

in any part must be even. Thus such parts, and in particular primitive divergents of this type, are excluded for charged mesons. But the general treatment of divergences in that paper is not complete, even for charged mesons, as there are secondary divergences from parts built up of $\delta\lambda$ -vertices which have not been considered.³ These difficulties arise only in high orders, (f^8), so that with the direct use of Dyson's work⁵ it is now simple to set up in this way the combined interaction of charged spinless mesons, nucleons (scalar interaction), and the electromagnetic field.

The Hamiltonian for pseudoscalar mesons is

$$H = \sum H_i \quad (i=1, 2, 3),$$

where

$$H_1 = \frac{ie}{\hbar c} A_\mu \left(\phi^* \frac{\partial \phi}{\partial x_\mu} - \frac{\partial \phi^*}{\partial x_\mu} \phi \right) - \left(\frac{ie}{\hbar c} \right)^2 A_\mu^2 \phi^* \phi,$$

$$H_2 = if \bar{\psi} \gamma_5 (\tau_- \phi^* + \tau_+ \phi) \psi + \delta\lambda \phi^{*2} \phi^2,$$

$$H_3 = ie \bar{\psi} \gamma_\mu \tau_F \psi A_\mu.$$

The n_μ -dependent and mass renormalization terms have not been included. For scalar mesons (γ_5) $_{\alpha\beta}$ is replaced by $-i\delta_{\alpha\beta}$.

The primitive divergents must satisfy

$$\frac{2}{3} E_n + E_m + E_p < 5.$$

This cannot be satisfied unless at least one term on the left-hand side is zero. Using the charge conservation rule, it can be shown that the genuine primitive divergents are the same as for the separate pairs of interacting fields,^{1,2,5} but new internal structures are now possible. Since, for example, a proton-photon vertex part may depend on both e and f , the renormalized constants are double power series in e and f .

Considerations are now restricted to S -matrix elements of fourth order in the coupling constants. The $\delta\lambda$ -term is chosen to cancel the (logarithmic) divergence from the scattering of mesons by mesons. All quadratic and linear divergences are removed by mass renormalization. In order that all the remaining (logarithmic) divergences can be removed by the renormalization of the constants e and f it is necessary that certain conditions be satisfied. The charge renormalization is determined by the infinite constants in $\Delta F'$, $D F'$ and the meson photon 3-vertex parts. It must be shown that this renormalization is (i) identical with that determined by the infinite constants in $S F'$, $D F'$, and Γ_μ (the proton-photon interaction), and (ii) that it also removes the divergences from C -parts. Each of these conditions contains terms in e^2 and f^2 , so they are together equivalent to four conditions. It must also be shown (iii) that the infinities from parts with two external neutron lines and one external photon line exactly cancel, since there is no simple vertex of this type and no renormalization is possible.⁶ It has been checked by direct calculation that all these conditions are satisfied both for scalar and pseudoscalar mesons.

Although this result appears to be purely fortuitous in our work, it certainly suggests that some general principle is operative which might make renormalization effective to any order for this three-field mixture.

The author would like to thank Mr. Salam for stimulating discussions.

¹ P. T. Matthews, preceding letter.

² P. T. Matthews, Phil. Mag. **41**, 185 (1950).

³ The author is further indebted to Dr. Feldman for pointing out these errors.

⁴ W. H. Furry, Phys. Rev. **51**, 125 (1937).

⁵ F. J. Dyson, Phys. Rev. **75**, 486, 1736 (1949).

⁶ These graphs give the neutron magnetic moment and were discussed by K. M. Case, Phys. Rev. **76**, 1 (1949).

The Radioactivity of Am²⁴²

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THE neutron irradiation of 475-yr. Am²⁴¹ results in the production of a pair of isomers,^{1,2} Am^{242m} with 16-hr. half-life known to decay by β^- -emission, and a long-lived ground state,

TABLE I. Principal x-ray lines in decay of Am^{242m}.

X-ray line designation	Level transition	Measured energy	Relative intensity (corrected)*	Calculated energy	
Pu	$L\alpha_1$	$L_{III} - M_V$	14.33 \pm 0.01	68	14.31
	$L\beta_2$	$L_{III} - N_{IV}$	17.32 \pm 0.02	22	17.29
	$L\beta_1$	$L_{II} - M_{IV}$	18.34 \pm 0.02	54	18.30
	$L\gamma_1$	$L_{II} - N_{IV}$	21.52 \pm 0.05	18	21.43
	$L\alpha_1$		14.66 \pm 0.02	39	14.66
Am	$L\beta_2$		18.90 \pm 0.07	8	18.84
	$L\beta_1$				
	$L\gamma_1$				
	$L\alpha_1$		15.01 \pm 0.01	100	15.00
Cm	$L\beta_2$		18.13 \pm 0.04	36	18.08
	$L\beta_1$		19.48 \pm 0.02	107	19.38
	$L\gamma_1$		22.78 \pm 0.07	33	22.68

* Estimated corrections for crystal and counter window absorption and reflection intensity variation with energy.

^b Not resolvable from other lines.

Am²⁴², which is also a β^- -emitter and has weak alpha-branching. Since fairly strong activities can be prepared by pile neutron irradiation of small quantities of Am²⁴¹, it became feasible to examine the radiations, particularly those of Am^{242m}, with beta-ray and x-ray spectrometers. When this was done only was the β^- -transition characterized, but good evidence was obtained that in addition this isomer undergoes appreciable branching decay by isomeric transition and by electron capture.

The electron-capture branching results in the long-lived plutonium isotope, Pu²⁴², which was found with a mass spectrograph upon examination of the plutonium fraction from a sample of americium which had been subjected to long neutron irradiation.³

Much of the information on the modes of decay of Am^{242m} was derived from a bent crystal x-ray spectrometer used to analyze the L x-ray mixture. This instrument and its use in measuring the L -series x-rays accompanying radioactive decay in the heavy element region are described in another publication.⁴ When the irradiated Am²⁴¹ was observed with the x-ray spectrometer, L -series x-rays of curium (96), americium (95), plutonium (94), and neptunium (93) were identified, among which the $L\beta_1$, $L\beta_2$, $L\alpha_1$, and $L\gamma_1$ -lines were in greatest abundance. The curium, americium, and plutonium x-rays decayed with a 16-hr. half-life but the neptunium x-ray intensity did not decrease over a period of several days. The origin of the x-rays are presumably as follows: those of curium from an internal conversion process following the β^- -transition, the americium x-rays from internal conversion in the isomeric transition of Am^{242m}, the plutonium x-rays following electron-capture decay, and the neptunium x-rays (which did not decay) following the internal conversion of a γ -ray known to exist in the alpha-decay¹ of Am²⁴¹. Table I lists the principal x-ray lines observed, their measured energies and relative intensities, and the energies calculated using the Moseley relation in a manner described elsewhere.⁴ The complete x-ray spectra and other decay properties of Am²⁴² will be discussed more fully at a later date. It is worth noting, however, that for a particular level vacancy, the ratios of intensities of different lines are about the same (as they should be), and that the relative incidence of L_{II} and L_{III} lines are roughly equal both for the plutonium and curium x-rays, but those from the isomeric transition (americium x-rays) show the L_{III} level x-rays to be about five times as abundant as those from the L_{II} level. Selection rules are probably in force here which are responsible for the selection of the L_{III} level (a $p_{3/2}$ state) and for a long lifetime of the metastable state.

The β^- -ray spectrum of Am^{242m} was taken with a 255°-shaped magnetic field β -ray spectrometer using a resolution of three percent. The Fermi-Kurie plot of the continuous β^- -spectrum showed an end point of 628 ± 5 kev. Two sets of L conversion lines were seen corresponding to gamma-rays of 38 kev and 52 kev, assuming the former to be a plutonium γ -ray and the latter from curium. In attempting to assign these gamma-rays to the several transitions, the differences between L_{II} and L_{III} lines were matched with expected differences of the edges for plutonium, americium, and curium. The assignments on this basis are not

conclusive, since the differences in spacing between successive elements are about the same as the experimental uncertainties of electron energies. Tentatively, as indicated above, the 52-keV γ -ray is assigned to the β^- -transition and the 38-keV γ -ray to the electron-capture transition. This leaves unaccounted for the origin of the x-rays of americium, so if the above assignments are to be taken seriously, the conversion electrons accompanying the isomeric transition must lie among the Auger electrons. There is some evidence that this is the case.

Lead and copper absorption curves showed no hard γ -rays or K x-rays and only the 50-keV soft γ -ray. When compared with the abundance of the conversion electrons, this γ -ray appears to be about 50 percent converted.

From arguments (not all consistent) based on relative abundances of x-rays, conversion electrons, and the β^- -particles, Am^{242m} appears to decay about 60 percent by β^- -emission, 20 percent by L-electron capture, and 20 percent by isomeric transition. All three of the modes of decay give rise to L-series x-rays which, when properly assigned and abundances measured, should aid materially in arriving at a decay scheme and in shedding light on the nuclear processes which result in the particular x-rays of this interesting nucleus.

The β^- -particle of the ground state of Am^{242} has also been measured, but the accuracy of the end point has not yet been determined with desirable accuracy. The value obtained is 580 ± 30 keV which is consistent with the supposed 52 keV associated with the isomeric transition of Am^{242m} .

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² W. M. Manning and L. B. Asprey, loc. cit., Paper No. 22.7.

³ Thompson, Street, Giorso, and Reynolds, University of California Radiation Laboratory Report, UCRL-657 (June, 1950), to be submitted for publication.

⁴ Barton, Robinson, and Perlman, to be submitted for publication.

On Sommerfeld's Surface Wave

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NUMEROUS papers on the theory of propagation of electromagnetic waves over plane earth have appeared, following Sommerfeld's celebrated paper¹ of 1909. Though it has been realized by subsequent authors that Sommerfeld's discussion of his basic equation for the vector potential of a vertical electric dipole in the presence of the earth is not quite satisfactory, this equation itself was generally accepted. However, in 1947 Epstein² proposed a new solution. As I have pointed out elsewhere³ Epstein's expression for the vector potential is incompatible with the physical situation because it is singular along the whole axis of the dipole, whereas Sommerfeld's solution is regular outside the dipole. In fact, Epstein's solution is nothing but Sommerfeld's solution minus the surface wave.

The surface-wave problem was reconsidered by Kahan and Eckart. In their first note⁴ these authors accepted Epstein's solution as being the only one compatible with Sommerfeld's radiation condition,⁵ though they only showed that the surface wave does not fulfill this condition. It is, of course, immaterial whether some part (e.g., the surface-wave term) of Sommerfeld's solution does or does not satisfy the radiation condition. The behavior of the complete solution is conclusive.

In a second note,⁶ Eckart and Kahan come to the conclusion that Epstein's solution is incorrect, though they fail to mention that they were of a different opinion in their first note.⁴ They now accept Sommerfeld's original solution and point out that Sommerfeld's evaluation of the integral along the branch cut is in error. They stress that a correct evaluation would have yielded an expression that contains the surface wave with negative sign, so that the final result would have coincided with Weyl's result,⁷ the

negative surface-wave term being cancelled by the positive term due to the residue of the pole of the integrand. This explanation and clarification of the controversy is not at all new but has been known since 1937 through the work of Wise⁸ and Rice.⁹

In two longer papers,^{10,11} of which the last is apparently an English version of the first, Kahan and Eckart elaborate their previous discussions. In view of the foregoing arguments it will be evident that a detailed analysis of these papers is unfruitful. Let it be sufficient to mention, therefore, that the only new and interesting part of these two papers consists in an attempt to prove a uniqueness theorem on the basis of Sommerfeld's radiation condition, for real-valued wave numbers k_1 and k_2 . Unfortunately, their proof breaks down, as one can demonstrate from Eq. (24) of reference 11. The left-hand member of this equation is a space integral of which the imaginary part is zero. The right-hand member is a surface integral of which the real part is zero. It is important to note that Eq. (24) is only valid in the limit $R \rightarrow \infty$. It is true that both members of Eq. (24) tend to zero as $R \rightarrow \infty$. Consequently, Ru_1 and Ru_2 tend to zero if R tends to infinity. This is the only important conclusion that can be drawn from Eq. (24). It cannot be inferred that u_1 and u_2 vanish identically, because the left-hand member, though equal to zero, consists of a sum of positive and negative terms. Whereas in many problems Sommerfeld's condition $\lim_{R \rightarrow \infty} R(\partial u / \partial R - iku) = 0$ is sufficient, and $Ru \rightarrow 0$ superfluous, this does not hold in the presence of an infinite plane earth, as is apparent from Rellich's paper.¹² If the earth is infinite, Sommerfeld's conventional form of the radiation condition does not apply at all, and even Rellich's theorem¹² is not applicable to a plane earth.

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On the Transport of Aluminum Atoms by a Gas

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A SYSTEM which provides for the continuous flow of radioactive gas¹ between a duraluminum bombardment chamber at the cyclotron and a 14-cm radius of curvature magnetic spectrometer² has been in operation in this laboratory for several months. Because of the distance between the cyclotron building and the physics laboratory where the spectrometer is located, it is necessary to circulate the gas between these two buildings through underground copper pipes. The total length of pipe between the bombardment chamber at the cyclotron and the beta-ray spectrometer is 600 feet. An experimentally measured time of 16 seconds is required for the gas to travel through this length of pipe.

One of the most interesting facts discovered to date while using this system is that an activity which can be attributed to Al^{28} can be carried through the system in appreciable quantities. The fact that the activity belongs to aluminum has been verified in a number of ways. The maximum beta-ray end-point energy (2.8 Mev according to our measurements) and the half-life (127 seconds) of this activity as measured at the magnetic spectrometer end of the system agree quite closely with previously reported³ values for Al^{28} . The activity appears to be produced because of the duraluminum construction of the bombardment chamber and the window separating the main vacuum system of the cyclotron from this chamber. The cyclotron beam (10 Mev, 100 μamp . for