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Cascade Showers in Lead*

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A study has been made of the energy and angular distributions of the electrons produced when the x-ray beam from the 322-Mev Berkeley synchrotron falls on a slab of lead one-half inch in thickness. A cloud chamber containing the piece of lead was in a magnetic field of 1800 gauss. Measurements were made on 1286 electron secondaries having energies greater than three Mev. The energy and angular distributions of these electrons are in satisfactory agreement with the theory.

I. INTRODUCTION

THE diffusion equations of cascade theory[1] describe the course of an electron-photon shower as it progresses through matter. When the initial boundary conditions are introduced these equations determine the average number of the electrons and gamma-rays of a given energy as a function of thickness in the material. The lateral[2] development of the shower has been studied extensively and in particular the lateral spread and angular distributions of shower particles have been obtained under different simplifying assumptions. Other properties of cascade showers that have also been studied theoretically are the fluctuations in the number of particles as a function of thickness resulting from a single primary of a given energy.

Before the advent of high energy electron accelerators the experimental investigation of these shower problems was restricted to experiments on the soft component of cosmic rays. Many experiments[3] have been performed which measured the counting rate or ionization as a function of thickness of material. The transition (or shower) curves that are obtained rise rapidly to a maximum and then decrease more slowly as the thickness of absorber is increased. These results are in qualitative agreement with the theory but suffer from a number of difficulties: (1) it is necessary to integrate over the not too well-known primary spectrum in order to compare with the theory; (2) the experiments are subject to various large geometrical corrections; and (3) the hard component must be separated out.

A more direct comparison with experiment may be obtained from cloud-chamber data. From a study of fifty showers, Hazen[4] has been able to compare with the theory the number of particles at the maximum of the shower as a function of the total number of particles under eight 0.7 cm lead plates. Nassar and Hazen[5]

TABLE I. The observed number of tracks in 10 Mev and 10° intervals. These numbers have been corrected for the omission of tracks having dip angles greater than 45°. The lowest energy group has been normalized relative to the second to correct for the omission of electrons between three and ten Mev in the second half of the experiment. This group has also been corrected for the fact that the interval includes only seven Mev instead of ten.

E(Mev)$\diagdown\theta$	0–9	10–19	20–29	30–39	40–49	50–59	60–69	70–79	80–89	Total
3–9	43.5	95.0	79.2	155	55.5	65.5	48.7	7.52		548±33
10–19	42	82	76	63	42	30.5	24.3	11.5	2.0	373±20
20–29	47	69	51	29	12	16.5	5.8			230±15
30–39	42	43	32	8	3	4.5		1.9		134±12
40–49	36	24	12	4	1					77±9
50–59	21	17	7	1						46±7
60–69	21	16	7		1					45±7
70–79	26	7	2							35±6
80–89	19	8	1							28±5
90–99	11	4	5							20±4.5
100–109	13	7								20±4.5
110–119	11	4								15±4
120–129	12	1								13±4
130–139	9	1								10±3
140–149	8	0								8±3
>150	13	1								14±4

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1 H. S. Snyder, Phys. Rev. **76**, 1563 (1949); B. Rossi and K. Greisen, Rev. Mod. Phys. **13**, 240 (1941).
2 G. Molière in W. Heisenberg, *Cosmic Radiation* (Dover Publications, New York, 1946); S. Z. Belenky, J. Phys. U.S.S.R. **8**, 347 (1944); J. Roberg and L. W. Nordheim, Phys. Rev. **75**, 444 (1949).
3 J. C. Street and R. T. Young, Phys. Rev. **46**, 823 (1934); **47**, 572 (1935); C. G. Montgomery and D. D. Montgomery, Phys. Rev. **48**, 786 (1935).
4 W. E. Hazen, Phys. Rev. **66**, 254 (1944).
5 S. Nassar and W. E. Hazen, Phys. Rev. **69**, 298 (1946).

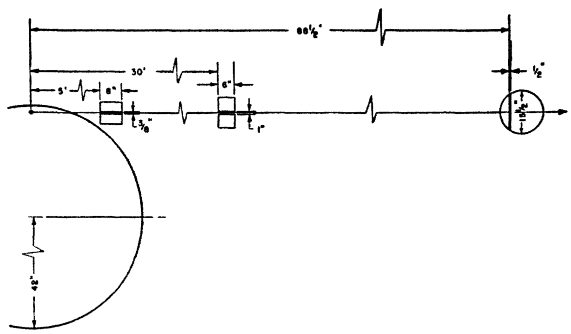

FIG. 1. The geometry of the experiment. Two lead collimators were used between the synchrotron and the cloud chamber.

have also determined the shape of the shower curve but in addition they have measured the energy spectrum of the electrons at the maximum of the shower as well as the fluctuations in the number of particles. Their results are certainly consistent with the theory but are unsatisfactory in two ways: (1) the energy of the incident electron is never experimentally determined, and (2) the number of showers observed is rather small.

When the 322-Mev Berkeley synchrotron began to operate, the systematic and controlled measurement of these quantities became possible. Blocker *et al.*[6] have determined the shape of the shower curve for lead, copper, aluminum, and carbon. They have measured the current from an ionization chamber as a function of thickness of material and have obtained the transition curves with extreme accuracy. We have sought to measure the energy spectrum at the maximum of the shower in lead; i.e., at the point where the maximum ionization occurs which is under approximately one-half inch of lead.

II. EXPERIMENTAL DETAILS

A cloud chamber, described in a previous paper,[7] in a magnetic field of 1800 gauss was located in the x-ray beam of the Berkeley synchrotron and $88\frac{1}{2}$ feet from its target. Two collimators were used. The first was a $\frac{1}{8}$ in.$\times\frac{3}{8}$ in. horizontal slot located five feet from the synchrotron target, and the second was a $\frac{1}{16}$ in.$\times\frac{3}{4}$ in. slot, 30 feet from the target and at the same vertical height as the center of the illuminated region of the cloud chamber. These produced a spray of electrons emerging from the lead and occupying an area about $\frac{1}{4}\times 2$ in. The x-ray beam traversed the $\frac{3}{4}$ in. quartz wall of the synchrotron donut, $88\frac{1}{2}$ feet of air and the $\frac{1}{4}$ in. glass wall of the cloud chamber before impinging on a half-inch lead plate inside the chamber. (See Fig. 1.)

The energy and angular distributions of the electrons which emerge from the $\frac{1}{2}$-in. thick lead plate have been measured by reprojection.[7] Besides the radius of curvature ρ, two angles, α and β, were measured. α is the dip angle or the angle that the start of the track makes with the horizontal. β is the angle that the start of the track makes with the plane defined by the beam direction and the vertical. The energy of the electron is then given by $E=300\ H\rho\cos\alpha$ and the scatter angle $\theta=\cos^{-1}(\cos\alpha\cos\beta)$.

The photographs measured were selected on the basis of quality and population. For example, a photograph that contained fifteen tracks was easy to measure, whereas one having twenty-five was measurable only in cases in which the photography was exceptional. Each photograph represented, of course, a single pulse from the synchrotron and, indeed, a single pulse of extremely low intensity. The tracks have in all cases been selected and measured by two independent observers and from their reproducibility we believe that the errors in the angles are about $\pm 2°$ and in the radii of curvature, ± 5 percent. Multiple scattering by the gas (a mixture of argon and helium) at the magnetic field used (1800 gauss) produces a standard error of approximately ± 6 percent over the whole energy range.

In the first part of the experiment all of the tracks corresponding to electrons above three Mev were

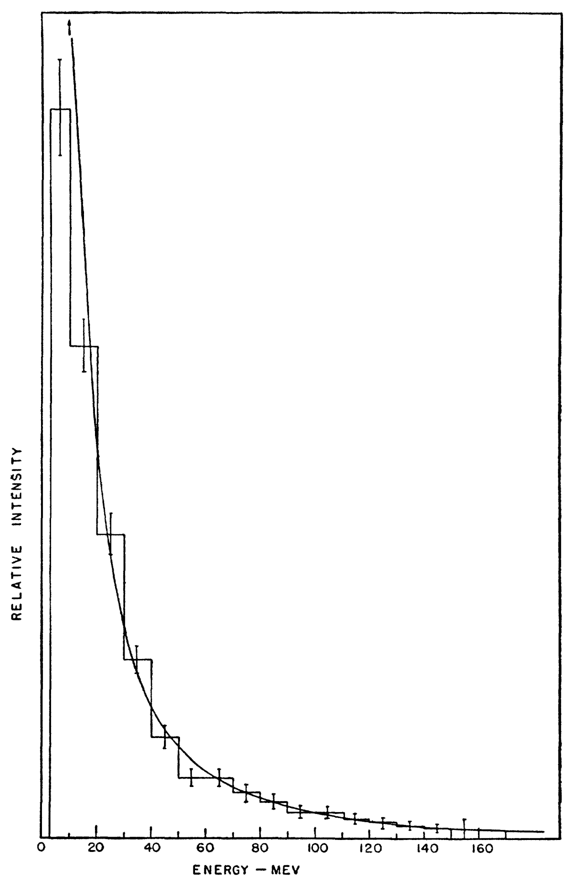

FIG. 2. The differential energy spectrum of the electrons. The standard deviations on the histogram are based only on the number of tracks measured. The smooth curve is the theoretical result obtained from Approximation B of Rossi and Greisen and has been normalized for the best fit with the experimental points.

[6] Blocker, Kenney, and Panofsky, Phys. Rev. 79, 419 (1950).
[7] Brueckner, Hartsough, Hayward, and Powell, Phys. Rev. 75, 1274 (1949).

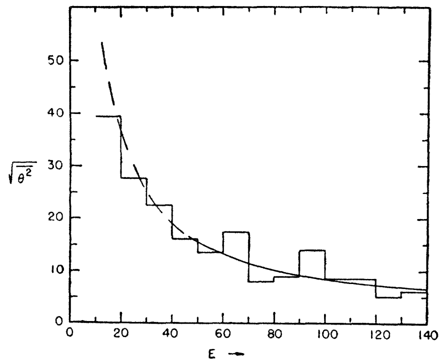

FIG. 3. The root mean square angle *versus* energy of the electrons. The smooth curve is taken from the paper by Roberg and Nordheim.

included; later, because of the preponderance of low energy electrons, we set the lower limit at 10 Mev and have normalized the data accordingly. With these limitations all tracks were measured if their dip angles were less than 45°; a geometrical correction based on the assumption of azimuthal symmetry was made for the omitted tracks:

$$\pi/[2 \sin^{-1}(\sin 45°/\sin\theta)].$$

Since the scatter angles of the electrons result from their Coulomb scattering in the lead, they are a strong function of the energies of the electrons. Thus, for example, the geometrical correction mentioned above is necessary only below 40 Mev and is really important only below 20 Mev. Another result of this energy dependence is that it has effectively extended the upper limit of the energies that could be measured, for the high energy electrons come out from the lead plate essentially in the forward direction and traverse the diameter of the cloud chamber giving about 30 cm of track on which to make an otherwise very difficult curvature measurement.

III. RESULTS

We have measured a total of 1286 tracks. Table I shows the numbers of tracks in 10 Mev and 10° intervals. These numbers have been corrected for the omission of tracks with dip angles greater than 45° and the lowest energy group has been normalized relative to the second to correct for the omission of electrons between three and 10 Mev in the second half of the experiment. This group has also been corrected for the fact that the interval includes only seven Mev instead of 10.

Figure 2 shows a histogram of the measured energy distribution. The standard deviations are based only on the number of tracks measured. Mr. Walter Aron has very kindly calculated for us the energy spectrum of the electrons by applying the initial condition of a $1/E$ gamma-ray spectrum in Approximation B of Rossi and Greisen. The x-ray spectrum of the synchrotron differs from thin target bremsstrahlung spectrum because of pair production in the target and the differential absorption of the x-rays, by the synchrotron's target and quartz donut, the air between the synchrotron and the cloud chamber, as well as the quarter-inch glass wall of the chamber. Powell[8] has shown that the effect of all these corrections is to reduce the intensity of the x-rays almost uniformly over the whole spectrum, and since we are interested in relative intensities only, the corrections to the theory are unnecessary. The limitation of the Rossi and Greisen representation is that the asymptotic cross sections, which break down at low energies, are used. Aron has corrected this difficulty by increasing the shower unit to 0.783 cm from the asymptotic value 0.5 cm. This value was obtained from the analysis of the curves of Blocker *et al.*[6] The theoretical curve has been normalized for best fit with the experimental one. The agreement is really more than satisfactory. The low energy group is expected to be low due to the large fraction of shower particles which travel backwards as is apparent from the shape of the angular distribution.

Figure 3 shows a plot of the r.m.s. angle of scattering as a function of the energy. The rather large deviations from a smooth curve above 50 Mev result from the limited number of events at high energies. (See Table I.) The fluctuations in the number of particles in the shower are themselves large and have a large effect on the r.m.s. angle for all energies.

The smooth curve results from the calculations of Roberg and Nordheim.[2] They have calculated the mean square angle of scattering as a function of energy from the lateral spread of the shower, taking into account the Coulomb scattering of the emergent electron and its ancestors. Although the calculation was intended primarily for small scattering angles in which the angular distribution is taken to be gaussian, the extrapolation to large angles appears to fit the observations.

A calculation by Belenky[2] does not include the small angle approximation and should, therefore, be more applicable to the case of lead. We have compared our results with the distribution function of Belenky and find them to be consistent, though the number of events observed in the experiment is not great enough to permit any definite conclusion concerning the various available calculations.

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[8] W. M. Powell (to be published).