$$\sin\delta_{-l-1}=-\tfrac{1}{2}(E+1)^{\frac{1}{2}}\int_0^\infty 2\cdot V(r)\cdot r\cdot\left\{j_l(kr)\cdot g_l(r)-\left(\frac{E-1}{E+1}\right)^{\frac{1}{2}} j_{l-1}(kr)\cdot f_{-l-1}(r)\right\}\cdot dr. \quad (72\text{-b})$$

The zeroth approximation to the wave functions will cause Eqs. (67) to yield a first-order approximation which will agree with the Born approximation in the Dirac theory except that $(e^{2i\delta_l}-1)/2i$ is replaced by $\sin\delta_l$.

A result similar to relation (66) will hold for potentials that go like Ze^2/r at infinity. We simply replaced in (66), the free particle solutions g_l^0 and f_l^0 by the Coulomb solutions and the potential $V(r)$ by the deviation from the pure Coulomb potential Ze^2/r.

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The Evaluation of the Collision Matrix

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Dyson's systematic approach to the reduction of the Heisenberg S-Matrix into a sum of "graph" terms can be simplified. A notation is introduced and an algebraic theorem is proved, which allow one to handle the reduction problem quite easily and in the same manner for any type of field.

I. INTRODUCTION

THE Feynman technique[1] for calculating transition probabilities is now so widely used that a full and satisfactory understanding of it by a wider circle is desirable. An important step in this direction has been Dyson's[2] direct derivation of the method from a simple expression for the S-matrix. One feels, however, that the *pedagogical* value of Dyson's proof is slightly marred by certain omissions and obscurities; moreover, some of the algebraic considerations seem more involved than should be necessary. The purpose of this note is to supply a simple and straightforward proof. The case of the electron-positron field interacting with a quantized electromagnetic field is sufficiently general to allow us to demonstrate all of the features of the method.

Let then $\psi(x)$ be the Dirac field operator at the space-time point x, $\psi^\dagger$ the hermitian-conjugate of ψ, $\bar{\psi}=\psi^\dagger\beta$ the adjoint, and $A_\mu(x)$ the μ-th component of the electromagnetic potential. The S-matrix for the system can be written:

$$S=1+S_1+S_2+\cdots \quad (1)$$

where S_n, the term of the n-th order in the electron charge ϵ is expressed by a multiple integral over a product of field factors ψ, $\bar{\psi}$, A [Eq. (18) below]. Our problem is the reduction of S_n to a sum of terms *à la* Feynman. To this end we notice that our fields ψ and A are linear combinations of creation and destruction operators. For example, $\psi(x)=\sum_r a_r\psi_r(x)$ where the ψ_r's are the normalized representatives of the states of a free Dirac particle, and a_r is a destruction (creation) operator if r is a positive (negative) energy state. Collecting all the positive energy states together into a term $u(x)$ and the negative states into a term $\bar{v}(x)$ we can write:

$$\psi(x)=u(x)+\bar{v}(x), \quad \bar{\psi}(x)=\bar{u}(x)+v(x), \quad (2)$$

where u ($\bar{u}$) destroys (creates) electrons, and v ($\bar{v}$) destroys (creates) positrons.[3] Similarly, we can write

$$A_\mu(x)=a_\mu(x)+a_\mu^\dagger(x), \quad (2')$$

where $a_\mu(a_\mu^\dagger)$ destroys (creates) photons.[4] Substituting such expressions into a product of fields, we can expand each product into a sum of products in which each factor is either a creation or a destruction operator.

Following an idea of Houriet and Kind,[5] we then proceed to rearrange such a product so as to carry all creation operators to the left of all destruction operators, writing for instance: $u(x)\bar{v}(y)=-\bar{v}(y)u(x)$, $u(x)\bar{u}(y)=\{u(x),\bar{u}(y)\}-\bar{u}(y)u(x)$ where the anticommutator $\{u,\bar{u}\}=u\bar{u}+\bar{u}u$ is a c-number. Thus one may transform a product of n creation and destruction operators into the "ordered" product of the same factors, plus extra terms in which some pairs of factors have been replaced by their commutators or anticommutators while the remaining factors are "ordered" in the above sense. The advantage of this is that when we take the matrix element of an ordered product between a final and an

[1] R. P. Feynman, Phys. Rev. **76**, 749 (1949).
[2] F. J. Dyson, Phys. Rev. **75**, 486 (1949).
[3] We set $\bar{u}=u^\dagger\beta$, $v=\bar{v}^\dagger\beta$ or $\bar{v}=\beta v^\dagger$; the asymmetry in the definition of $\bar{u}$ and $\bar{v}$ shall not cause any trouble here.
[4] $a_\mu^\dagger$ is the hermitian conjugate of a_μ (of $-a_\mu$ if $\mu=4$). As regards the treatment of timelike photons, see Section IV.
[5] A. Houriet and A. Kind, Helv. Phys. Acta **22**, 319 (1949).

initial state, all the destruction operators must destroy particles actually present initially, and all creation operators must create particles actually present finally. The virtual creation-followed-by-destruction processes of the customary perturbation theory have been "digested" in the commutators and anticommutators, that appear in the course of the rearrangement. The main problem to be solved in carrying out this idea is one of algebraic technique, i.e., the development of a convenient notation to handle the rearrangement process easily and express the result in a compact way. This is achieved, we think, in the Theorems 1 and 2 below.

II. ALGEBRAIC PRELIMINARIES

Any "simple" factors in a product, like $\psi(x)$, $\bar{\psi}(y)$, $u(x)$, $\bar{u}(y)$, $\cdots$, $A(x)$, $a(y)$, $\bar{a}(z)$ will be denoted also by a capital letter like U, V, W, $\cdots$.

In the case of anticommutative fields (not considered by Houriet and Kind) the rearrangement of a product leads to changes in sign, as in the simple examples considered before. We call S-product, and denote by semicolons on both sides of the product, an "ordered product" symbol defined in such a way that the changes in sign are automatically taken into account. The symbol is then defined by the following rules:

Rule A: Distributive law:

$$:U\cdots V\psi(x)W\cdots Z: = :U\cdots Vu(x)W\cdots Z: + :U\cdots V\bar{v}(x)W\cdots Z:.$$

The S-product can thus be decomposed into a sum of S-products containing creation and destruction operators only.

Rule B: An S-product of the latter type is defined by:

$$:UV\cdots Z: = \delta_P XY\cdots W, \tag{3}$$

where the ordinary product on the right contains the same factors U, V, $\cdots$ ordered in the previously specified manner (creation operators to the left, destruction to the right) and δ_P is the signature, ± 1, of the permutation [between left-hand side and right-hand side of Eq. (3)] *of the electron-positron factors only.*

The δ_P factor takes care of the above-mentioned sign changes. The creation (or destruction) operators may be freely permuted amongst themselves, without affecting the right-hand side of Eq. (3).

Examples of S-products:

$$:\psi(x)\bar{\psi}(y): = :u(x)\bar{u}(y): + :u(x)v(y): + :\bar{v}(x)\bar{u}(y): + :\bar{v}(x)v(y): = -\bar{u}(y)u(x) + u(x)v(y) + \bar{v}(x)\bar{u}(y) + \bar{v}(x)v(y) \tag{4}$$

$$:\bar{\psi}(y)\psi(x): = :\bar{u}(y)u(x): + \cdots = \bar{u}(y)u(x) + v(y)u(x) + \bar{u}(y)\bar{v}(x) - \bar{v}(x)v(y).$$

Clearly

$$:\psi(x)\bar{\psi}(y): + :\bar{\psi}(y)\psi(x): = 0 \tag{5}$$

while the anticommutator:

$\psi(x)\bar{\psi}(y) + \bar{\psi}(y)\psi(x) \equiv \{\psi(x), \bar{\psi}(y)\}$ is in general $\neq 0$.

Note also that:

$$:\psi(x)\psi(y): = \psi(x)\psi(y), \quad :\bar{\psi}(x)\bar{\psi}(y): = \bar{\psi}(x)\bar{\psi}(y).$$

The reader can easily derive the result:

Rule C: Factors within an S-product symbol can be permuted "as if" all anticommutators $\{\psi, \bar{\psi}\}$, $\cdots$ and commutators $[A, A]$ etc., were zero.

One can write the Dirac current as an S-product

$$s_\mu = i\epsilon :\bar{\psi}\gamma_\mu\psi: = i\epsilon \sum_{\alpha\beta} (\gamma_\mu)_{\alpha\beta} :\bar{\psi}_\alpha\psi_\beta:, \tag{6}$$

where ϵ is the electron charge and γ_1, $\cdots$, γ_4 are the Dirac matrices. Writing the current as an S-product is equivalent to the customary prescription for removing the infinite charge of the negative sea.

Another kind of ordering that is useful occurs in the chronological product or T-product, with time running from right to left. This can be used, of course, only if the factors U, V, $\cdots$ of the product are specifically labeled with a time.

Rule D: The T-product is defined by:

$$T(UV\cdots Z) = \delta_P XY\cdots, \tag{7}$$

the factors U, V, $\cdots$ being rearranged in chronological order in the ordinary product on the right. Our T-product differs from Dyson's P-product in having the sign factor δ_P [same meaning as in Eq. (3)]. This simplifies the definition of the contraction symbol, Eq. (8) below.

Example:

$$T(\psi(x)\bar{\psi}(y)) = \psi(x)\bar{\psi}(y) \quad \text{if} \quad x_0 > y_0 \qquad = -\bar{\psi}(y)\psi(x) \quad \text{if} \quad x_0 < y_0. \tag{7'}$$

One has:

$$T(\psi(x)\psi(y)) = \psi(x)\psi(y), \quad \text{etc.}$$

Rule C′: The T-product obeys a rule quite similar to Rule C.

A further complication of the T-product occurs in the expression for the S-matrix, Eq. (18) below. If two or more "simple" factors in the product are labeled with the same time, the T-symbol does not prescribe their order. In the useful case, however, it turns out that either the order is irrelevant (independent fields) or it is prescribed by the S-rule as in (6). A symbol like

$$T(UV:WXY:\cdots Z), \tag{7''}$$

for example, will then mean that W, X, Y (bear the same time-label and) appear, in the form of an S-product, as a single factor in the chronological ordering. This may be called a mixed T-product.

We now turn to the basic problem of transforming the T-product in Eq. (18) into a sum of S-products. To this end we adopt Houriet and Kind's idea of a "contraction" symbol to represent the commutator or anticommutator which arises in switching two factors. The contraction will be denoted by appending a dot

superscript to both factors.[6] Thus, in the simplest case we write:

$$T(UV)=U^{\cdot}V^{\cdot}+:UV: \tag{8}$$

Houriet and Kind's definition, however, handles the reduction of an *ordinary* product into an ordered one. The definition (8) handles directly the reduction of a T-product, which is what we need, and avoids the complicated reassembly process which is otherwise necessary to arrive at the simple Feynman formulas. Contractions between independent fields, such as ψ and A, are zero. But also:

$$\psi^{\cdot}(x)\psi^{\cdot}(y)=\bar{\psi}^{\cdot}(x)\bar{\psi}^{\cdot}(y)=0. \tag{9}$$

On the other hand:

$$\psi^{\cdot}(x)\bar{\psi}^{\cdot}(y)=\{u(x),\,\bar{u}(y)\}=\sum_{r>}\psi_r(x)\bar{\psi}_r(y)\ \text{ if }\ x_0>y_0$$

and (10)

$$\cdots=-\{\bar{v}(x),\,v(y)\}=-\sum_{r<}\psi_r(x)\bar{\psi}_r(y)\ \text{ if }\ x_0<y_0,$$

where $r>$ (or $r<$) means that the sum extends over the positive (or negative) energy states only. The right-hand side of Eq. (10) is clearly the "mystical" kernel function which is variously designated with K_+ by Feynman,[7] $\frac{1}{2}S_f$ by Dyson,[2] etc.

We shall omit the $+$ in K_+ and write instead the spinor indices explicitly. Here are the main formulas:

$$\psi_{\alpha}^{\cdot}(x)\bar{\psi}_{\beta}^{\cdot}(y)=K_{\alpha\beta}(x-y), \tag{11a}$$

$$\bar{\psi}_{\alpha}^{\cdot}(x)\psi_{\beta}^{\cdot}(y)=-\psi_{\beta}^{\cdot}(y)\bar{\psi}_{\alpha}^{\cdot}(x)=-K_{\beta\alpha}(y-x), \tag{11b}$$

$$A_{\mu}^{\cdot}(x)A_{\nu}^{\cdot}(y)=\tfrac{1}{2}\delta_{\mu\nu}D_F(x-y), \tag{11c}$$

where D_F is defined, for example, by Dyson.[8]

One last convention about symbols. We want to give a meaning to an S-product with one or more contractions. In order to distinguish different contractions in a product, we shall use different superscripts, such as double dots, triple dots, etc. If a contraction is marked between two adjacent factors, it has the value given by (11) and may be regarded as a c-number; the remaining factors form the S-product proper. Contractions between non-adjacent factors are defined by:

Rule C″: Factors within an S-product with one or more contractions (including the contracted factors) obey a rule[9] similar to C. For example, if $U, V, \cdots$ etc. are all anticommutative fields, one has:

$$:U^{\cdot}VW^{\cdot\cdot}X^{\cdot\cdot}Y^{\cdot}Z:=-(U^{\cdot}Y^{\cdot})(W^{\cdot\cdot}X^{\cdot\cdot}):VZ: \tag{12}$$

$$:TU^{\cdot}VW^{\cdot\cdot}XY^{\cdot}Z^{\cdot\cdot}:=+(U^{\cdot}Y^{\cdot})(W^{\cdot\cdot}Z^{\cdot\cdot}):TVX:. \tag{13}$$

[6] Houriet and Kind's symbol is a line connecting the two factors like a string attached at both ends. It is very convenient for handwriting, but has been abandoned here for typographical reasons.

[7] See reference 1, Eq. (17).

[8] See reference 2, Eq. (42).

[9] The self-consistency of this rule is insured by Eq. (11b). Verify also that any two arrangements leaving the contracted fields together will give the same value.

And now let us prove the basic theorem. First let us show that if $:UV\cdots XY:$ is an S-product, and Z a factor *labeled with a time which is earlier than any of the times for* $U, V, \cdots, Y$ then

$$:UV\cdots XY:Z=:UV\cdots XY^{\cdot}Z^{\cdot}:+:UV\cdots X^{\cdot}YZ^{\cdot}:+\cdots+:U^{\cdot}V\cdots XYZ^{\cdot}:+:UV\cdots XYZ:. \tag{14}$$

It is clearly sufficient to prove Eq. (14) under the assumption that each simple factor is either a creation or a destruction operator, and furthermore that Z is a creation operator (if Z is a destruction operator the equation is trivial). Owing to the C-rules, the validity of Eq. (14) is invariant against a permutation of $UV\cdots XY$, and we may choose the order in such a way that $U, V, \cdots, X, Y$, are already an S-product *as they stand.* Let us then prove (14) on the assumption that $U, V, \cdots X, Y$ are all destruction operators; any number of creation operators may then be added on the left. The proof follows by induction. First (14) is true if there is only one factor:

$$YZ=T(YZ)=Y^{\cdot}Z^{\cdot}+:YZ:.$$

Then assume (14) is true for n factors, and multiply by another destruction operator T on the left:

$$T:UV\cdots XY:Z=T:UV\cdots XY^{\cdot}Z^{\cdot}:+\cdots+T:UV\cdots XYZ:.$$

Since $U, V\cdots Y$ are destruction operators, the colon between T and the rest can be moved to the left of T everywhere except in the last term. Now $:UV\cdots XYZ:=\delta_P ZUV\cdots XY$, δ_P being defined as usual. Since again (T being assumed "later" than Z)

$$TZ=T(TZ)=T^{\cdot}Z^{\cdot}+:TZ:=T^{\cdot}Z^{\cdot}+\delta_Q ZT$$

(Q being the permutation of electron positron operators in passing from TZ to ZT) one finds easily for the last term:

$$\begin{aligned}T:UV\cdots XYZ:&=\delta_P TZ(UV\cdots Y)\\&=\delta_P(T^{\cdot}Z^{\cdot}UV\cdots Y+\delta_Q ZTU\cdots Y)\\&=:T^{\cdot}UV\cdots YZ^{\cdot}:+\delta_P\delta_Q ZTU\cdots Y.\end{aligned}$$

Now $\delta_P\delta_Q=\delta_R$ where R belongs to the rearrangement from $ZTU\cdots Y$ to $TU\cdots YZ$, so that the last term is $=:TU\cdots YZ:$. This completes the proof for $(n+1)$ factors.

We may further generalize Eq. (14). If we mark a contraction between U, V *in all terms*, the equation will still be valid, since $U^{\cdot}V^{\cdot}$ will be merely a factor added on the left. Similarly one can add several such factors. Then, thanks to the C-rules, one can rearrange the order, and Eq. (14) is seen to hold even when any number of contractions (the same in every term!) is marked within $UV\cdots XY$.

Theorem 1. A T-product can be transformed into a sum of S-products as follows:

$$T(UV\cdots XYZ)=:UV\cdots XYZ:+:U^{\cdot}V^{\cdot}W\cdots WYZ:+:U^{\cdot}VW^{\cdot}\cdots XYZ:+\cdots+\cdots+:U^{\cdot}V^{\cdot\cdot}W^{\cdot\cdot}\cdots X^{\cdots}Y^{\cdot}Z^{\cdots}: \tag{15}$$

where the sum on the right includes all possible sets of contractions one can indicate. Of course, one can omit terms with contractions which are identically zero, such as $\psi^{\cdot}\psi^{\cdot}$, $\psi^{\cdot}A^{\cdot}$, etc.

Equation (8) is, of course, the simplest case of (15). The general case follows easily by induction. Suppose (15) is proved for n factors. Multiply on the right by an Ω belonging to an earlier time than any other factor:

$$T(UV\cdots Z)\Omega = :UV\cdots XYZ:\Omega + :U^{\cdot}V^{\cdot}W\cdots XYZ:\Omega + \cdots$$

they apply Eq. (14) to each term on the right and notice that

$$T(U\cdots Z)\Omega = T(UV\cdots Z\Omega).$$

The theorem is then seen to hold for $n+1$ factors. The restriction on the time label for Ω can be removed by rearranging the order of factors, using the C-rules.

Theorem 2. A mixed T-product, such as (7″) can be decomposed in a manner similar to Eq. (15) but omitting contractions between factors already S-ordered [for example contractions $W^{\cdot}X^{\cdot}$, $W^{\cdot}XY^{\cdot}$, $X^{\cdot}Y^{\cdot}$ in (7″)].

The reader can construct the general proof, after we have examined the case (7″) as an example. We may as usual assume that each factor W, X, Y is either a destruction or a creation operator. We then consider (7″) as the limit of:

$$T(UVWXY\cdots Z),$$

where the time label of the creation operators amongst W, X, Y is assumed later (by an infinitesimal amount) than that of the destruction operators. Equation (15) may then be applied. Furthermore the contractions to be omitted according to the statement of Theorem 2 are actually zero, so that the theorem follows.

III. PHYSICAL APPLICATIONS

The somewhat tedious details we have developed for the sake of clarity will not, we hope, obscure the fact that the whole method is quite simple, indeed somewhat trivial. Nevertheless it allows one to operate with considerable freedom on rather complex expressions.

Let us use for convenience the abbreviations:

$$\mathbf{A} = -i\gamma_\mu A_\mu, \quad \text{that is} \quad A_{\alpha\beta} = -i(\gamma_\mu)_{\alpha\beta}A_\mu \tag{16}$$

and

$$\bar{\psi}\mathbf{A}\psi(x) \quad \text{for} \quad \sum_{\alpha\beta}\bar{\psi}_\alpha(x)A_{\alpha\beta}(x)\psi_\beta(x). \tag{17}$$

As Dyson has shown, the n-th order term S_n in Eq. (1) can be written as a multiple integral over n points $x, y, \cdots, z$ in space time[10]

$$S_n = [(-i\epsilon)^n/n!]\int T(:\bar{\psi}\mathbf{A}\psi(x)::\bar{\psi}\mathbf{A}\psi(y):\cdots:\bar{\psi}\mathbf{A}\psi(z):) \times (dx)(dy)\cdots(dz), \tag{18}$$

[10] We use units $\hbar=1$, $c=1$.

where $(dx)=dx_1dx_2dx_3dx_0$, etc.$\cdots$. This integral involves a mixed T-product which can be reduced in the desired manner by means of Theorem 2. The rest of the analysis then proceeds very much in the same manner as in Section 7 of Dyson's paper,[2] Dyson's "associated pairs" of mutually annihilating factors being replaced here by the modified Houriet and Kind "contractions" that we have defined. There are, however, several simplifications. Thanks to the C-rules we can handle the S-products of Eq. (15) with great ease, so that nothing is to be gained now by reducing the procedure to a set of mechanical recipes. Here are, then, not recipes but a few suggestions as to how best to conduct the evaluation in any given case.

Suppose then, that (18) is decomposed according to Theorems 1 and 2 into a sum of terms, each term being an integral of an S-product with a number of contractions. For example, a term in S_3 will be:

$$[(-i\epsilon)^3/3!]\int :\bar{\psi}^{\cdot\cdot}\mathbf{A}\psi^{\cdot}(x)\bar{\psi}^{\cdot}\mathbf{A}^{\cdot\cdot}\psi(y)\bar{\psi}\mathbf{A}^{\cdot\cdot}\psi^{\cdot\cdot\cdot}(z): \times (dx)(dy)(dz). \tag{19}$$

If one applies a permutation to the $\bar{\psi}\mathbf{A}\psi$ groups (allowing the contractions to follow their factors according to rule C'') then one changes the names of the variables x, y, z, to restore their former order in the product, one will get in general another possible term in the development of S_3. The number of such "equivalent" terms is in the case of (19) precisely 3!, so that one can allow for them simply by omitting the factorial on the denominator. In the general case, however, the $n!$ permutations of the $\bar{\psi}\mathbf{A}\psi$ groups will not lead to $n!$ distinct terms[11] since a given set of possible contractions may be left completely unchanged by a subgroup of g permutations (automorphisms of the set), g being equal for two "equivalent" sets.[12] In this case, therefore, the number of distinct equivalent terms of a given kind will be $n!/g$, and their total contribution to S_n will be:

$$[(-i\epsilon)^n/g]\int S\text{-product}\cdot(dx)\cdots(dz), \tag{20}$$

the S-product being any one of the equivalent S-products, with contractions.

A "graph" is now simply a concise way of writing an S-product of the kind which interests us. In fact, once the points $x, y, \cdots, z$ are marked it is implicitly understood that there is a factor $:\bar{\psi}\mathbf{A}\psi:$ for each point. If there is a contraction between $A(x)$ and $A(y)$, say, draw

[11] Compare the similar discussion in reference 5, where, however, the expression "equivalent" has a slightly different meaning. The argument is reproduced here with appropriate modifications for the convenience of the reader.

[12] If P is an automorphism of the set s (we can write: $Ps=s$) and T transforms s into the equivalent set s' ($s'=Ts$) then: $s'=Ts=TPs=TPT^{-1}s'$, hence TPT^{-1} is an automorphism of s'. This establishes a one-to-one correspondence of a well-known type between the automorphisms of the two sets.

a dashed line between x and y and omit the A symbols. If there is a contraction between $\bar{\psi}(x)$ and $\psi(y)$, draw a directed full line from x to y and omit the ψ-symbols. The free factors ψ, $\bar{\psi}$, A, can be similarly disposed of by means of lines with a loose end, namely directed lines "into" the diagram (factor ψ), "from" the diagram (factor $\bar{\psi}$) or undirected dashed lines (factor A). For example (19) will be represented by the graph in which a directed line runs inwards to y, then from y to x, then from x to z, and thence outwards; a dashed line connects y and z.

At this stage, one will raise the question of *signs*. There are, indeed, signs involved in the values of the contractions, Eqs. (11a) and (11b), and in the δ_P-factors in Eq. (3), etc. One can, however, without memorizing any recipes, reduce such complications to a minimum, if one writes the $\bar{\psi}A\psi$-groups in a suitable order (this does not affect the over-all result). If an open polygon runs from point 1 to 2, to 3, $\cdots$, to m, one will write these points in the same order in the S-product, thus:

$$:\bar{\psi}\mathbf{A}\psi^{\cdots}(m)\cdots\cdots\cdots\cdots\bar{\psi}^{\cdots}\mathbf{A}\psi^{\cdot\cdot}(3)\bar{\psi}^{\cdot\cdot}\mathbf{A}\psi^{\cdot}(2)\bar{\psi}^{\cdot}\mathbf{A}\psi(1):.$$

All the contractions will then be between adjoining factors in the order of Eq. (11a) with no signs involved. If the points 1, $\cdots$, m, are in a closed loop, there will be a further contraction between $\bar{\psi}(m)$ and $\psi(1)$. One can carry over $\bar{\psi}(m)$ to the right of $\psi(1)$ by jumping an odd number of ψ or $\bar{\psi}$-factors. This will give a further factor: $-K(1, m)$ with one minus sign. In the end result there is a factor (-1) for each loop.[13] As regards the free factors ψ, $\bar{\psi}$ there will be, in the interesting cases, two, or at most, four of them. All questions of sign will be easily handled (after one has decided whether one wants to use the u or the $\bar{v}$ part of a ψ, the $\bar{u}$ or the v part of a $\bar{\psi}$) simply by using the easily remembered C-rules.

[13] This accounts for the $(-)^l$ factor in Dyson's formula (51) (reference 2). The $(-1)^n$ arises from a difference in sign between Dyson's S and our K kernel.

IV. LONGITUDINAL WAVES

In electrodynamics, longitudinal waves always require special considerations. In our case we must specify first that the destruction part a_μ is meant in the sense: part with an $e^{-i\omega t}$ $(\omega>0)$ dependence. As applied to a_4, this definition really implies *creation* of time-like photons; since these photons have negative energies, the term destruction is, nevertheless, quite proper. This is, at any rate, the convention leading to Eq. (11c).

The next and more important point is the following one. In Dyson's treatment, there is an ambiguity in the vacuum expectation value of $A_\mu(x)A_\nu(y)$ and one must show that this ambiguity is irrelevant. This has been done recently by Dyson.[14]

In our case, there is no ambiguity in the value of the contraction of $A_\mu(x)A_\nu(y)$, but the problem arises, of course, in another way. Consider, for example, a transition described by a graph involving a certain contraction between A_μ and A_ν. At first sight one can get a contribution to the *same* transition also from another term in the S-matrix, containing instead the S-product of A_μ and A_ν. This would not normally be the case but the complication arises out of the presence of an undetermined number of longitudinal and timelike photons both in the initial and final state. One has then to show that these extra terms are really zero. We have just learned, however, that this question is treated in detail in a forthcoming paper by Coester and Jauch[15] to which the reader may be referred.

[14] F. Dyson, Phys. Rev. **77**, 421 (1950).

[15] This paper uses Houriet and Kind's method which is closely related to ours. I am indebted to Dr. K. M. Watson for showing me the manuscript in his possession. This paper has meanwhile appeared, Phys. Rev., **78**, 149 (1950).