

electrode E , which compensate the Lorentz force due to the motion of the protons in the magnetic field. Beyond R , the path becomes a slowly shrinking spiral with the dee voltage chosen slightly above the minimum of about 100 v, necessary to allow clearance of the injection plate I of 0.25 mm thickness after the first revolution. ν_R is determined from the dee frequency ν at which protons reach the inner probe P , protruding into the dee cavity. Since the magnetic field has to be shimmed to very great homogeneity and, therefore, could not provide magnetic focusing, we relied principally upon phase focusing to hold the protons within the dee cavity; it proved, indeed, sufficient to give probe currents of the order of 10^{-12} amp. after about 500 revolutions. This arrangement has the following advantages.

(1) Independently of the arc pressure, the pressure in the dee chamber can be made low enough so that gas collisions and ionization do not affect the measurement.

(2) Contrary to accelerating operation, the decelerating cyclotron can be operated not only at a dee frequency in the vicinity of $\nu = \nu_R$, but also of $\nu = n\nu_R$, where n is an odd integer. For a given number of revolutions, the higher multiples provide a correspondingly more accurate timing of the protons and the resonance band width is thereby reduced by $1/n$. A further reduction of the band width is due to the circumstance that the energy change of the protons per revolution becomes appreciably less when they have reached a radius sufficiently small so that the transit time between the dees becomes comparable to the period of the dee oscillation. Consequently, with the dee voltage still high enough to allow clearance of the injection plate, the number of revolutions which are necessary to reach the probe at a given position increases with increasing n , so that additional gain in resolution is obtained.

The measurements were carried out at a field $H \approx 5300$ gauss where the frequency of rotation is $\nu_R \approx 8.1$ Mc, and we have successfully operated the dees near all odd multiples of ν_R from the first to the eleventh. The consistency of the results obtained for the higher multiples gave a gratifying confirmation of our accuracy. Observations were made by applying a slight modulation with a period of 4 sec. to the dee frequency and by photographing the trace on an oscillograph whose vertical deflection gave a measure of the instantaneous probe current. Figure 2 shows a typical trace, taken around $\nu = 9\nu_R$ with the same frequency interval covered twice as the small modulating condenser goes through a full revolution. The shape of the traces has been qualitatively understood in terms of the focusing action and its variation with ν . The relative half-width has been found to be about $1/200$ for $n=1$; the expected gain in resolution was confirmed by the corresponding values $1/4000$ and $1/10,000$ found for $n=5$ and $n=11$, respectively. A marker was produced on each trace by the beat with a frequency meter, thus providing a highly precise record of the dee frequency. A small nuclear induction head, movable within the dee chamber, gave the nuclear resonance frequency ν_N of protons in water; the signal from this head was also used to hold H constant during runs and to ascertain homogeneity between the monitoring position and the inner region of the dees.

To insure an accurate result we have searched carefully for systematic errors: analysis showed that relativistic corrections were negligible; both by analysis and by experiment it was further shown that space-charge effects, collisions of the protons, fringing fields from the injection electrode, and distortions of the oscillating dee field by the probe likewise did not affect the accuracy of the result. Taking all errors into account, we can at present state our result to be

$$\mu_P = (2.79245 \pm 0.0002) \mu_n. \quad (4)$$

The performance of our apparatus indicates that considerably higher accuracy, valuable also for relative mass determinations of the light atoms, will be attainable through some modifications; operation with very large values of n seems here particularly promising.

While this work was in progress, Hipple, Sommer, and Thomas² reported a new determination of the Faraday, based upon a similar

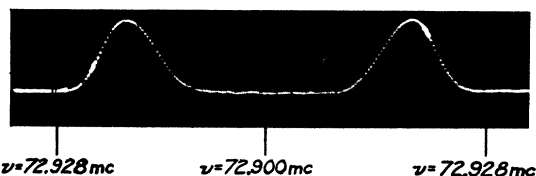


FIG. 2. Oscillogram of detector probe current versus dee frequency near the ninth multiple of ν_R .

principle for measuring ν_R by their accelerating "omegatron"; the value for μ_P derived from their measurement agrees with (4). We shall not enter into a discussion here of the connections between our result, the value of the Faraday, the ratio of the masses of proton and electron through the experiment of Purcell and Gardner,³ and other fundamental constants.

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² Hipple, Sommer, and Thomas, Phys. Rev. **76**, 1877 (1949).

³ J. H. Gardner and E. M. Purcell, Phys. Rev. **76**, 1262 (1949).

Calculation of the Absorption and Emission Spectra of the Thallium-Activated Potassium Chloride Phosphor

FERD E. WILLIAMS

General Electric Research Laboratory, Schenectady, New York

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PREVIOUS interpretations of solid-state luminescent phenomena have been either qualitative or phenomenological.¹ Therefore, it is important to calculate quantitatively the properties of a simple phosphor from first principles. The calculation of the peak absorption and emission energies for KCl:Tl has already been reported.² Improvements in the theory have permitted the determination of the absorption and emission spectra and their temperature dependence.

It is first recognized that the KCl:Tl phosphor is an ionic crystal³ and that the luminescence arises from transitions characteristic of Tl^+ substituted in dilute concentration for K^+ ions.⁴ The radial charge densities of free Tl^+ in the ground 1S_0 state and in the excited $^3P_1^0$ state are evaluated using the Sommerfeld modification⁵ of the Fermi-Thomas method for the core and the Hartree self-consistent field method⁶ for the two outer shell electrons. From these wave functions and from the known ionic radius r_0 , polarizability α , and repulsion energy constant ρ for the ground state,³ these parameters are evaluated for the Tl^+ in the excited state interacting with Cl^- . The method of Kirkwood⁷ is used to determine α . The variation of repulsion energy with interatomic distance a is shown to be equal to the variation of S^2/a with a , where S is the overlap integral: $\int \Psi_{Tl^+} \Psi_{Cl^-} dV$. The Tl^+ in the 1S_0 and the $^3P_1^0$ states are substituted in dilute concentrations for the K^+ in the KCl, and the change in total energy of the system is calculated as a function of the change in the $Tl^+(000)$ nearest $Cl^-(100)$ distance Δr with the condition that the remainder of the lattice rearranges to minimize the total energy. A good approximation to this condition requires only displacement $\Delta r'$ of the $K^+(200)$ radially from the Tl^+ to the position $\Delta r_m'$ of minimum energy. Only symmetric displacements in Δr and $\Delta r'$ are considered. Madelung, exchange repulsion, van der Waals, and ion-dipole interactions are included. For the $^3P_1^0 Tl^+$, the Coulomb overlap interactions determined from the free ion wave functions are included. After transformation to the $\Delta r_m'$ plane, the total energy versus the configuration coordinate Δr is plotted in Fig. 1 for the 1S_0 and $^3P_1^0$ states. It should be noted that the equilibrium Tl^+ nearest Cl^- distance is less for the excited than for the ground state. The energies are fitted to the plotted parabolas by mini-

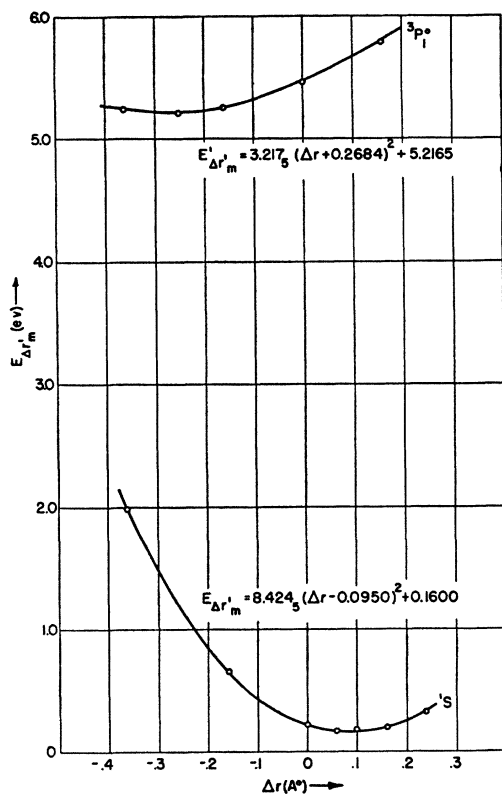


FIG. 1. Variation in total energy versus configuration coordinate for the $1S_0$ and $3P_1^0$ states of Tl^+ in KCl.

mizing the squared deviations. The absorption spectrum is computed by recognizing that the various atomic configurations of the system in the ground state have probabilities in accord with a Boltzmann function:

$$P(\Delta E) = C \exp[-(E - E_0)/kT] d(\Delta r)/d(\Delta E),$$

where $P(\Delta E)$ is the probability of the transition energy ΔE and C is a normalization constant. The emission spectrum is similarly determined using $E' - E_0'$. The calculated spectra are compared with experiment⁸ on Fig. 2 for 298°K. The agreement is satisfactory. The broader theoretical emission spectrum may arise from using free ion wave functions for calculating the Coulomb overlap correction, or from neglecting the rearrangement energies of other

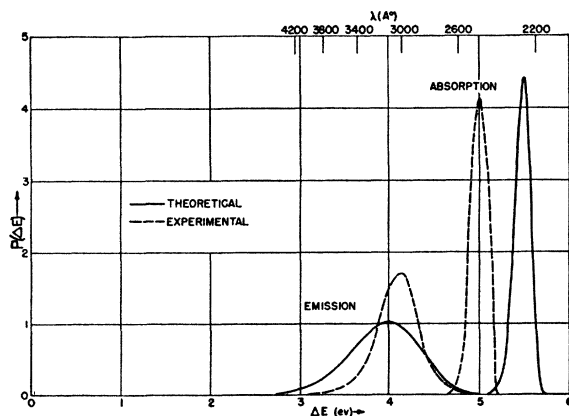


FIG. 2. Theoretical and experimental absorption and emission spectra of KCl : Tl.

neighbors for the configurations of large $|\Delta r|$. The theoretical temperature dependences of the spectra are also found to be in good agreement with experiment, the half-widths of both absorption and emission being halved in going to 80°K.

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Photo-Disintegration of O^{16}

MELVIN A. PRESTON*

National Research Council of Canada, Chalk River, Ontario, Canada
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THE reaction $O^{16}(\gamma, \alpha)C^{12}$ has been studied theoretically using an α -particle model of the O^{16} nucleus. Since the identity of charge and mass centers eliminates dipole transitions, the effect has been assumed to be quadrupole. The experiments of Millar and Cameron¹ show that the cross section of this reaction as a function of γ -energy is fairly sharply peaked and has a maximum between 16 and 18 Mev. Since this is well above the Coulomb barrier of C^{12} , it has been considered reasonable to treat the emitted particle as a plane wave and to apply a barrier penetrability factor for the lower energies. We assume that the wave function can be written as $\phi(r)\Psi$, where ϕ is a function of the coordinates of the ejected particle and Ψ depends on those of the other three. We consider the effect of the γ -ray on only the ejected particle, and we are led to matrix elements of the type $\int \Psi_f^* \exp(ip \cdot r/\hbar) \alpha \alpha \Psi_0 \phi_0(r) d\tau$; Ψ_f is the wave function of the ground state of C^{12} ; this should overlap Ψ_0 considerably. Moreover, it is reasonable to neglect the interaction of the γ -ray with the other particles, at least until there is sufficient energy to emit further α -particles; this threshold is at 18 Mev. Even then this interaction can be neglected until the cross section of the competing process is appreciable; results of Millar and Cameron indicate that this is still small at $E_\gamma = 21$ Mev. We put the integral of $\Psi_f^* \Psi_0 = 1$; actually, it is smaller, but is energy independent in the region considered. [This model does not permit the calculation of the $(\gamma - n)$ and $(\gamma - p)$ processes, but their presence does not affect our results.]

We obtain

$$\sigma(\theta) = \pi(27m^2/\hbar^3c^3)E_\gamma^3 p \left| \int_0^\infty r^4 j_2(pr) \phi_0(r) dr \right|^2 \sin^2\theta \cos^2\theta$$

($m =$ proton mass). For energies below the Coulomb barrier ($E_\gamma < 12$ Mev), σ is multiplied by the penetration factor. For ϕ_0 , we have used the forms: $\exp(-r/b)$, $\exp(-r^2/b^2)$ and modified Wheeler wave functions. Following Wheeler,² we have thought of O^{16} as a tetrahedron of four α -particles with a wave function

$$\prod_{i=1}^4 \exp[-\beta(1-\alpha)R_i^2] \prod_{i<j} [\exp\{-\frac{1}{4}\beta\alpha R_{ij}^2\} - \exp\{-\frac{1}{4}\beta(\alpha+\epsilon)R_{ij}^2\}].$$

This insures that no two particles will come too close together, nor will any be too far from the center. By ignoring the vibrations of the other three particles a wave function, $\phi_0(r)$, has been deduced for the ejected particle. It is of the form $\exp(-r^2/b^2) \times f(r, a/b)$, in which a is the distance from a face centroid to a vertex of the tetrahedron. With this model it is reasonable to consider that the radius of the nucleus is $(3/4)(2a^2 + b^2)^{1/2} = r_0$.

Figure 1 shows the total cross section $\sigma(E_\gamma)$. All the curves have been arbitrarily normalized to the same maximum. For each type of ϕ_0 , the range b has been adjusted so that the maximum of σ occurs at $E_\gamma = 17.6$ Mev. Preliminary experimental points of