Energy Dependence of Proton-Proton Scattering, 18.8 to 31.8 Mev*

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(Received June 22, 1950)

Measurements have been made of the absolute differential cross section of proton-proton scattering at 90° in the center-of-mass system. The energy of the incident protons was varied from 31.8 Mev down to 18.8 Mev, and an energy interval of ±1.0 percent was selected by means of a deflecting magnet. Apparatus using a proportional counter and a triple coincidence method was used to reduce the background. Measurements were made simultaneously, using apparatus for measuring the angular distribution at 31.8 Mev and a single charge integrator for collecting the incident proton beam.

The cross section at 90° in the center-of-mass system varies approximately as the reciprocal of the energy of the incident protons over the range investigated. Data are presented to show this dependence. Comparison is made with a Yukawa potential well, and with the tensor model selected by Christian and Noyes.

I. INTRODUCTION

MEASUREMENTS of the proton-proton differential cross section at 31.8 Mev[1-3] showed that the shape of the scattering cross-section curve was compatible with a pure S-wave. However, the magnitude of the cross section was approximately 1.3 times the value extrapolated from the low energy data[4,5] assuming a square well of depth 10.5 Mev, of range e^2/mc^2, and S-wave scattering only. A more singular potential well, such as a Yukawa well has been selected[6] to be consistent with the angular distribution observed at 31.8 Mev, assuming that the D-wave scattering predicted for such a potential does not appear.

Measurements have been made by Herb *et al.*[4] over the range 0.860 Mev to 2.392 Mev; by Blair *et al.*[5] over the range 2.42 Mev to 3.53 Mev, and by others[7-12] up to energies of 14.5 Mev. Measurements have been made in the region of 240 Mev by Oxley[13] and in the region of 170 Mev to 340 Mev by Chamberlain and Wiegand.[14] The Berkeley 32-Mev linear accelerator has features which allow investigation of the region below 32 Mev.

II. METHOD

It was convenient to use the angular distribution apparatus[1] that was used at 31.8 Mev, to extend the measurements down to 25 Mev. The thickness of the aluminum window separating the hydrogen from the argon filled proportional counter limited the low energy range to 25 Mev for scattering angles of 90° in the center-of-mass system. To extend the measurements to still lower energy, a 90° coincidence method was used.

This apparatus was arranged so that the incident beam first passed through the angular distribution scattering chamber, then through the 90° coincidence chamber, and finally on into the charge integrator. This procedure allowed a direct comparison of the two geometries in the high energy region, and normalization of the 90° coincidence data to the 31.8-Mev data.

The 90° coincidence apparatus is shown in Fig. 1. To reduce the number of background counts, one of the 90° proportional counters was arranged as a double counter telescope. Thus scattered protons were counted by triple coincidences. The coincidence geometry is defined by the entrance apertures of each of the counters and the position of the incident beam. These apertures were of rectangular cross section 1 in.×2 in. and located at a distance of $7\frac{1}{2}$ in. from the center of the scattering region. The scattered beam which could be accepted by the coincidence geometry was 45°±4° maximum and 45°±2° mean angle in the laboratory system of coordinates.

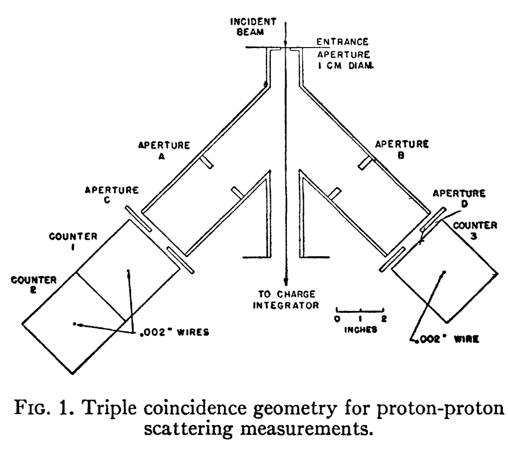

FIG. 1. Triple coincidence geometry for proton-proton scattering measurements.

* This work was sponsored under the auspices of the AEC.
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[5] Blair, Freier, Lampi, Sleator, and Williams, Phys. Rev. **74**, 553 (1948).
[6] Chew and Goldberger, Phys. Rev. **75**, 1637 (1949).
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[11] Dearnly, Oxley, and Perry, Phys. Rev. **73**, 1290 (1948); J. Rouvina, private communication.
[12] F. Faris and B. Wright, private communication.
[13] C. L. Oxley, Phys. Rev. **76**, 461 (1949).
[14] O. Chamberlain and C. Wiegand, Phys. Rev. **79**, 81 (1950).

TABLE I. Measured values of $(d\sigma/d\Omega)_{90° \text{ c.m.}}$ as a function of the energy of the incident protons.*

Energy (Mev)	$\frac{1}{E}\times10^3$	Run	Q coulombs $\times10^{-12}$	$\left(\frac{d\sigma}{d\Omega}\right)$ c.m. millibarns	Statistical error %	RMS N %	Assigned absolute probable error %	Relative errors
31.8	31.45	A		14.3	±0.7		±1.8	0
31.8	31.45	1C	2250	14.4	±2.2	±2.4		
31.45	31.8	1T	2250	14.5	±2.8	±3.1		
31.45	31.8	2T	856	14.3	±4.1	±2.6		
31.8	31.45	3A	1285	14.4	±0.9	±1.3		
31.8	31.45	3C	2185	14.3	±1.4	±1.5		
31.45	31.8	3T	3850	14.35	±2.4	±1.9		
31.45	31.8	Normalized Triple		25.45	–	–	±2.2	0
25.45	39.30	4A	2890	18.32	±0.7	±1.1		
25.45	39.30	4C	2890	18.4	±1.2	±1.4		
25.45	39.30	Weighted Mean		18.36	–	–	±2.1	±1.4
25.2	39.68	4T	5780	18.7	±1.7	±1.9	±2.6	±2.0
21.9	45.66	6T	1925	22.8	±2.5	±2.1	±2.3	±2.1
18.8	53.19	7T	1713	27.2	±2.5	±2.2	±2.4	±2.2

* Runs designated by "A" are with angular distribution singles geometry, runs designated C are with double coincidence angular distribution geometry, and runs designated T are triple coincidence geometry.

III. ENERGY SELECTION AND MEASUREMENT

The proton beam from the Berkeley 32-Mev linear accelerator[15] was collimated and then deflected in an analyzing magnet. Approximately 6 meters of further collimation gave a beam of 1 cm diameter and an angular divergence of ±0.001 radians. The energy spectrum was measured by observing the current at the charge integrator as a function of the deflecting magnet field.

The absolute energy was determined by measuring the deflection[16] of the proton beam in the magnetic field. The energy of the coincidence protons scattered at 90° in the center-of-mass (c.m.) system was also measured by determining their range in aluminum. Using the range data[17] extrapolated from low energies, the energy of the incident protons for the above deflecting magnetic field was observed to be 31.6±0.3 Mev.

Incident protons of energies less than 31.8 Mev were obtained by operating the linear accelerator in a manner different from the normal adjustment. Since beam currents of approximately 10^{-12} ampere were adequate to give a satisfactory counting rate, it was possible to adjust the energy of the injected protons and the voltage gradient at the output end of the linear accelerator so that a good beam of protons with energy as low as 18.8 Mev would result. By adjusting the magnetic field of the analyzing magnet to the appropriate value, protons with an energy spread of ±1.0 percent were incident at the scattering chamber. The mean energy of the incident protons was calculated from the deflection of the protons in the magnetic field, and these values, corrected for the energy loss in the Nylon foil and hydrogen, are the values used in Table I. Changes in the magnetic field over the range of energies used could be measured with an accuracy of $\pm\frac{1}{3}$ percent. Thus the mean energy could be determined to $\pm\frac{2}{3}$ percent.

IV. COUNTERS

Each proportional counter consisted of a rectangular chamber $1\frac{1}{2}$ in.×3 in.×4 in. with a 0.002 in. diameter tungsten wire mounted along the major axis (Fig. 1). The scattered protons entered a 0.002-in. duraluminum foil mounted on the broad side of the counter. In the double counter these protons went on into the second counter, with no foil separating the two counters.

The signals from each of these counters were amplified by an amplifier having a gain of 3000 and a band width of 0.9 Mc. The output of each amplifier was coupled to a discriminator and gate circuit that generated a 0.7-μsec. pulse. These gated pulses associated with the various counters were then coupled into a triple coincidence circuit which recorded the coincidence scattered protons.

V. CORRECTIONS

The resolving time of the counter circuits was measured with a double pulse signal generator and observed to be 4 μsec. The counting rate of each counter was kept at a sufficiently low rate so that the correction for counts being missed by the triple coincidence circuit was less than $\frac{1}{2}$ percent. This low counting rate also resulted in the corrections for accidental coincidences being less than $\frac{1}{4}$ percent.

The individual counting rates were kept approximately constant by varying the magnitude of the incident beam current. Thus the corrections to the relative cross section are very small due to counter resolving time.

As an additional check on the accidental coincidence rate, double coincidences were measured between one of the telescope counters and an auxiliary counter, measuring protons scattered from the same incident

[15] Bradner, Crawford, Gordon, and Woodyard, Phys. Rev. **73**, 534A (1948).

[16] R. R. Wilson and E. C. Creutz, Rev. Sci. Inst. **17**, 385 (1946).

[17] Livingston and Bethe, Rev. Mod. Phys. **9**, 263 (1937).

beam. This counter was arranged so that it had approximately the same counting rate as the other counter, but scattered protons could not be in real coincidence.

All protons incident at the counters have greater than 8.0-Mev energy in the laboratory system. Multiple scattering of 8.0-Mev protons by 20 cm of hydrogen has been estimated using Williams formula[18] and found to have a root-mean-square scattering angle of 0.07°. This correction when applied to the coincidence geometry is estimated to be less than $\frac{1}{2}$ percent.

The root-mean-square multiple scattering angle in two of the 0.002 in. duraluminum foils (chamber exit foil and counter entrance foil) is estimated to be 1.5° at 8.0 Mev. The counter aperture was sufficiently large, compared with the entrance aperture, so that protons scattered by these foils would be counted, unless they were scattered at an angle of greater than 6.0°. Assuming a Gaussian distribution, the probability of scattering greater than 7.0° is 6×10^{-5}. No correction was made for multiple scattering by the foils.

An estimate of the correction for multiple nuclear scattering is made as follows. The cross section for single nuclear scattering of 8 Mev protons by protons was observed[9] to be 60.5×10^{-27} cm^2 at 90° c.m. Assuming the cross section is an order of magnitude greater than this in the forward direction in the laboratory system, the probability of an 8.0-Mev proton making an elastic collision in the 20 cm of hydrogen and possibly not being counted is estimated to be less than $\frac{1}{3}$ percent. No correction was made for multiple nuclear scattering.

The position of the incident proton beam was observed at the charge integrator by means of a photographic emulsion. It was observed that multiple scattering in the hydrogen did not cause a significant number of protons to be missed by the Faraday cage.

An investigation has been made of the effects of secondary electrons on the measurements of the charge integrated by the Faraday cup. The potential of the electron suppressor cylinder[1] was made variable. Measurements of protons scattered from helium were taken as a standard and the angular distribution apparatus[1] was used to obtain good statistics, a standard deviation of ±0.6 percent.

The total number of protons scattered into this apparatus for an indicated charge of 371×10^{-12} coulomb was 4.2 percent less for zero potential on the electron suppressor cylinder than for a potential of negative 100 volts.

The observed difference in the number of scattered protons for negative 420 volts was 1.1 percent greater, for protons of 31.8-Mev incident energy, and 0.7 percent less for protons of 25.9-Mev incident energy. An additional magnetic field of 75 gauss at the aluminum foil resulted in an observed decrease of 0.3 percent in the number of scattered protons. It is concluded that the bias potential of negative 135 volts used during the proton-proton scattering experiment was sufficient to suppress most of the secondary electrons from the aluminum foil and from the Faraday cage, over the range of 25.9 Mev to 31.8 Mev. Lower energy ranges have not been investigated. During the scattering experiments, the pressure was measured in the region of the Faraday cage and was always less than 10^{-5} mm of mercury.

VI. PROCEDURE

The scattering chamber was evacuated and observed to be vacuum tight. Hydrogen was then admitted through a palladium tube until the pressure was slightly greater than one atmosphere. The excess hydrogen was then permitted to escape from the chamber via an oil-lock tube which regulated the pressure and prevented back diffusion of air into the system. The pressure was

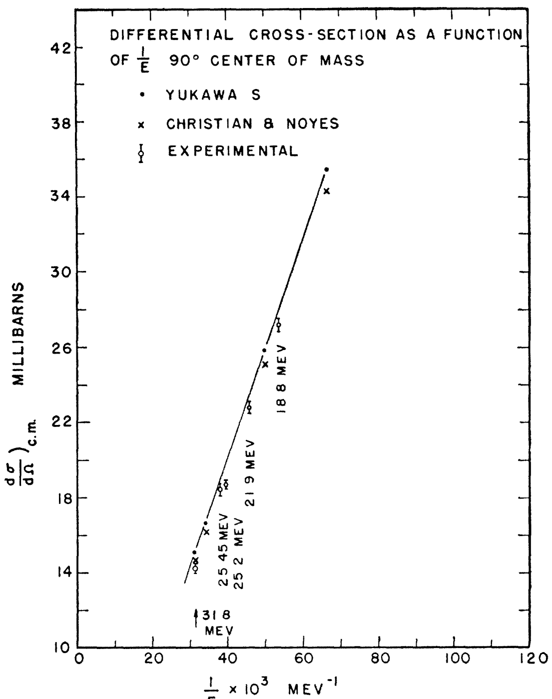

FIG. 2. Variation of the absolute differential cross section for proton-proton scattering at 90° center of mass as a function of the reciprocal of the energy of the incident protons.

calculated from the measurements of the height of the oil in this oil-lock column, the density of the oil, and the barometer reading. The temperature of the hydrogen was determined by measuring the temperature of the scattering chamber. It was possible to operate the proportional counters well up on the plateau for counting protons and still have a negligible number of background counts, using the triple coincidence method. The threshold for counting gamma-rays was determined for each counter, and then each counter was operated well above this point.

[18] E. J. Williams, Proc. Roy. Soc. **169**, 531 (1939).

TABLE II. Calculated values of the differential cross section for proton-proton scattering at 90° c.m.

Energy (Mev)	$1/E\times10^3$	$(d\sigma/d\Omega)_{90°}$ c.m. Christian and Noyes	Millibarns Yukawa S	Yukawa S and D	Square S	Square S and D
5	200	100.21	100.37	99.63	99.60	99.64
10	100	52.05	53.51	52.73	50.70	52.73
15	66.6	34.23	35.40	34.33	31.72	34.33
20	50	24.95	25.84	24.58	21.85	24.58
29.4	34.01	16.11	16.62	15.15	12.47	15.15
32	31.25	14.59	15.00	13.51	10.89	13.51

TABLE III. Calculated values of the function $\bar{K}$ using the method of Blatt and Jackson.

Energy Mev	Christian and Noyes	Square S	$\bar{K}$ Yukawa S	$\bar{K}$ Yukawa S and D
5	6.018	6.052	6.025	6.052
10	8.318	8.540	8.113	8.235
15	10.424	11.165	10.053	10.354
20	12.416	13.932	11.894	12.524
29.4	15.908	19.536	15.248	16.660
32	16.843	21.188	16.188	17.846

Each run was made by opening a beam shutter and operating until a charge of 920.7×10^{-12} coulomb was collected on the Faraday cage. These runs were of approximately 20 minutes duration for protons of 31.8-Mev incident energy. This corresponds to a beam current of approximately 10^{-12} ampere. The incident beam current was reduced at lower energies in order that the counting rate would be approximately constant. The proton plus background counts of each counter were recorded as well as the triple coincidence and the accidental double coincidence counts. The corrections described above were then made.

VII. RESULTS

The proton-proton differential cross sections for 90° in the center of mass coordinates are given in Table I, Figs. 2 and 3. The energies of the incident protons, and the reciprocals of the energies are likewise tabulated.

The tabulated errors for the measured cross section are the statistical errors which were calculated by using the reciprocal of the square root of the number of counts. The probable errors due to temperature and pressure measurement were $\pm\frac{1}{3}$ percent. The probable error of the absolute cross section due to charge measurement was measured at 31.8 Mev and observed to be less than $\pm\frac{1}{2}$ percent, and the assigned probable errors of the absolute cross sections are tabulated.

The curve of Fig. 2 is normalized for the measured cross section of 14.3×10^{-27} cm^2 at 31.8 Mev, and all other points are plotted relative to this value. This procedure was more convenient than that of carefully evaluating the triple coincidence geometry.

The mean energy of the incident protons tabulated in Table I is the value calculated from the deflection in the magnetic field. The probable error in energy for each adjustment of the magnetic field is $\pm\frac{2}{3}$ percent.

Table II is a summary of the calculated values of the differential cross section for proton-proton scattering at 90° c.m. The third column is a tabulation of the values calculated by Christian and Noyes,[3] assuming a tensor model.

The curve of Fig. 2 is plotted through the values listed in Table III calculated by Christian and Noyes, assuming a Yukawa well of range $=1.417\times10^{-13}$ cm, and depth of 49.35 Mev. These are the parameters determined by Chew and Goldberger[6] selected to be con-

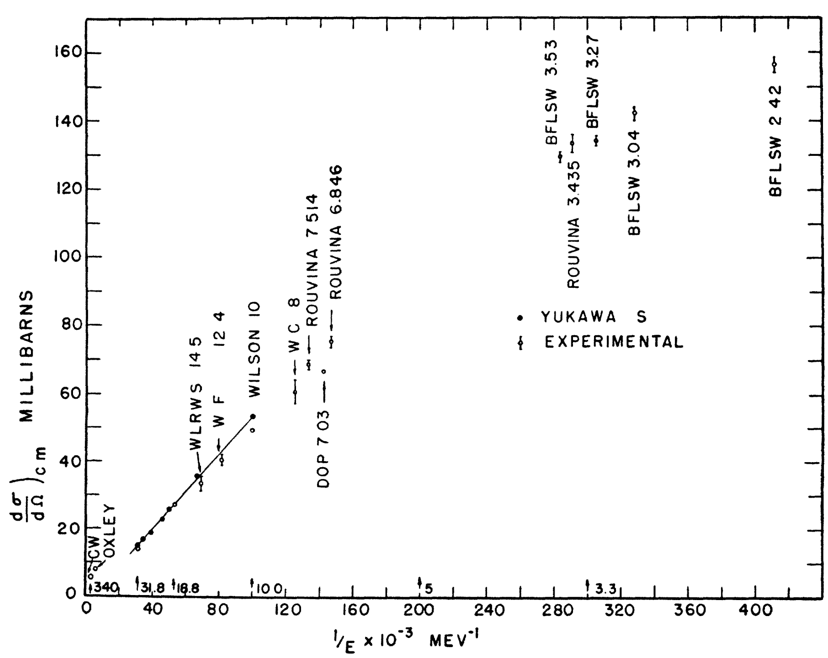

FIG. 3. Summary of proton-proton scattering data, 90° c.m. as a function of the reciprocal of the energy of the incident protons.

TABLE IV. Summary of measured values of the differential cross section for proton-proton scattering at 90° c.m. and calculated values of $\bar{K}$ assuming the measured cross sections.

Ref.	E(Mev)	Error %	$\frac{1}{E}\times 10^3$	$\left(\frac{d\sigma}{d\Omega}\right)$ 90° c.m.	Probable error %	Calculated $\bar{K}$
5	2.42			156.6	±2.1	4.86 ±0.05
5	3.04			142.1	±1.8	5.15 ±0.06
5	3.27			134.3	±1.4	5.24 ±0.06
5	3.53			129.4	±1.4	5.36 ±0.07
19	2.42			—		4.90 ±0.05
19	3.04			—		5.17 ±0.05
19	3.28			—		5.26 ±0.06
19	3.53			—		5.41 ±0.06
19	4.2			—		5.83 ±0.30
20	4.94	±0.04		99.57		6.02 ±0.18
11	7.03	±0.06		66.5		7.63 ±0.11
9	8.0	±0.1		60.46	±3.5	8.00 ±0.40
10	14.5	±0.7		33.4	±2.	10.78 ±0.80
11	3.435	±1.4	291.1	133.4	±2.4	5.30 ±0.067
	6.846	±0.58	146.1	146.48	±1.4	6.86 ±0.122
	7.514	±0.53	133.1	68.87	±1.3	7.18 ±0.130
12	12.4	±2.0	80.64	40.3	±2.0	9.683 ±0.2
Present work	18.8	±0.7	53.19	27.2	±2.4	11.67 ±0.27
	21.9	±0.7	45.66	22.8	±2.3	12.91 ±0.27
	25.2	±0.7	39.68	18.7	±2.6	14.65 ±0.33
	25.45	±0.7	39.30	18.36	±2.1	14.84 ±0.27
	31.45	±0.7	31.79	14.45	±2.2	16.73 ±0.30
	31.8	±0.7	31.45	14.3	$\pm^{1.8}_{1.5}$	16.76 $\pm^{0.25}_{0.21}$
1	31.8	3	31.45	14.3	$\pm^{1.8}_{1.5}$	16.76 $\pm^{0.25}_{0.21}$
2	29.4	±0.7	34.0	16.0	±3.0	15.84 ±0.39
13	250.		4.0	"8"		
14	170.		5.88	4.0		
	340.		2.94	4.0		

sistent with the angular distribution observed at 31.8 Mev, assuming that somehow the D-wave scattering predicted for such a potential does not appear. Figure 3 is a summary of the measured values of the differential cross section for proton-proton scattering at 90° c.m. as a function of the reciprocal of the energy of the incident protons.

Table IV and Fig. 4 are a summary of the values of the function $\bar{K}$ defined by Blatt and Jackson.[19] The straight line labeled Blatt and Jackson, Fig. 4, is the best straight line fit for $K=3.755+0.4603E$ determined from low energy proton-proton scattering data.[20]

It is concluded that at present no potential model has been discovered having any effective range long enough to fit the low energy data, that also predicts negligible D-wave scattering at 32 Mev. Hence, there is no existing theoretical basis for such an S-wave model. The tensor model of Christian and Noyes[3] has been arranged to satisfy the experimental data at 250 Mev and 340 Mev, as well as the low energy data.

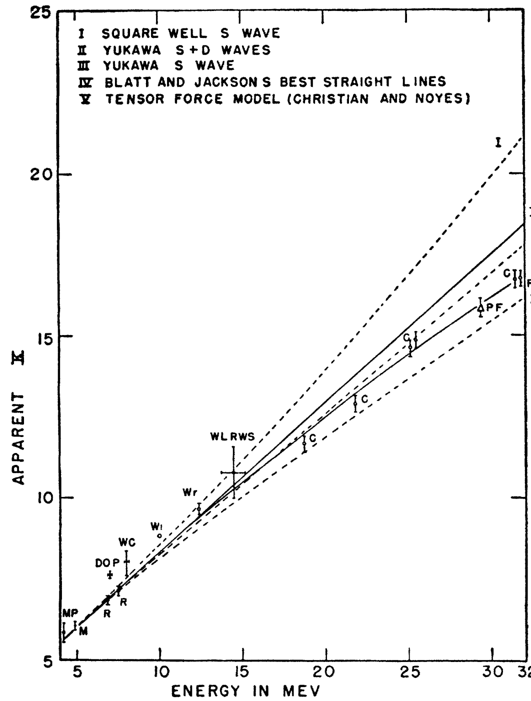

FIG. 4. Variation of the function $\bar{K}$ as a function of the energy of the incident protons.

It is a pleasure to express my thanks to Professors Luis Alvarez and W. K. H. Panofsky for encouragement to continue this problem. Also, I am grateful to Professor Geoffrey Chew, Mr. Richard Christian, and Mr. H. P. Noyes for stimulating discussions, and for calculation of the values of the function $\bar{K}$. The linear accelerator crew under the direction of Mr. Robert Watt played an important role in making all the necessary adjustments for changing the energy and for satisfactory operation of the linear accelerator. The design and maintenance of the counting apparatus was done by G. O. Essex and A. J. Stripeika under the supervision of H. D. Farnsworth.

[19] J. D. Jackson and J. M. Blatt, Rev. Mod. Phys. **22**, 77 (1950).
[20] See also R. E. Meagher, Ph.D thesis, University of Illinois, 1949; Phys. Rev. **78**, 667 (1950).