Neutron Refraction in Ferromagnets

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THE disagreement between the results of Halpern[1] and of Ekstein[2] concerning the index of refraction of neutrons in ferromagnets can be summarized as follows. (1) Halpern claims that the change in neutron index of refraction is determined by the scattering amplitude in the forward direction. (2) Ekstein points out that Halpern's calculation of the forward scattering amplitude is incorrect and that the magnetic scattering of a neutron by an ion vanishes in the forward direction. (3) Ekstein shows that the change in index of refraction does not vanish but is determined by the magnetic field in the magnet.

I shall show here that *all* of these statements are correct. The procedure is based on a general theory of the multiple scattering of waves to be published soon. The chief result of this theory is that the coherent wave $\langle\psi(\mathbf{r})\rangle$ obeys an equation

$$(E-H-c\bar{T})\langle\psi(\mathbf{r})\rangle=0, \tag{1}$$

$$\bar{T}=\int_\tau T(\mathbf{r}_j)n(\mathbf{r}_j)d\mathbf{r}_j, \tag{2}$$

where $n(\mathbf{r}_j)$ is the density of scatterers. Here c is a correction factor for the relation between the "effective" and total neutron wave and for present purposes may be set equal to unity. $T(\mathbf{r}_j)$ is the transition operator[3] whose matrix elements $T_{ba}(\mathbf{r}_j)$ are proportional to the scattering amplitude from direction a to direction b for a scatterer at position $\mathbf{r}_j$. These differ by a phase factor from the case in which the scatterer is located at the origin:

$$T_{ba}(\mathbf{r}_j)=\exp[i(\mathbf{k}_a-\mathbf{k}_b)\cdot\mathbf{r}_j]T_{ba}(0). \tag{3}$$

$\bar{T}$ represents the sum of the scattering amplitudes of the scatterers in a volume τ equal to the volume of quantization. It follows from (2) and (3) that $\bar{T}$ is already diagonal and independent of τ if plane waves quantized in τ are used as the eigenfunctions. Since $E=\hbar^2k^2/2m$ and $H=-\hbar^2\nabla^2/2m=\hbar^2k'^2/2m$, the propagation constant k' in the magnet is related to the vacuum case by:

$$k'^2=k^2-c(2m/\hbar^2)(\bar{T})_{aa}. \tag{4}$$

This establishes the first point that the index of refraction k'/k is determined by the forward scattered amplitude.

Since both Halpern and Ekstein treat the scattering in Born approximation, we can replace T by the corresponding interaction potential:

$$T(\mathbf{r}_j)=V(\mathbf{r}-\mathbf{r}_j)-\mu\boldsymbol{\sigma}\cdot\mathbf{B}(\mathbf{r}-\mathbf{r}_j), \tag{5}$$

where $V(\mathbf{r}-\mathbf{r}_j)$ is the pseudopotential of the jth nucleus and $\mathbf{B}(\mathbf{r}-\mathbf{r}_j)$ is the magnetic field associated with the corresponding ion. The forward scattered amplitude of a single scatterer is then proportional to:

$$\lim_{\tau\to\infty}\int_\tau[V(\mathbf{r}-\mathbf{r}_j)-\mu\boldsymbol{\sigma}\cdot\mathbf{B}(\mathbf{r}-\mathbf{r}_j)]d\mathbf{r}. \tag{6}$$

The second term vanishes in the limit, since the complete space integral of a solenoid vector $\mathbf{B}$ vanishes,[4] providing $\lim r\mathbf{B}\to 0$ as $r\to\infty$. This establishes the second point that the Born approximation to the magnetic forward scattered amplitude vanishes.

We shall now show that (4) yields a non-vanishing magnetic contribution to the index of refraction providing we calculate $(\bar{T})_{aa}$ rather than $(T_{aa})_{Av}$. In other words, we must add the contributions of the individual scatterers before calculating the forward amplitude. Usually the two procedures would be in agreement, but here they differ because of the long-range nature of the magnetic forces:

$$(\bar{T})_{aa}=\frac{1}{\tau}\int_\tau d\mathbf{r}\int_\tau d\mathbf{r}_j[V(\mathbf{r}-\mathbf{r}_j)-\mu\boldsymbol{\sigma}\cdot\mathbf{B}(\mathbf{r}-\mathbf{r}_j)]n(\mathbf{r}_j) \tag{7}$$

$$=\lim_{\tau\to\infty}\frac{1}{\tau}\int_\tau d\mathbf{r}[V_t(\mathbf{r})-\mu\boldsymbol{\sigma}\cdot\mathbf{B}_t(\mathbf{r})] \tag{8}$$

$$=V_{Av}-\mu\boldsymbol{\sigma}\cdot\mathbf{B}_{Av}, \tag{9}$$

where $V(\mathbf{r})$ and $\mathbf{B}_t(\mathbf{r})$ represent the total potential and total magnetic field at $\mathbf{r}$ and V_{Av}, $\mathbf{B}_{Av}$ are the corresponding macroscopic average potential and field. Since the eigenvalues of $\boldsymbol{\sigma}\cdot\mathbf{B}_{Av}$ are $\pm|\mathbf{B}_{Av}|$ for neutrons polarized parallel or antiparallel to the magnetic field, we obtain Ekstein's result:

$$k'^2=k^2-(2m/\hbar^2)[V_{Av}\pm\mu|\mathbf{B}_{Av}|] \tag{10}$$

and establish the third point.

Equation (8) embodies carefully the principle that an index of refraction should be computed for a medium of infinite volume. The space integral of $\mathbf{B}_t(\mathbf{r})$ does not vanish for an infinite magnet because $r\mathbf{B}_t(\mathbf{r})$ does not vanish as $r\to\infty$.

[1] O. Halpern, Phys. Rev. **76**, 1130 (1949). Halpern, Hamermesh, and Johnson, Phys. Rev. **59**, 981 (1941).
[2] H. Ekstein, Phys. Rev. **78**, 731 (1950); **76**, 1328 (1949).
[3] M. Lax, Phys. Rev. **78**, 306 (1950).
[4] J. A. Stratton, *Electromagnetic Theory* (McGraw-Hill Book Company, Inc., New York, 1942), p. 111.

Cation Distribution in Copper Zinc Ferrite

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A NOTE in this Journal by Brockman[1] on the effect of low temperature heat treatment on the Curie temperature of copper zinc ferrite calls for comment.

The experimental results are not disputed; in fact, they agree with some qualitative observations made at an earlier date by the author of the present note.

The interpretation seems to me to be open to question, however. In view of the strong preference of zinc for the tetrahedral position, as shown by the fact that a quenching from 1200°C brings about only a weak ferromagnetism in pure zinc ferrite, and given the great sensitivity of the saturation magnetization of pure copper ferrite to variations in the heat treatment,[2] it seems to be much more likely that changes in the distribution of the copper atoms over the available positions in the lattice are responsible for the observed variations. Obviously, accurate measurements of the saturation magnetization would give the answer to this problem at once.

[1] F. G. Brockman, Phys. Rev. **77**, 841 (1950).
[2] L. Néel, Comptes Rendus **230**, 190 (1950).

Crystals and Geiger Counters for Scintillation Counting*

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IN three previous communications,[1-3] the writers have discussed the detection of alpha-, beta-, and gamma-rays by recording scintillations from crystals in photo-sensitive Geiger counters. A survey has been conducted wherein a large number of inorganic crystals have been produced with emphasis upon the silver-activated alkali halides. Some organic materials have also been considered. Using a scintillation Geiger counter as a detector, effects have been observed as shown in Table I.

TABLE I.

Scintillator	Particles detected in the photon Geiger counter
NaCl-Ag	alpha, beta, gamma
NaBr-Ag	alpha, beta
NaBr-Tl	alpha
Powdered durene	alpha
KCl-Ag	alpha
LiCl-Ag	alpha
LiBr-Ag	alpha

NaCl-Ag remains thus far the most satisfactory crystal for general purposes.

As often mentioned in the earlier communications, the counters have been of the gauze cathode-type and have been usually sensitized by a method described by Scherb.[4] However, counters are now being produced without sensitization of any kind which are sensitive enough to detect scintillations induced by alpha- and beta-rays. The discharge at liquid air temperatures does of course increase the counter sensitivity and hence efficiency of detection. It has always been necessary to sensitize the counters for detection of gamma-rays.

Several remarkable characteristics of treated photon counters have been observed in a qualitative fashion. For example, after the discharge treatment, the counters will withstand very high counting rates over a long period of time (greater than five hundred thousand counts per minute) without "breaking down." It is possible that points and rough spots on both the wire and the cathode are removed by the discharge. Although accurate observations have not been carried out, the counter sensitivity appears to remain constant over long time intervals. It also appears that photo-sensitivity increases as the diameter of the cathode of the counter is decreased. It is assumed, of course, that the operating voltage is kept constant by adjusting the counter pressure. The increased activity is explained by the fact that with smaller diameters, a stronger electric field is maintained adjacent to the cathode of the counter so that more of the soft ultraviolet photoelectrons are attracted into the counting volume of the counter.

Experiments with various counter geometries and crystals are continuing.

* Assisted by the joint program of the ONR and AEC.
[1] C. E. Mandeville and H. O. Albrecht, Phys. Rev. **79**, 1010 (1950).
[2] C. E. Mandeville and H. O. Albrecht, Phys. Rev. **80**, 117 (1950).
[3] C. E. Mandeville and H. O. Albrecht, Phys. Rev. **80**, 300 (1950).
[4] M. V. Scherb, Phys. Rev. **73**, 86 (1948).

The Detection of Gamma-Ray-Induced Scintillations from Crystals in a Photo-Sensitive Geiger-Müller Counter*

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IN two previous communications,[1,2] the writers have discussed the detection in photon Geiger counters of scintillations from alpha-particles and beta-rays on NaCl-Ag. The photo-sensitive counters have been constructed of wire gauze cathodes and Corning 9741 glass. The cathode materials have been usually copper and tinned steel. Sometimes, the counters have been sensitized by a discharge technique previously described by Scherb,[3] formerly of this laboratory.

It has been shown[2] previously that short-lived fluorescent pulses are also produced in the deep ultraviolet when NaCl-Ag is irradiated by gamma-rays.[2] Scintillations from gamma-rays on NaCl-Ag have been detected until recently only in a 1P28 photomultiplier tube.[2] However, sufficient quantum efficiency has now been obtained to detect them in a photo-sensitive Geiger counter.

Photon counters have now been produced which are far more sensitive to the ultraviolet than are those used for detection of scintillations from alpha-particles and beta-rays. One of these very sensitive counters was placed within concentric cylinders of NaCl-Ag and Textolite, Textolite being innermost, and was irradiated by the million-volt gamma-rays of Sc^{46}. The wall thickness of the NaCl-Ag cylinder was 1.27 cm, and the wall thickness of the Textolite was 0.32 cm. When the Textolite shield was removed, the counting rate was observed to increase by a factor of 2.5. Precautions were taken to ascertain that the change in counting rate was a genuine effect and was not one brought about by stray light admitted with removal of the shield. The measurements were repeated subsequently with the use of a paper shield rather than Textolite, and the same result was obtained. When the cylinder of NaCl-Ag was removed, the effect disappeared. When the photon counter was replaced by an untreated one in a Nonex envelope, no effect was observed. All of these tests of the experimental arrangement point to the fact that the increased counting rate could be explained only by counting ultraviolet fluorescence when the crystals of NaCl-Ag were irradiated by gamma-rays.

The factor of 2.5 in increase of efficiency was obtained in a relatively poor geometric arrangement. For example, the inner diameter of the cylinder of NaCl-Ag was several times greater than the diameter of cathode of the photon counter. This unfavorable situation resulted from the shape of the counter at its ends and from the fact that sufficient space was allowed for easy removal of the Textolite light shield. It is estimated that were the side walls of the cylinder of NaCl-Ag contiguous with the counter, an efficiency factor of 4 or 5 would have been obtained.

The limit of photo-sensitivity has not yet been reached. It is anticipated that the efficiency factor for gamma-rays will be increased.

* Assisted by the join program of the ONR and AEC.
[1] C. E. Mandeville and H. O. Albrecht, Phys. Rev. **79**, 1010 (1950).
[2] C. E. Mandeville and H. O. Albrecht, Phys. Rev. **80**, 117 (1950).
[3] M. V. Scherb, Phys. Rev. **73**, 86 (1948).

A Variational Principle for Time-Dependent Problems

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RECENTLY variational methods have been used particularly by Schwinger and his school[1] in connection with different kinds of physical problems.

It seems worth while to show that a similar variational principle holds for problems concerning the evolution of a quantum-mechanical system due to the action of an arbitrary time- and space-dependent potential. Let $f(\mathbf{x}, t)$ be the initial state of the system, $\psi(\mathbf{x}, t)$ the perturbed wave function, and $g(\mathbf{x}, t)$ an arbitrary final state. Then the probability amplitude for the transition from the state f to the state g is stationary with respect to small variations of $\psi(\mathbf{x}, t)$. This is easily seen using Feynman's[2] integral formulation, as the variational principle is derived by transformation of the differential equation into an integral equation containing a suitable Green's function.

Let us restrict ourselves for simplicity to the case of the motion of only one electron in a given potential $A(\mathbf{x}, t)$. (A is the four-vector matrix defined by Feynman[3].) If at time t_1 the electron is in the state f, the transition amplitude for finding the electron in the state g at time t_2 is

$$\alpha=\int \bar{g}(2)\beta K^{(A)}(2,1)\beta f(1)d^3\mathbf{x}_1 d^3\mathbf{x}_2. \tag{1}$$

The potential acts on the electron transforming the initial wave function $f(1)$ into $\psi(2)$ which obeys the integral equation

$$\psi(2)=f(2)-i\int K_+(2,1)A(1)\psi(1)d\tau_1. \tag{2}$$

In a similar way we define the wave function $\chi(2)$ as

$$\bar{\chi}(2)=\bar{g}(2)-i\int \bar{\chi}(1)A(1)K_+(1,2)d\tau_1. \tag{3}$$

If g and f are orthogonal states, using Eqs. (2) and (3), Eq. (1) becomes

$$a=-i\int \bar{g}(2)A(2)\psi(2)d\tau_2=-i\int \bar{\chi}(2)A(2)f(2)d\tau_2. \tag{4}$$