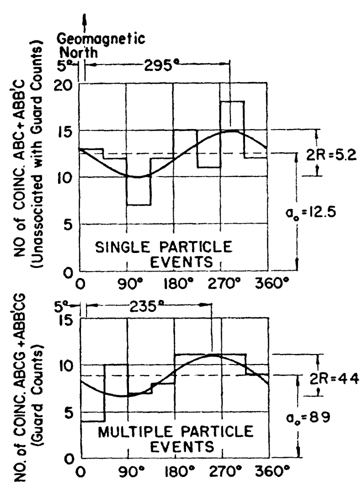

FIG. 2. Histograms of single particle events and multiple particle events respectively, of telescope ABC in 45° segments of azimuth taken for a period of 167 seconds above the atmosphere. The smooth curves are least-square fits to the histogram and are of the form $y=a_0+R\cos(\phi+\delta)$.

[9] Two low efficiency single counters (identical with counter B' of the telescope) were mounted along the axis of the rocket near the tip of the instrumentation nose cone; their counting rates were telemetered to ground.

[10] This result can be deduced from Vallarta's theory of the main cone under the assumption of a reasonable number-spectrum; the zenith angle dependence is very insensitive to the value of spectrum exponent chosen for this computation.

[11] J. A. Van Allen and S. F. Singer, Phys. Rev. **78**, 819 (1950).

[12] A similar excess of experimental intensity at large zenith angles over intensity calculated using geomagnetic theory has been observed previously at $\lambda=41°$N and ascribed to albedo by S. F. Singer, Phys. Rev. **77**, 729 (1950).

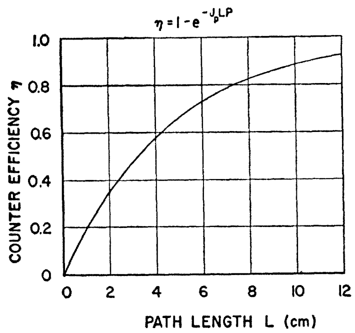

FIG. 3. Efficiency of low pressure (2.5 cm Hg) hydrogen-filled counter B' (or S') *vs.* path length L of a traversing particle of minimum ionization.

atmosphere a value close to $J_{p\,\mathrm{min}}$, we must assume the absence of a large fraction of electrons of energy in excess of 30 Mev, mesons in excess of 6 Bev, and protons in excess of 60 Bev. The last possibility is, of course, extremely unlikely, purely on the basis of energy considerations. Because of the steep rise of the primary specific ionization curve at energies below the minimum, we can exclude with more definiteness even a small percentage of mesons with energies less than 100 Mev, and protons with energies less than about 1 Bev. Electrons of energy less than 10 Mev are excluded as being too soft to pass through the telescope. *These considerations strongly suggest that the bulk of the albedo radiation consists of relatively high energy (>1 Bev) protons and/or 10–30 Mev electrons.* Mesons in the lower ranges of energy can be excluded on the basis of their short lifetimes.

It might be pointed out again that the present data are not consistent with the assumption that a great number of low energy (about 100 Mev) protons[13] are produced in the initial encounters of the primaries, so as to constitute a large fraction of the cosmic-ray flux above the atmosphere *at the equator*. On the other hand, the data on efficiency (Table IV) show a maximum efficiency near the Pfotzer maximum, a phenomenon previously found[14] at $\lambda = 41°$N. This result can be explained reasonably as being due to low energy protons which have been generated some distance below the top of the atmosphere.

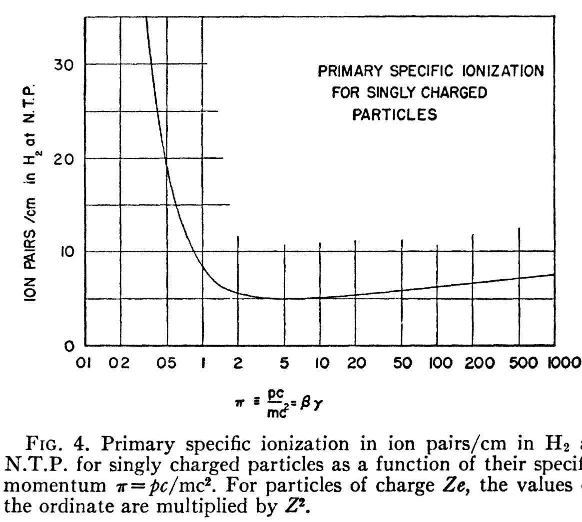

FIG. 4. Primary specific ionization in ion pairs/cm in H_2 at N.T.P. for singly charged particles as a function of their specific momentum $\pi = pc/mc^2$. For particles of charge Ze, the values of the ordinate are multiplied by Z^2.

[13] Assuming 35 percent of the radiation to consist of 100 Mev protons (specific ionization from Fig. 4 of $4J_{p\,\mathrm{min}}$, corresponding to a counter efficiency from Fig. 3 of 0.85), the telescope efficiency would have to be $0.35 \times 0.85 + 0.65 \times 0.38 = 0.55$, which is clearly much higher than the observed value of 0.38 ± 0.04.

This expedition has been made possible by the continuing support of the U. S. Navy Bureau of Ordnance and by the cooperation of many other branches of the Naval organization; in particular, the Naval Unit of the White Sands Proving Ground, the USS Norton Sound, the USS Agerholm, the USS Richard B. Anderson, and the Naval Ordnance Laboratory.

Special thanks are due Captain T. A. Ahroon, USN, and the other officers and men of the USS Norton Sound for the conduct of the expedition and the rocket firing.

The author is indebted to Professor M. S. Vallarta and to Dr. J. A. Van Allen for discussion of results. Messrs. W. A. Bowen and E. Maple of the Naval Ordnance Laboratory collaborated on the magnetometer phase of the operation. Mr. L. W. Fraser of this Laboratory and Mr. R. F. Ohlemacher of the New Mexico College of Agriculture and Mechanic Arts contributed essentially in the accomplishment of this experiment.

APPENDIX. PROPERTIES OF BURSTS PRODUCED IN LEAD

We wish to present some data of an exploratory experiment which relate to the shielded low efficiency telescope *RST* (Fig. 1)

TABLE III. Angular distribution of intensity.

1 (Net) directional intensity (at av. zenith angle 45°) (sec.$^{-1}$-cm^{-2}-steradian^{-1})	2 Telescope *ABC*	3 A-10 telescope (reference 2)
Average	0.0403±0.0035	λ.0410±0.0021
Minimum	0.032 ±0.005	0.031 ±0.004
Maximum	0.048 ±0.005	0.048 ±0.004
Asymmetry $2R/a_0$	0.42 ±0.28	0.41 ±0.16
Geomagnetic azimuth of maximum	295° ±40°	268° ±15°

TABLE IV. Single counter data on cosmic-ray plateau.

	1 Low efficiency counter	2 Low efficiency counter (shielded with 0.5 cm Pb)	3 Efficient counter (A10)[a]
Counting rate/sec.	4.18±0.16	4.75±0.40	9.7±0.17
Geometric factor (isotropic)	189.5	189.5	185
Av. directional intensity (particles-sec.$^{-1}$ cm^{-2} steradian^{-1})	0.022±0.001	0.025±0.002	0.0525±0.0009
Efficiency	0.42±0.02	0.48±0.04	~1.00
Efficiency at Pfotzer maximum	0.46	0.63	~1.00

[a] See reference 1.

[14] S. F. Singer, Phys. Rev. **77**, 730 (1950).

TABLE V. High altitude plateau data of telescope *RST*.

1 Time interval after launching	2 Time (sec.)	3 Number of coincidences (mutually exclusive) *RST*	4 *RSS'T*	5 *RSTX*	6 *RSS'TX*
65–241	167	43	32	48	67
245–295 (rocket pointing nose down)	50	22	8	8	13

TABLE VI. Summary of results (telescope *RST*).

Time interval	Directional flux (av. zenith angle of 45°)—sec.$^{-1}$ cm^{-2} steradian^{-1} Gross (with guard counts)	 Net (without guard counts)	Efficiency Gross	 Net
65–241	0.079±0.006	0.031±0.004	0.52±0.05	0.43±0.06
245–295	0.070±0.010	0.041±0.007	0.41±0.07	0.27±0.08

and the effects of the lead block on the unshielded telescope *ABC*. Statistics are not good enough to give any more than qualitative ideas, but on the whole they do strengthen the evidence presented by the unshielded telescope. Telescope *RST* was identical in all respects to telescope *ABC*, except for the 2 cm thick lead block covering its aperture. The sea-level calibrations on telescope *RST* (obtained without the lead shield) agreed within the statistical error with those of telescope *ABC* (column 1 of Table II). It can be seen from the data obtained when the rocket was pointing nose down (time interval 245 to 295 sec., Table VI) that, when the lead block is *below* telescope *RST*, the net flux and net efficiency of telescope *RST* agree very well with those of telescope *ABC* (Table II).

The pertinent data are presented in Tables V and VI.

It was originally hoped that the arrangement of lead block and counters would demonstrate two experimental features:

(A) *A directional burst detector:* telescope *RST* (and particularly the shower counts *RTX*) would show a very pronounced east-west asymmetry, much larger than the unshielded telescope *ABC*, for two reasons: (a) the lead would stop soft albedo entering from above and (b) produce bursts predominantly preserving the direction of the primary rays. This was not found to be the case; on the contrary, the azimuthal asymmetries of both net counts and gross counts of telescope *RST* were insignificantly different from zero.

(B) The other purpose was to demonstrate the use of a low efficiency counter as a device to indicate *narrow showers* which could not have been resolved as multiple events by Geiger counters of reasonable diameters.

It appears that the experimental efficiency (Table VI) is considerably higher than in the unshielded telescope indicating that heavily ionizing particles or narrow bundles of lightly ionizing particles, which originate in the lead block, are traversing counter *S'*. This result is supported qualitatively by the high efficiencies of the shielded low pressure counter (column 2 of Table IV).

We may finally note the effect of the lead block on the unshielded telescope *ABC*, i.e., on the guard counts *ABCG* and *ABB'CG*.

(a) The guard counting rate falls to a very low value when the rocket points nose down (Table I), indicating that the lead block produces most of the showers;

(b) A large azimuthal asymmetry, 0.50±0.34, is present (Fig. 2);

(c) The gross efficiency corresponds to particles of about minimum specific ionization (Table II).

We can harmonize results (A) and (B) with (b) and (c) above by assuming that the primary radiation in traversing the lead block produces both showers which preserve the original direction and showers which are mostly isotropic. Telescope *ABC* is not very sensitive to the latter type of event, since it is sufficiently far removed from the lead block to experience fairly low particle densities. It therefore exhibits asymmetry and a lower gross efficiency, whereas telescope *RST* does not. We cannot, however, account simply (i.e., in terms of known processes and known primary particles), for the *sign* of the asymmetry of the shower counts *ABCG*+*ABB'CG* (Fig. 2). With the lead block off to the side of telescope *ABC* (Fig. 1) the maximum of the guard counting rate should occur when telescope *ABC* points east if *positive* primaries produce showers predominantly in the *forward* direction. The precision of the present measurement is too low to allow us to draw any firm conclusions. We hope to be able to investigate this matter further experimentally.